SUB-SECTION X b

SEEPAGE PROBLEMS OF DAMS AND LEVEES

X b2 THE EFFECT OF A SLIGHTLY PERVIOUS TOP BLANKET ON THE PERFORMANCE OF RELIEF WELLS

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SYNOPSIS

This paper presents an approximate method for determining the effect of a slightly pervious top blanket on the performance of relief wells installed in an underlying pervious water-bearing deposit. An example is given showing the variation of seepage discharge for wells variously spaced with both slightly pervious and completely impervious top blankets. A solution is given for the case of an impervious top blanket with pervious water-bearing deposit of a limited landward extent blocked by an impervious formation.

During the past few years, the necessity of providing relief for seepage pressures in buried pervious foundations downstream or landward from water-retaining structures has received increasing attention. One method of providing such relief is by means of seepage wells. Mathematical and model analyses have been performed to determine the effectiveness of such wells 1), 2), 3) for the simple case of a semi-infinite, pervious foundation of uniform thickness and permeability covered by an upper, absolutely impervious blanket. The assumption of an impervious top blanket is not always permissible, and a means of estimating the effect of leakage through such a blanket is desirable. The writer has studied this problem recently and, while an exact mathematical solution has not been obtained, an approximate solution has been developed, which, in view of the necessary simplification of foundation conditions, is believed to be satisfactory for practical applications.

The solution can best be presented by means of an example (see figure 1-A). Bennett 4) has shown that the case given in figure 1-A without wells may be transferred to that shown in figure 1-B (without wells) without changing the seepage volume or the seepage pressure at the landside toe of the structure. This transformation is made by replacing the lengths of the leaking riverside and landside blankets by effective lengths of absolutely impervious blankets. The riverside blanket distance L_1 = 1000 ft is reduced to an effective distance L_1',

$$L_1 = \left[\frac{k_b}{k_f z_b z_f}\right]^{-\frac{1}{2}} \cdot \tanh\left[L_1 \left(\frac{k_b}{k_f z_b z_f}\right)^{\frac{1}{2}}\right] = \left[\frac{0.065 \times 10^{-4}}{0.013 \times 7.5 \times 100}\right]^{-\frac{1}{2}} \cdot$$

$$\tanh\left[1000\left(\frac{0.065 \times 10^{-4}}{0.013 \times 7.5 \times 100}\right)^{\frac{1}{2}}\right] = 822 \text{ ft.}$$

and the landside blanket distance L_3 = infinity in reduced to an effective distance L_3',

$$L_3' = \left(\frac{k_b}{k_f z_b z_f}\right)^{-\frac{1}{2}} = \left(\frac{0.065 \times 10^{-4}}{0.013 \times 7.5 \times 100}\right)^{-\frac{1}{2}} = 1223 \text{ ft.}$$

where

- K_b = permeability coefficient of top blanket
- k_f = permeability coefficient of pervious foundation,
- z_b = top blanket thickness,
- z_f = thickness of pervious foundation,

It is assumed that completely penetrating relief wells placed at the landside toe of the structure will have the same discharge rate and the same seepage pressure midway between wells at the base of the blanket in figure 1-A as in figure 1-B. Thus, with wells installed, the total seepage Q_t from the reservoir through the upstream blanket and directly from the reservoir as shown on figure 1-A equals Q_t in figure 1-B. Q_w is the well discharge in both cases, and Q_b the seepage into the tailwater in figure 1-B is equal to the seepage through the landside blanket of figure 1-A. Thus in both figures $Q_t = Q_w + Q_b$.

