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EARTH PRESSURE ON HORIZONTAL CIRCULAR CONDUITS

Department of Public Works AMSTERDAM
Section of Sewerage Works.

SUMMARY.

In consequence of the remarkable appearances of collapsed underground conduits in the forms of sewers, drains, culverts and inverted siphons a careful investigation resulted in the distinction of various types, in the first place mainly based on the character of the embankment and in the second place on the support system of the conduits.

Subterranean pipes may be distinguished into

1) Pipes in embankments (projecting conduits acc. to Marston).
a) Pipes directly embedded in the firm ground without further support (fig. 1a).
b) Pipes upon longitudinal supports. When a pipe is only sustained on a describent line of the pipe surface a "knife-edge support" is formed. This is exclusively of theoretical value. In practical engineering the pipe rests upon a broad longitudinal strip of the pipe surface thus forming a so-called saddle support Meanwhile, the support is mostly easily formed with beams as shown in fig. 1b, the pipe being sustained on a small strip of the beam.
c) Pipes on pile supports constructed at regular distances symmetrically under the pipe bottom line. Essentially we distinguish the same supporting cases as stated under b).
The types a) and b), having the same load for any cross-section of the pipe, are called biaxial or plane-loaded I) pipe structures. Type c) may consequently be called a tri-axial or space pipe structure.

2) Pipes in trenches (ditch conduits).
Exactly the same distinction of types a), b) and c) may here be made, as shown in fig. 1a^1, b^1 and c^1.

A clear and systematical classification has been set up in fig. 1; obviously type b" may be neglected as it is practically never used in pipe engineering, a" and c" being normal installation systems for inverted siphons.

The classification and the subsequent discussion II) are made up more particularly in relation to the soil condition of the low marshy North-Western provinces of the Netherlands and only type a''' makes an exception. In this case the pipe laying in a solid cohesive soil, needs no specially sheeted trench.

It is very peculiar that the earth loading scheme of the conduit depends highly on the conditions of the surrounding earth, the depth and width proportions of the trench and the character of the pipe supports, in other words depending mainly on the method of construction.

In the following the different types will be subjected to a further examination. In our discussions we did not neglect the cohesion, though this being a rather unreliable factor. Further we assume a transfer of the earth pressure on the pipe shell by means of the earth enclosed by the horizontal and perpendicular planes h and v (fig. 2) and the outer pipe surface. This supposition does not lead to serious errors, the earth remaining in static condition as a result of the symmetrical loading, and shearing stresses are almost excluded.

Type a.

The load system of fig. 2 may be used for small heights of the embankment i.e. up to about once the pipe diameter. An assumption of active (k_a) or passive (k_p) earth pressure upon the plane v is not according to reality. Quite recently shearing stresses were supposed to diminish in the long run, causing the horizontal intergranular stresses to increase to the value of the vertical stresses till the condition of the natural hydrostatic earth pressure has been attained. (Natural earth pressure).

However, we may sooner expect the neutral earth pressure. Meanwhile, it is seriously recommended to handle the k_a modulus, inducing larger values of earth pressure and increasing the cross-sectional bending moment values (ovalisation moments).

On the horizontal plane through the top of the pipe the weight of the backfill is acting, augmented with occasional traffic loads, which should be thoroughly introduced as spread loads p on the pipe wall.

Hence $$q_v = \sum_{\text{earth layers}} \gamma\, h + p \quad t/m^2 \qquad (1)$$

and $$q_h = k_a \cdot q_v \quad t/m^2 \qquad (2)$$

By dealing with nonhomogeneous earth masses the earth layers should be reduced to a height H for homogeneous soil with the well-known formula

$$H = h + \frac{p'}{\gamma} \qquad (3)$$

where p' = the uniformly distributed top load on the considered layer.

The upward pressure of the ground water causing a decrease of vertical and horizontal grain stresses should further be taken into account.

In the case of considerable fill-heights (up to 3 till 6 times the pipe diameter) the neatral earth pressure is introduced for horizontal pressure; and assuming the soil having the same qualities as an elastic material q_h takes over the value

$$q_h = \frac{1}{m - 1} \cdot q_v$$

where m = the reciprocal of Poisson's ratio of contraction.

