$\rho_m = 74.0$ cm

$\rho_b = 45.5$ cm

$\frac{\rho_m}{\rho_b} = 1.65$

The differential settlement is 28.6 cm and by the analysis of the settlement curves,(first part of the paper) we saw that:

$\rho_m = 74.2$ cm

$\rho_b = 47$ cm

$\frac{\rho_m}{\rho_b} = 1.58$

The differential settlement is 27 cm.

CONCLUSIONS

Kögler's method needs an adaptation to take conveniently care of the distribution of pressures in the soil.

The modulus of elasticity of the slab's concrete due to the plastic flow, which is a characteristic of this loading type, is noticeably smaller than that fixed by the mahority of codes for superstructures in general.

Adopting for the concrete a modulus of elasticity of about 10^5 kg/cm^2 it is possible satisfactorily to interprete the results of the measures of settlements of tank O.C.B. 9, using the constants of consolidation, in particular the coefficient of volume decrease m_{vs}, determined by the tests performed on the soil samples.

SUMMARY.

The settlement curves of the concrete slab foundation of an oil tank, 100 ft in diameter and 30 ft height, are known.

The analysis of these curves and of the results of the tests performed on samples of soil, which was well studied from the geothechnical point of view, according to documentation presented, loads us to the conclusion that such settlements agree - in general with those which would be expected if the modulus of elasticity of concrete was about 10^6 t/m^2.

This value agrees with the phenomenon of plastic flow of the concrete.

A computation method of slab foundations based on the modified Kögler's method which agrees satisfactorily with the results observed, is presented.

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DISCUSSION OF ASSUMPTIONS PERTAINING TO STRESS ANALYSES FOR SETTLEMENT COMPUTATION PURPOSES

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In computing building settlements due to the consolidation of clay-like subsoils it is generally assumed that the magnitude of the settlement is a function of the normal stress acting upon horizontal planes. To obtain an estimate of these normal stresses due to building loads the following assumptions are usually made:

a. The soil mass is assumed to have the properties of a homogeneous and isotropic mass that deforms in accordance with Hooke's Law.
b. The soil mass is assumed to be of seme-infinite extent and to be loaded by forced that act normally to its plane surface.
c. The structure producing the load is assumed to be relatively flexible in comparison with the soil that provides its support.

These assumptions are made to simplify the stress analysis and/or to make applicable to the stress analysis problem mathematical solutions that have been developed from the theory of elasticity.

In this paper it is the writer's purpose to examine the foregoing assumptions, to suggest some refinements in the procedure for making a stress analysis and to indicate the consequences of not employing these refinements in procedure.

I. DISCUSSION OF ASSUMPTIONS

1. Vertical Normal Stress vs. Major Principal Stresses and Bulk Stress.

As stated, in computing the magnitudes of settlement resulting from the consolidation of clay-like soils, the settlement is assumed to be a function of the vertical normal stress acting upon horizontal planes. The settlement associated with the consolidation of soil is dependent on the excess hydrostatic pressure that is created in the soil mass when it is loaded. It seems likely that the major principal stress or, perhaps, the sum of the principal stresses (bulk stress) might be a better measure of the excess hydrostatic pressure than the vertical normal stress.

The vertical normal stress acting upon horizontal planes,is, in fact, the major principal stress at all locations immediately below the point of application of the resultant of a system of normal forces. At all other locations the major principal stress acts upon planes that are inclined with respect to the horizontal. Since, of all of the normal stresses that act on the various planes passing through a given point, the magnitude of the major principal stress is the greatest, the magnitude of the vertical normal stress will be smaller than the corresponding major principal stress at all locations except those immediately below the point of application of the resultant of the loads. Accordingly,settlements computed on the basis of vertical normal stresses will be smaller than those computed from major, principal stresses at all points except at the location of the resultant of the normal forces that are applied to the surface. At this point the settlement computed on the basis of the major principal stress. The settlement profile of a uniformly loaded surface as computed from vertical normal stresses in "disshaped". Therefore, since the settlement at the center of the area as computed from major principal stresses will equal that as computed from vertical normal stresses, while the settlement at the edges as computed form major principal stresses will be greater than that as computed from vertical normal stresses, the <u>differential</u> settlement between the center and edges of the loaded area will be smaller as computed on the basis of major principal stresses than that as computed using the vertical normal stresses as a measure of the hydrostatic excess pressures.