The writer has obtained a solution 5) for the conditions shown in figure 1-B with drain wells installed a distance S from the riverside entrance face of the pervious foundation and spaced at distance a apart. Using the upper surface of the top blanket as a datum plane, the piezometric head at the base of the top blanket at any point is

$$h_{xy} = \frac{H(L - y)}{L} - q \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh\frac{\pi}{L}(na - x) - \cos\frac{\pi}{L}(y + S)}{\cosh\frac{\pi}{L}(na - x) - \cos\frac{\pi}{L}(y - S)}\right]$$

As shown in figure 1-B, y is measured with the y-axis going through one of the wells; x is

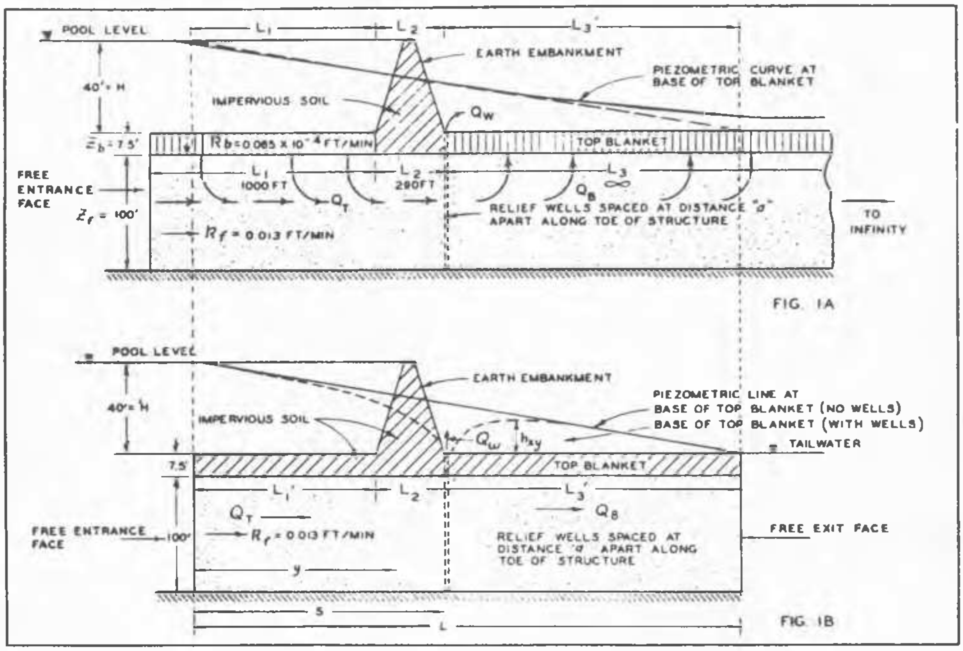

FIG. 1