As much a modulus is not acceptable for soils Terzaghi III) introduced the expression

$$q_h = k_n \cdot q_v \qquad (4)$$

in which k_n is known as the neutral earth pressure ratio and is an exclusively experimental value which should only be determined from field specimens at a laboratory of soil mechanics. According to Terzaghi the following values of k_n may be used for approximate computations

for dense sand	0.40 - 0.50
loose sand	0.45 - 0.50
clay	0.60 - 0.75.

According to Keverling Buisman IV), however, the neutral pressure in cohesive soils

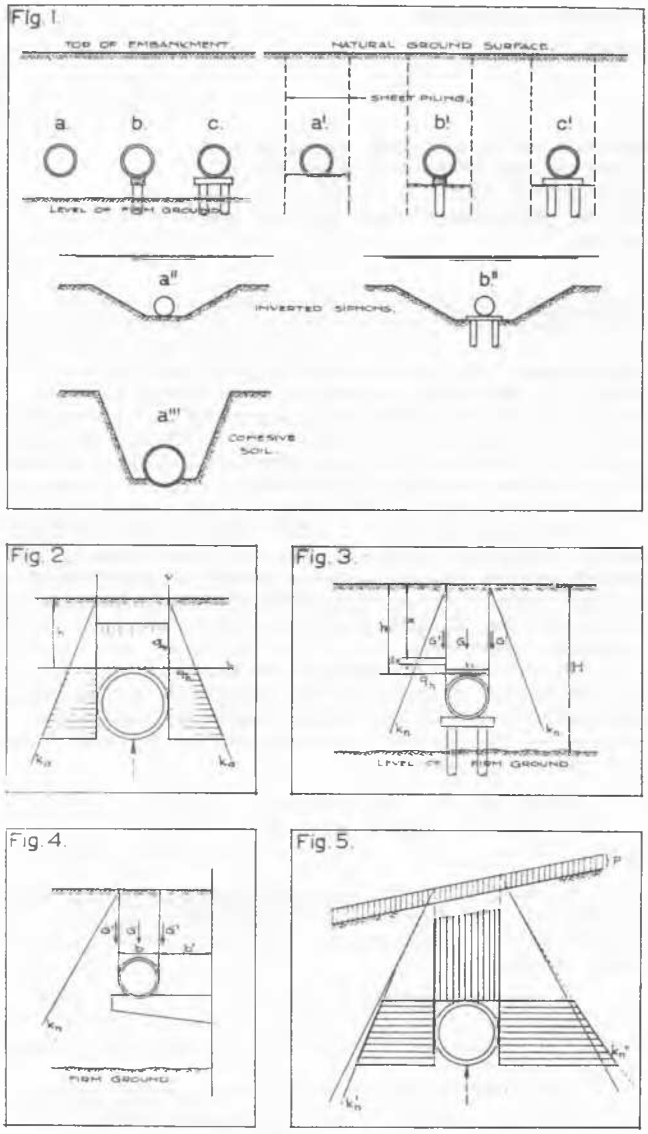

FIG. 1 2 3 4 5

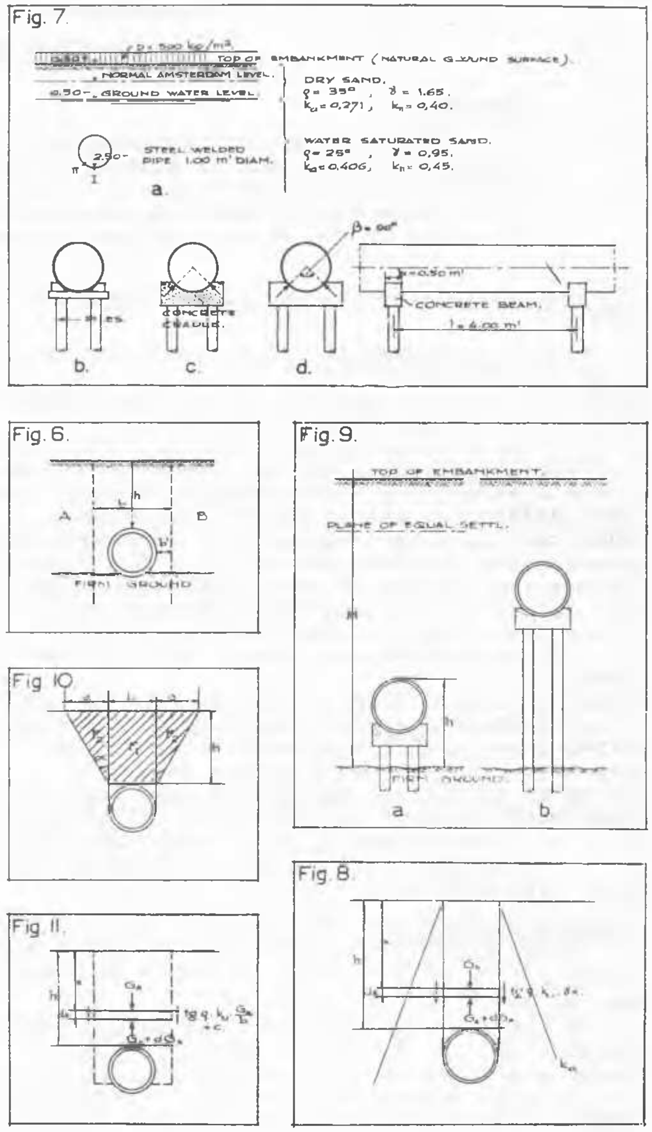

FIG. 6 7 8 9 10 11

approaches the active pressure, so that for clay we may reasonably pass over to $q_h = k_a . q_v$, this assumption giving less favourable loading schemes for the conduits.

Types b and c (fig. 3).

These are exceptionally dangerous systems as it is very difficult to determine the top load. The earth is inclined to settle slowly all over the height H, while the pile supports will almost remain unmoved. Then the topload will be formed by G + 2 G', where G' is the resultant of the frictional stresses in the virtual planes v produced by the settling earth of the outer embankments.

Hence $\quad dG' = k_n . \gamma x \; dx . tg \rho$

where ρ = angle of repose of soil

$$G' = \tfrac{1}{2} k_n \gamma h^2 tg\, \rho \;\; t/m' \text{ pipelength.} \qquad (5)$$

The top load will be

$$q_v = \frac{G + 2G'}{b} \qquad (6)$$

and the lateral horizontal pressure as in formula (4). V)

Figure 4 represents the case of a conduit resting on a beam being a part of a concrete wall. The earth between wall and conduit remains in suspension; G'' and the lateral pressure from the right on the conduit are formed by this "arching or bin effect" or "silo action" as termed in Western European nomenclature, and more closely considered in a following paragraph of this article. The loading of the conduit structure is obviously nonsymmetrical and the conduit is inclined to move towards the concrete wall.

Of course the values of k_n are given for horizontal levels of the embankment surface. In dealing with moderately sloped embankments the lateral pressure on the vertical planes v_1 and v_2 in fig. 5 will be according to the two dotted curves indicated by k_n' and k_n''. Our investigations did not go as far as the determination of these two moduli which we should prefer to call the coefficients of minor and major neutral earth pressure, and we are quite convinced that further investigation in this direction would be very useful.

Type a'.

We will now consider the conduits laid in trenches especially sheet piled trenches which mostly occur in deep-laid pipe engineering (fig. 6). Here, too, the "bin effect" will act after the trench have been refilled and the sheetpiles pulled out again. Between the two more or less solid masses A and B the fill material inclines to settle downwards.