The ratio of the magnitude of the bulk stress at a given depth below the periphery of a uniformly loaded area with respect to the bulk stress at the same depth below the center of the area is larger than the corresponding ratio of the vertical normal stresses at these locations. 1). Therefore, the differential settlement computed on the basis of bulk stresses will also be smaller than that computed on the basis of vertical normal stresses.

Settlement studies indicate that differential settlements, which are computed on the assumption that the settlement is a function of the vertical normal stress, are larger than the actual observed differential settlements. Since the differential settlement as computed from major principal stresses or as computed from bulk stresses would both yield smaller differential settlements than those computed from vertical normal stresses, it would appear desirable to investigate whether or not either of these stresses are the significant stresses in the consolidation process. Tri-axial apparatus equipped with a device to measure pore water pressures would be well suited for such an investigation.

2. Idealized Massed vs. Real Soil Mass

To obtain an estimate of the state of stress within a clay-like soil mass due to normal loads applied to its surface it is commonly assumed that the soil mass is homogeneous and isotropic and that it deforms in accordance with Hooke's Law. The Boussinesq solution 2) is applicable to such a mass when it is loaded in this manner. The application of this solution to settlement problems has been simplified by the development of influence tables and charts. 1)

A mass that is assumed to have the idealized properties for which this solution is applicable may represent reasonably well the properties of a uniform, clay-like soil provided it is subjected to loads of moderate intensity. If, however, the soil mass contains layers or lenses of granular material the properties of the mass as a whole will be altered so that the stress pattern for a given loading condition will deviate from that obtained from the Boussinesq solution. It seems likely that thin, horizontal layers of sand or silt, which may exist in an otherwise uniform deposit of clay-like soil, will serve to reinforce the mass so as to restrict horizontal displacements. A rigorous solution applicable to such a mass that is assumed to be reinforced internally so that horizontal displacements are entirely prevented was obtained by Westergaard 3).

The stress patterns for the vertical normal stress due to a load applied at a point as obtained from this solution are compared with that obtained from the well known Boussinesq solution in Fig. 1. It is to be noted that the vertical normal stress obtained from the Westergaard solution is a function of Poisson's ratio while that obtained from the Boussinesq is independent of this ratio. At any point below the point of application of the load the stresses obtained by both solutions are equal when Poisson's ratio is equal to 0.25; for a value of Poisson's ratio smaller than 0.25 the vertical normal stress as obtained from the Westergaard solution is smaller than the corresponding stress obtained from Boussinesq's solution. For a value of Poisson's ratio equal to zero (no lateral expansion) the vertical normal stress at any point immediately below the point of application of the load as obtained from Westergaard's solution is equal to two-thirds of the corresponding stress obtained from Boussinesq's solution. For a value of Poisson's ratio equal to 0.5 (no volume change) the vertical normal stress at any point below the point of application of the load as obtained from the Westergaard solution is infinite. Thus, it is seen that the properties of the mass for which Westergaard's solution is applicable are such that the pattern of vertical normal stresses may be either more favorable or less favorable, from the point of view of differential settlement of the loaded area, than the pattern obtained from the Boussinesq solution depending on the value of Poisson's ratio that is assumed to describe the soil mass. From a study of observed settlements as compared with computed settlements 4) it is the writer's opinion that the most realistic stress pattern is obtained from Westergaard's solution for the case of Poisson's ratio equal to zero.

To simplify the application of Westergaard's solution to settlement problems the writer 5) has prepared influence values for the case of Poisson's ratio equal to zero applicable to various loading conditions that are of frequent occurrence.

Burmister 6) has developed the theory that applies to the determination of stresses and displacements in layered systems, each layer of which is assumed to be homogeneous, elastic and isotropic. The application of this solution is made difficult because of the necessity of determining the relative values of the elastic properties of the various layers constituting the system. Approximate values of the stresses in such a mass can be obtained by assuming the layered system to be equivalent to a uniform mass having the properties of the predominant layer but having a modified thickness; the thickness of each layer being adjusted in accordance with its stiffness relative to that