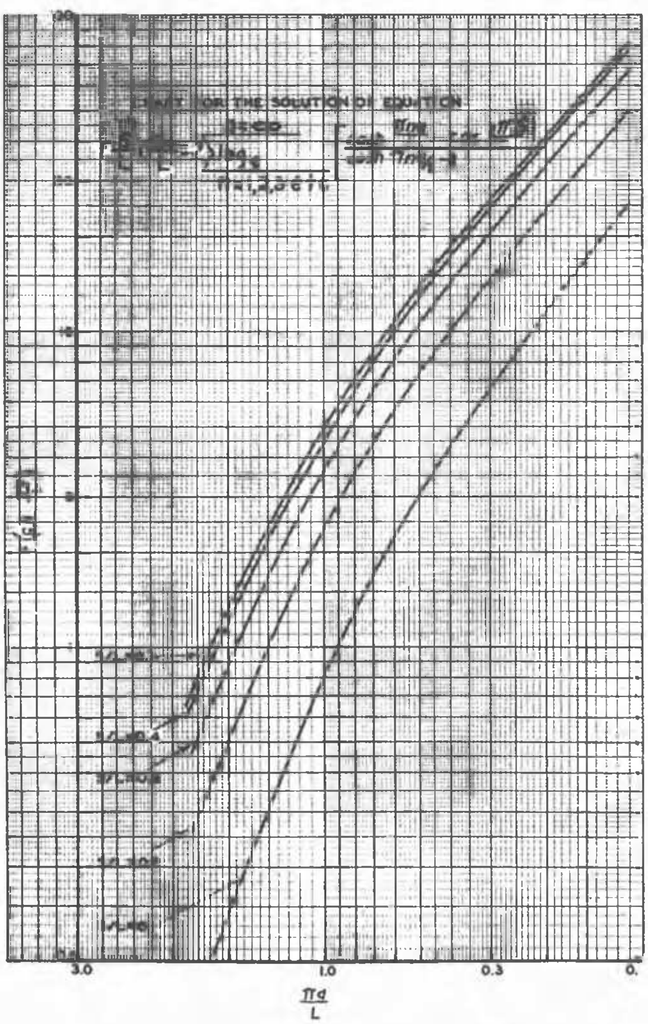

FIG. 2

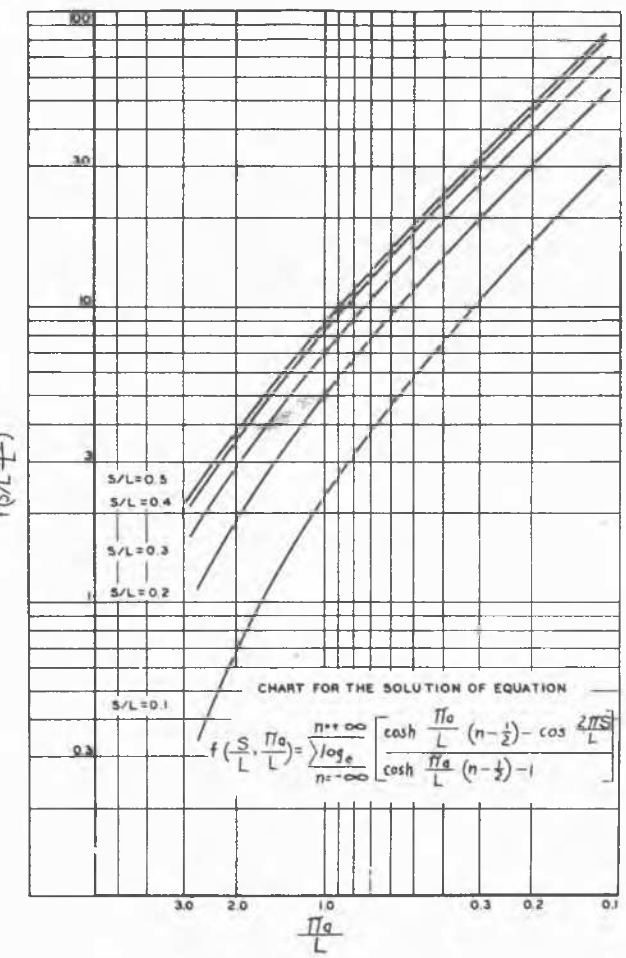

FIG. 3

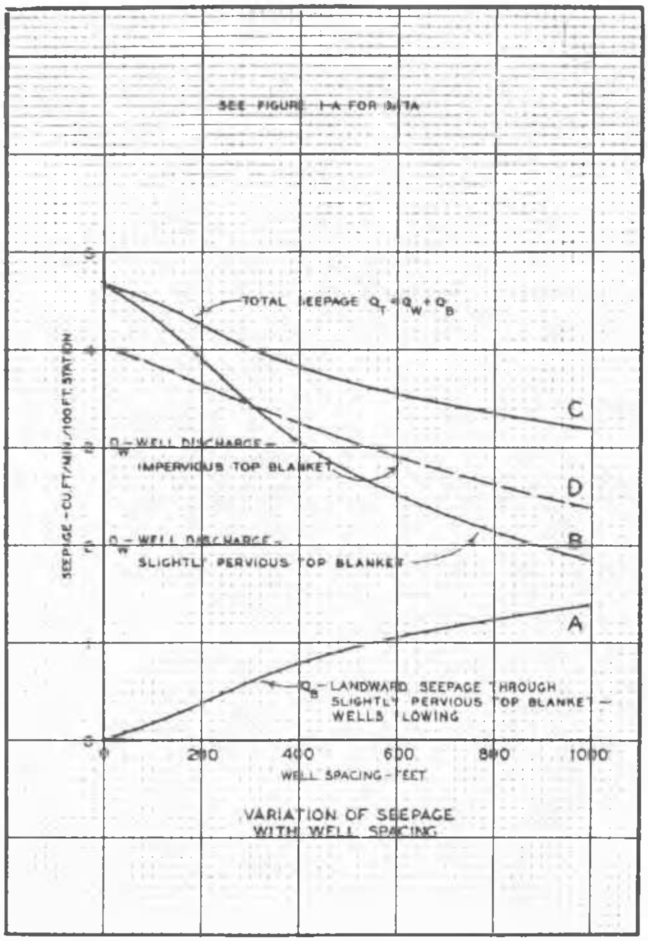

FIG. 4

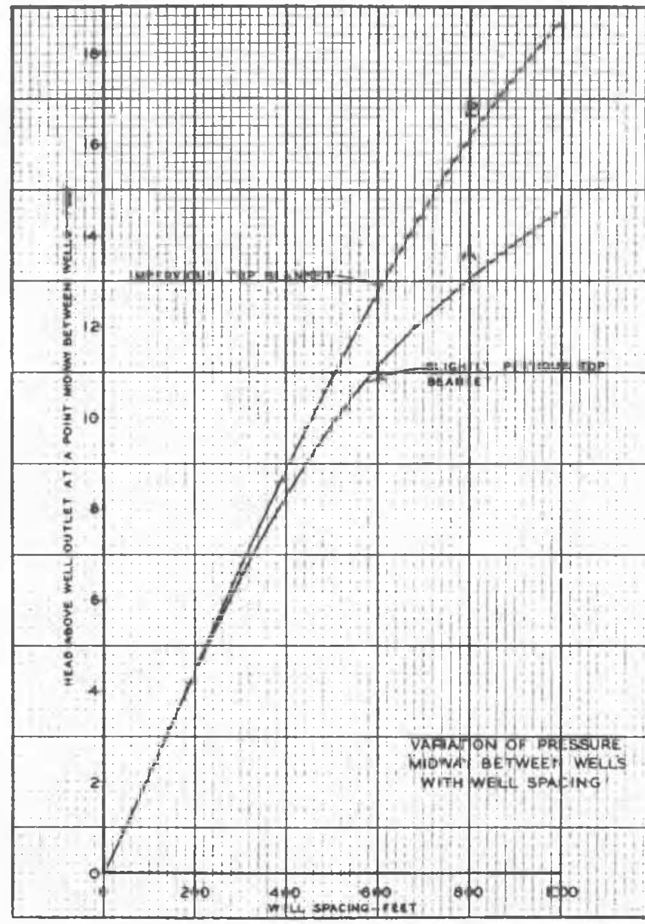

FIG. 5

measured normally to the plane of the section; and n takes on all integral values between minus and plus infinity, including zero.

$L = L_1' + L_2 + L_3'$ and $S = L_1' + L_2$; r_w is the well radius, and H is the net head between the reservoir and the discharge point of the wells which is assumed to be the top of the blanket. The constant q is

$$q = \frac{H\left(\frac{L-S}{L}\right)}{\log_e \left[\frac{1-\cos\left(\frac{2\pi S}{L}\right)}{1-\cos\left(\frac{\pi r_w}{L}\right)}\right] + 2\sum_{n=1,2,3}^{n=\infty} \log_e \left[\frac{\cosh\left(\frac{\pi n a}{L}\right) - \cos\left(\frac{2\pi S}{L}\right)}{\cosh\left(\frac{\pi n a}{L}\right) - 1}\right]}$$

Approximating $\left(1 - \cos\frac{\pi r_w}{L}\right)$ as $2\left(\frac{\pi r_w}{2L}\right)^2$, q may be expressed as:

$$q = \frac{H\left(\frac{L-S}{L}\right)}{\log_e \frac{1}{2}\left(\frac{2L}{\pi r_w}\right)^2 + \log_e\left(1-\cos\frac{2\pi S}{L}\right) + F\left(\frac{S}{L},\frac{\pi a}{L}\right)}$$

where

$$F\left(\frac{S}{L},\frac{\pi a}{L}\right) = 2\sum_{n=1,2,3}^{n=+\infty} \log_e \left[\frac{\cosh\left(\frac{\pi n a}{L}\right) - \cos\left(\frac{2\pi S}{L}\right)}{\cosh\left(\frac{\pi n a}{L}\right) - 1}\right]$$