The well-known formula for homogeneous soil VI)

$$q_h = \frac{\gamma F}{fO}\left(1 - e^{-\frac{fOk_a h}{F}}\right)$$

after introducing F = bl, O = 2l and f = tgρ, becomes

$$q_h = \frac{\gamma b}{2\,tg\rho}\left(1 - e^{-\frac{2 tg\rho k_a h}{b}}\right) \qquad (7a)$$

and

$$q_v = \frac{q\,h}{k_a} \qquad (7b)$$

The practical limit of height, at which the bin effect becomes effective, follows from the position of the plane of rupture, running from one trench edge down to the opposite trench wall and will be

$$h = \frac{b}{tg(45^\circ - \varphi)} = \frac{b}{\sqrt{k_a}} \qquad (8)$$

It is urgently desired to refill very thoroughly the trench up to the top of the conduit, if possible with sand dumped in water, or otherwise by tamping the fill material in order to require an increase of the horizontal pressure on the sides of the conduit. The value of this pressure is approximately to obtain from

$$q_h' = \frac{\gamma b'}{2\ tg} \qquad (9)$$

with reference to fig. 6.

Types b' and c'.

Whenever the conduit of fig. 6 is not to be installed on a firm bed on the natural ground, and supports are wanted on regular distances, as in case c described, there is actually no difference with the types of the preceeding paragraph. However, for practical problems it is strongly recommended not to take into account the lateral pressure q_h' of formula (9).

It is quite clear that the earth consolidates slowly and after some time the bin effect will vanish, meanwhile at the same time the vertical top pressure will be growing in reverse order of succession, until it has reached its utmost limit i.e. the full weight of the earth layers above top-level of the conduit. However, the lateral earth pressure will also increase considerably up to the neutral earth pressure due to the settlement of the fill material. In this case the conduit shell computed at less favourable bending moments will easily stand the new load condition.

With regards to the preceeding discussions we will now describe the influence of the separate types of structures on the computation of the conduits. We have computed the types a, b, c, b' and c' in fig. 1 for the conduit of fig. 7a. The following figures VII) were obtained:

Type a	M_I	= 350kgm,	with a sectional thrust N =	800 kg
" b	M_I	= 650 " ,	"	1100 "
" c	M_I	=1200 " ,	"	2300 "
" b'	M_{II}	= 300 " ,	"	1900 "
" c'	M_{II}	= 470 " ,	"	3000 "

The types b' and c' are supposed to have concrete saddle-supports as shown in fig 7c and 7d, the favourable qualities of these supports being proved from the above figures.

We may conclude that the manner of support has great influence upon the loading scheme of the conduit, and the same may be said of the method of installation. Conduits on pile supports will always have greater thickness of the shell. Also stresses in space-loaded conduits form a quite urgent problem, which should be carefully and extensively studied and should be solved theoretically and practically from laboratory models for pipes of different diameter, supporting distances, and of differ. ent material.

REFERENCES.

I. See: De Waterstaatingenieur of July 1931 p. 247: Prof.Ir. C.G.J. Vreedenburgh. Berekening der spanningen in den wand van vlak belaste buisleidingen etc.

II. Our discussions form the foundation of the recent Regulations for computation of conduits, in use at the Section of Sewerage Works, Department of Public Works Amsterdam, and the derived formulas are mostly from its engineering staff.

III. Charles Terzaghi: Principles of Soil Mechanics, Eng. News Record, vol. 95, 1925.

IV. Prof.Ir. A. Keverling Buisman , Grondmechanica, 2e druk, 1943, p. 199.

V. Thus so far our theory; we reject Marston's view on this matter as stated in Proceedings of American Society of Civil Engineers of June 1947, article of M.G. Spangler M.A.S.C.E. : Underground conduits. The differential equation, derived by Marston, has the form (see fig. 8):

$$G_x + dG_x = G_x + \gamma b dx + 2tg\rho . k_a . \frac{G_x}{b} . dx$$

As the compression forces act from the exterior masses towards the inner prism the 3rd term of the right part should be written more accurately: 2 $tg\rho . k_n . \gamma x dx$. Instead of $\frac{G_x}{b}$ we substitute the vertical earth stress in the exterior massif x, and for k_a the neutral pressure ratio is introduced.