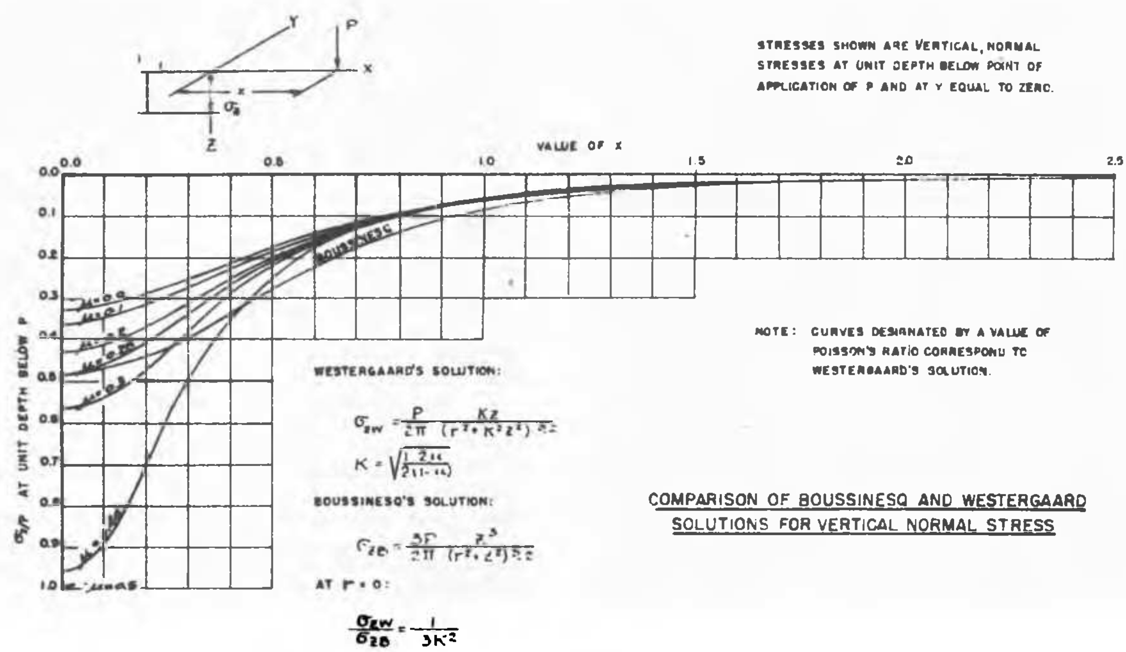

FIG. 1

of the predominant layer.

3. Assumed vs. Real Boundary Conditions

The foregoing solutions from the theory of elasticity are strictly applicable to masses of semi-infinite extent that are bounded by a plane surface. In general, building loads are not transmitted on the plane that is coincident with the ground surface nor does the mass, which is reponsible for the settlement, extend to an infinite depth.

The effect of the presence of a rigid bed and a flecible but inextensible bed at a given depth below the loaded surface has been investigated by Biot 7). The pressure of a rigid bboundary results in a mateiral increase in vertical normal stress in the vicinity of the boundary immediately below the point of application of the load; the presence of an inextensible, flexible layer produces a minor effect.

A solution from the theory of elasticity that takes into consideration a discontinuous surface is not yet available. For stress computation purposes it is commonly assumed that the plane of loading is coincident with the ground surface; that is, the soil located above this plane is assumed not to exist. If the soil above the plane of loading offers little resistance to deformation as compared with that of the underlying soil, the effect of its presence on the stress pattern in the soil mass located below the plane of loading is thought to be of minor importance. The greater the relative stiffness of the soil, which is located above the plane of loading, as compared with that of the underlying soil, the greater will be the error in the stress pattern that is obtained by ignoring in this manner the effect of a discountinuity in the surface of the soil mass. In any case it seems likely that the stress pattern obtained by neglecting the effect of a discontinuity in the surface will be such as to result in a smaller computed differential settlement than that which would be obtained if the effect of the discountinuity could be taken into account properly.

4. Flexible vs. Rigid Structure

As a first approximation in computing building settlements it is generally assumed that a structure is perfectly flexible and, accordingly, that it will not redistribute the load of the building to the underlying soil as settlement takes place. Actually, unless a structure is, in fact, perfectly flexible, it will redistribute its load as differential settlements develop. The amount by which the differential settlement is decreased due to the stiffness of the structure will depend upon the relative stiffness of the structure as compared with that of the soil supporting it. From a study (8 of the effect of variations in stiffness of a structure on the settlement pattern the writer is of the opinion that structures of conventional design are, in general, so flexible as compared with the types of soil to which a direct transfer of load would be permitted that the settlement pattern obtained by assuming the structure to be perfectly flexible will approximate closely that which would be obtained if the effect of the stiffness of the structure were taken into account.