Curves indicating the variation of $F\left(\frac{S}{L},\frac{\pi a}{2}\right)$ against $\frac{\pi a}{L}$ for various values of S/L are shown on figure 2. The head midway between wells at (x, y) = (a/2, S) may be determined by substituting these coordinate values in the preceding equation for h_{xy}, or as follows:

$$h_{a/2,S} = H\left(\frac{L-S}{L}\right) - q\, f\left(\frac{S}{L},\frac{\pi a}{L}\right)$$

where

$$f\left(\frac{S}{L},\frac{\pi a}{L}\right) = \sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh \frac{a}{L}\left(n-\frac{1}{2}\right) - \cos\frac{2\pi S}{L}}{\cosh\frac{\pi a}{L}\left(n-\frac{1}{2}\right) - 1}\right]$$

Curves indicating the variation of $f\left(\frac{S}{L},\frac{\pi a}{L}\right)$ against $\frac{\pi a}{L}$ for various values of S/L are shown on figure 3. The individual well discharge Q_w is $Q_w = 4\pi q k_f z_f$. The gradient at the tailwater discharge face in figure 1-B is $i_w = \left(\frac{H}{L} - 4\pi q\frac{S}{aL}\right)$; therefore, the discharge Q_b beyond the wells and into the tailwater per unit well spacing a is $Q_b = k_f\, i_{tw}\, z_f a$, which is equal to the seepage through the slightly pervious top blanket of figure 1-A.

The variations of seepage with well spacing a for the conditions shown in figure 1-A are shown on figure 4, based on the assumption that well resistance and velocity head losses are negligible. Curve A indicates the leakage through the slightly pervious blanket with wells in operation. Curve B indicates the seepage from the wells, while curve C indicates the combined seepage from both blanket and wells. Curve D indicates the well seepage based upon conditions shown in figure 1-A, except that the top blanket is assumed to be absolutely impervious. Shown on figure 5 are curves for seepage pressure heads at the base of the top blanket midway between the relief wells. Curve A is for the case of well installation with a slightly pervious top blanket, while curve B is for the case of well installation with an impervious top blanket. As would be expected, the additional avenue of seepage escape through the leaking top blanket produces a greater total seepage and a reduced pressure head midway between wells compared to the case of an impervious top blanket.

It was assumed in the above discussion that the landside portion of the foundation extended to infinity in the plane of the section shown in figure 1-A. The presence of an impervious deposit blocking the landward extent of the pervious foundation would limit the distance L_3 and would not permit the assumption of L_3 equal to infinity. The writer has obtained a solution for this case for the condition of a pervious foundation, fully penetrated by relief wells, spaced uniformly along a line parallel to the entrance and covered by an impervious top blanket. The piezometric head at the base of the top blanket, using the top of the blanket as a datum plane, is:

$$h_{xy} = H + q' \sum_{n=-\infty}^{n=+\infty} \log_e \left(\left[\frac{\cosh \frac{\pi}{2L}(na-x) - \cos \frac{\pi}{2L}(y+S)}{\cosh \frac{\pi}{2L}(na-x) - \cos \frac{\pi}{2L}(y-S)} \right] \cdot \left[\frac{\cosh \frac{\pi}{2L}(na-x) - \cos \frac{\pi}{2L}(y+2L-S)}{\cosh \frac{\pi}{2L}(na-x) - \cos \frac{\pi}{2L}(y-2L+S)} \right] \right)$$

and

$$q' = \frac{-H}{\sum_{n=-\infty}^{n=+\infty} \log_e \left[\frac{\cosh\left(\frac{\pi na}{2L}\right) - \cos\left(\frac{\pi S}{L}\right)}{\cosh \frac{\pi na}{2L} - \cos \frac{\pi r_w}{2L}} \right] \cdot \left[\frac{\cosh \frac{\pi na}{2L} + 1}{\cosh \frac{\pi na}{2L} + \cos \frac{\pi S}{L}} \right]}$$