Hence $dG_x = \gamma b\ dx + 2\ tg\rho . k_n . \gamma x . dx.$

leading to the pressure on the horizontal plane through the top of the conduit.

$$G_x = \gamma bh + tg\rho . k_n . \gamma h^2 \qquad (10)$$

being essentially similar to formula (6). We will also remark that no theory of stresses exists without consideration of the deformations of the structure and the surroundings connected to it. In the above-mentioned article of Spangler quite a part has been dedicated to the settlements of the surrounding earth in regard to that of the inner prism above the top of the pipe, the displacements of the supports and the deflection of the pipe. However, we do not degree fully with the acceptance of the "plane of equal settlement". Besides the fact that the used formulas are rather disputable - f.i. Hooke's law's application on soil is not usual on the European continent - the plane of equal settlement will only be actual when the proportions between depth of conduit and depth of firm ground level below top of embankment remain within reasonable values.
For common pipe systems type b of fig. 9 is more frequent and it is clear that in this case the plane of equal settlement does not act below the top of embankment but quite

high above. As for tunnelling purposes (pressure tunnels for hydro-electric plants) circular tunnel pipes constructed deep under the surface Marston's theory is not applicable. We may then conclude that for ordinary pipe engineering the deformations need not to be considered.
The formulas (6) and (10) may be made more convenient for practical pipe computaion as follows:

$$G = \gamma b h + 2\gamma\, k_n\, tg\rho \cdot \frac{h^2}{2} =$$

$$= \gamma\left(b h + 2\, k_n\, tg\rho \cdot \frac{h^2}{2}\right) =$$

$$= \gamma\left\{b h + 2 \cdot \tfrac{1}{2}(h \cdot k_n \cdot tg\rho)\, h\right\}$$

The bracketed product $h.k_n.tg\rho$ strikes us for having a length as dimension (1) and may be written in the form

$$h.tg\,\alpha = a,$$

the meaning of which clearly shown in fig. 10.

Hence $G = \gamma(hb + 2.\tfrac{1}{2}.ah)$
$G = \gamma(F_1 + 2F_2)$ (11)

in other words: the vertical earth pressure on the pipe is formed by the shaded area in fig. 10. Tracing the limits of the angle α we consider that
k_n varies from 0.4 till 0.70.
ρ " " 25° " 40°.
hence $tg\rho$ will vary from 0.186 till 0.588
and α = about 10° till 30°. (12)
In practice we can use the following values:

Sand saturated with water (dense sand)	$\alpha=13^\circ$
Wet or loose sand	$\alpha=18^\circ$
Waterlogged clay	$\alpha=18^\circ$
Wet clay	$\alpha=30^\circ$

VI. See lit. note IV page 199. The here developed theory proves that (7a) is only of approximate value.
Where the cohesion is not involved in formula (7) we will derive the following equation with reference to fig. 11:

$$G_x + dG_x = G_x + \gamma\, b dx - 2\, tg\rho.k_a.\frac{G_x}{b}dx - 2cdx$$

wherein c = ultimate cohesive resistance.
The solution of this differential equation is:

$$G = \frac{b(\gamma b - 2c)}{2\, k_a\, tg\rho}\left(1 - e^{-\frac{2\, tg\rho . k_a . h}{b}}\right) \quad (13)$$

expressing nothing more than the approximate form of (7a) including the cohesion. For practical problems, however, we strongly suggest not to count on the cohesion.

VII. As stated above case b is not practically imaginable with a knife-edge bearing, and in the case of c the cross-pieces will always be provided with slight recesses and two fixed wood supports as shown in fig.7b. The computed figures for types b and c are therefore somewhat exaggerated. For the types c and c' must also be reckoned on additional longitudinal bending stresses.

APPENDIX WITH ADOPTED LETTER SYMBOLS IN THE TEXT.

k_a = modulus of active earth pressure.
k_n = " " neutral " "
q_v = assumed uniformly distributed load on top level of conduit.
q_h = load coefficient on vertical side planes of conduit.
γ = specific weight of earth.
c = ultimate cohesive resistance of the earth.
e = base of natural logarithms.
ρ = angle of internal friction of soil, assumed to be aequivalent with angle of repose.
b = for projecting conduits equal to horizontal breadth of conduit,
and for ditch conduits equal to width of trench.
f, F, 0 and 1 figures used by Keverling Buisman in his theory of bin effect.

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