If, however, the superstructure as well as the substructure are deliberately stiffened so as to minimize differential settlement, the effect of the stiffness on the settlement pattern can be determined conveniently by a method of successive approximations as follows:

a. Determine the building loads assuming no distortion of the frame.
b. Compute the settlement pattern using the loads as found in step (a).
c. Determine the load transfer resulting from the distortion of the frame caused by the differential settlement determined in sept (b).
d. If the shift in load found in step (c) is significant, compute the settlement using the corrected loads.
e. If the settlement pattern as found in step (d) is significantly different from that found in step (b), determine the change in loading resulting from the distortion of the building caused by the differential settlements computed in step (d).
f. Repeat step s (d) and (e) successively until the changes in loading due to the distortion of the building become of negligible importance.

The changes in settlement pattern will become smaller and smaller as this procedure is continued. However, since the settlements as found in steps (b), (d), etc., cannot be determined with any refined degree of accuracy, one would not in most cases be justified in extending the analysis beyond step (d).

II. COMPARISON OF OBSERVED AND COMPUTED SETTLEMENTS.

From the foregoing discussion, it is to be noted that some of the various assumptions, which are made in a stress analysis for settlement computation purposes, lead to a stress pattern, that will result in a greater computed differential settlement than that which may reasonably be expected to occur; others lead to a smaller computed differential settlement. Although the net effect of these assumptions may be such as to offset one another in a particular case, it is highly improbable that the errors will compensate one another in general.

In order to determine the net effect of these assumptions in a particular case, the writer,4) compared the actual settlement records of a building located in Boston, Massachusetts, with the corresponding computed settlements.

The subsoil conditions at the location of this building as disclosed by numerous dry sample borings that were made at the site are in general uniformly the same. The typical profile 9) consists of approcimately 120 feet of unconsolidated sediments as follows: 20 ft. of sand and gravel fill; 20 ft. of organic clay, peat, fine sand and shells; 7 ft. of hard, yellow clay; 8 ft. of medium, blue clay and 65 ft. of soft, blue clay with some sand and gravel existing in lenses and layers.

The free ground water surface occurs five to ten feet below the ground surface.

The load of the building under study was transmitted by means of Gow caissons to the hard yellow clay stratum existing 40 feet below the surface.

Consolidation tests were performed on representative samples of the soft blue clay underlying this building. These samples were obtained by undisturbed sampling methods. The weight of the building (41,000 tons) and the weight of the soil that had been excavated to provide basement space (35,000 tons) were carefully estimated. A load plan, as shown in Fig. 2, was prepared showing the variation in intensity of the building loads throughout the area occupied by the building. The intensity of pressure varies from section to section as shown on the load plan because of the differences in the number of stories of the various sections of the building. The vertical normal stresses in the soft clay produced by the net effect of the increase in pressure due to the building load and the decrease in pressure due to the weight of the soil that was removed to provide basement space were computed both from Westergaard's solution and from Boussinesq's solution. In applying these solutions the simplifying assumptions that have been enumerated and discussed were made. The settlements due to consolidation were computed using the data obtained from the consolidation tests on the assumption that the excess hydrostatic pressure created in a loaded mass of saturated soil is a function of the vertical normal stress.

Settlement observation points were installed at approcimately 50 different locations in this building. The settlement records of these points showed that the maximum settlement occurred at the location designated Point No. 6 in Fig. 2 and the minimum settlement, which was approcimately equal to one-half the maximum, occurred at the location designated Point No. 1.

The ratios of the observed settlement to the corresponding computed settlements as determined at these six locations are also shown in Fig. 2. The values of two ratios are given at each of these locations; the one, that as obtained from stresses computed from Westergaard's solution and designated by the subscript W, the other, that as obtained from Boussinesq's solution and designated by the subscript B. It is to be noted that the settlements computed from Westergaard's solution, using an assumed value of Poisson's ratio equal to zero, show in general a somewhat better, though not a significantly better, agreement with the corresponding observed settlements. A comparison of the observed settlements with the corresponding computed settlements also shows that, in general, the observed settlements vary in magnitude from a value equal to that computed to a value equal to one-half of that computed. Furthermore, it is to be noted that the location at which the observed settlement is equal to approcimately one-half of the computed settlement is at the point where the greatest settlement was observed to have occurred, while the location at which the observed settlement is approximately equal to the observed settlement is at the point where the minimum settlement was observed to have occurred. Since the maximum observed settlement was found to be approximately equal to double the magnitude of the minimum observed settlement, the magnitude of the computed maximum <u>differential</u> settlement within the loaded area as computed for this building is significantly greater than that which was observed to have occurred.