For the case in which the pervious foundation is blocked landward with a slightly pervious top blanket, a solution may be had by using the transformed case, except the transformed length L_3' is

$$L_3' = \left[\frac{k_b}{k_f z_b z_f} \right]^{-\frac{1}{2}} \cdot \left[\tanh L_3 \left(\frac{k_b}{k_f z_b z_f} \right)^{\frac{1}{2}} \right]^{-1}$$

Objections to the installation of relief wells have been raised on occasion, based upon the belief that such wells will increase the total seepage. Such an increase will occur for heads less than the critical design head, as indicated on figure 6. Curves A, B, and C in-

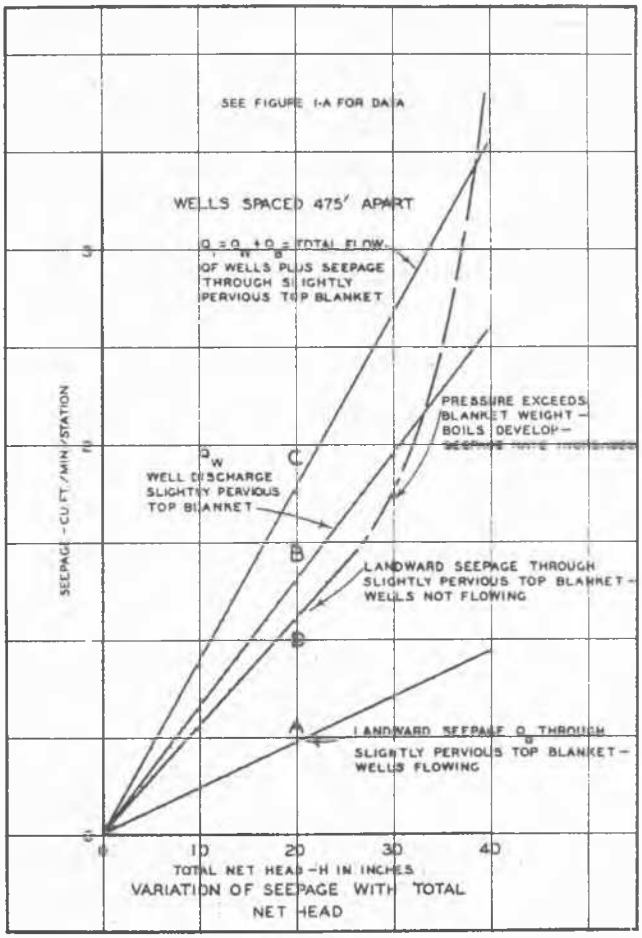

FIG. 6

dicate the variation, with net head H of blanket seepage, of well seepage and total seepage for a well installation as indicated on figure 1-A, while curve D indicates the blanket seepage for the case of no wells. After the total net head H has passed the critical value, boils will develop through the blanket when no wells are present, and the seepage will approach that for the well installation and may exceed it. This is shown by the dotted portion of curve D of figure 6. Thus, for the design head the difference in seepage for the two cases, with and without wells, may not be greatly different. The safety of the foundation of the structure as far as landside boils, piping, and landside sloughing are concerned, is far greater for the case with wells installed than for the case without wells. Further, even for net heads less than critical, curves A, C, and D on figure 6 indicate that, while the total seepage that must be disposed of is greater with wells discharging, the landward seepage through the blanket is much less than for the case with no wells. This is a very important advantage if the landward area is being farmed.

In conclusion, the use of the method of transformation presented above is sufficiently accurate for practical application, although it is not mathematically exact because of the necessary simplifying assumptions. In some cases where a very close spacing of wells is used, the well discharge will be somewhat greater than for the impervious top blanket case.
In general, however, the well discharge and seepage pressure head at base of blanket midway between wells will be less if the top blanket

is slightly pervious as compared to that if the top blanket is impervious. For most practical cases, the calculation of well discharges and pressures midway between wells, using the results of Muskat and Jervis (see references 1,2, and 3) are conservative, except for cases where the landward extent of the pervious foundation is limited by an impervious deposit.

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