III. CONCLUSIONS

Since the subsoil conditions underlying this building were found to be reasonably uniform throughout the area occupied by the building, it seems reasonable to conclude that the qualitive discrepencies between the observed and computed relative to the determination of the excess hydrostatic stress. A comparison of the observed and computed settlements for a given building showed that the net effect of assuming

1. that the excess hydrostatic stress is a function of the vertical normal stress,
2. that the soil mass has the properties of a homogeneous mass that deforms in accordance with Hooke's law,
3. that the soil mass responsible for the settlement is of semi-infinite extent and that the soil above the plane of loading does not exist, and
4. that the structure producing the load is relatively flexible as compared with the soil that provides its support

results in a settlement pattern in which

1. the observed settlements vary in magnitude from a value equal to that computed to a value equal to approcimately one-half of that computed for corresponding points,
2. the location at which the observed settlement is approximately equal to the computed settlement is at the point where the minim-

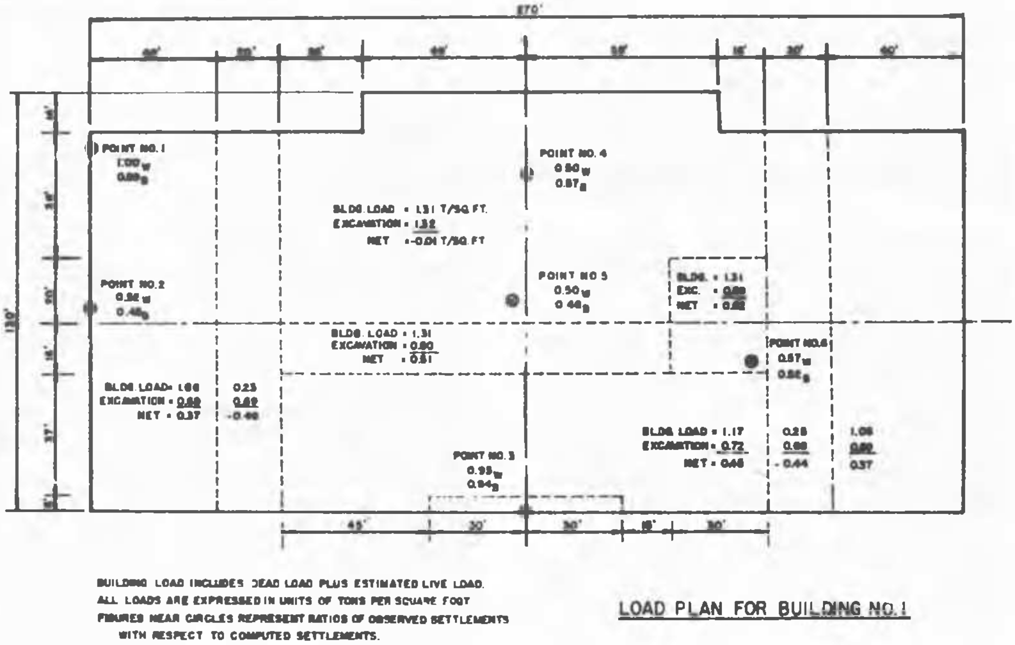

FIG. 2

um settlement was observed to have occurred while the location at which the observed settlement is equal to approximately one-half of the computed settlement is at the point where the maximum settlement was observed to have occurred.

Since the maximum observed settlement was found to be approximately equal to double the magnitude of the minimum observed settlement, the magnitude of the computed maximum differential settlement within the loaded area as computed for this building is significantly greater than that which was observed to have occurred.

It was also noted that the settlements computed from Westergaard's solution, using an assumed value of Poisson's ratio equal to zero showed in general a somewhat better, though not a significantly better, agreement with the corresponding observed settlements, than did the settlements as computed from Boussinesq's solution.

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