INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

SIMSG   ISSMGE

SUB-SECTION Ie

SHEARING STRENGTH AND EQUILIBRIUM OF SOILS

CONTRIBUTION TO THE SHEARING THEORY

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1. INTRODUCTION

The theory of failure by MOHR, developed from the COULOMB theory of internal friction, has been well proved by its application to coherent and incoherent soils. The following shearing theory for saturated soils is founded on this hypothesis of failure. In some parts it has been developed parallel to the studies of KREY-TIEDEMANN 1) and JUUL HVORSLEV 2) and forms a wider basis for interpreting the actual shearing and triaxial tests. The theories developed by the two authors mentioned will be broadened especially by a stricter formulation of the conditions of moisture content and cohesion at failure. The relation between moisture content, cohesion and state of strength will also be considered. Prof. Dr. R. HAEFELI and Eng. W. SCHAAD 3) had the chief parts in developing and definitely formulating this theory. The conditions of shearing resistance and internal friction of unsaturated soils have been specially investigated by R. HAEFELI and we would refer to his publication in present proceedings. 4)

2. THEORETICAL BASES

In order to judge the phenomenon of failure of saturated soils, it is essential to distinguish between the "open" and the "closed" systems where both these terms describe the state of tension of the capillary water.

In the open system the deformation is accompanied by continuous modification of the moisture content which adapts itself to the instantaneous state of compaction of the material so that the pore water is neither under tension nor pressure. In this case the relation between shearing resistance S and normal pressure σ_i is represented by a single line passing through the origin and which may be considered in practice as a straight line over a limited range of pressures. This straight line given by line a in figure 1 makes an angle φ_s (the angle of apparent internal friction) with the axis of normal pressure σ. The designation "apparent" expresses that both the effects of friction and cohesion are included in this value. Each of these influences is proportional to the compaction pressure σ_i 5).

We speak of a closed system of pore water when the moisture content of the soil remains unchanged if a mechanical stress is applied. Internal and external forces are then partly transmitted by water either in tension or in compression. In this case the relation between shearing resistance S and normal stress σ_i is represented by the lines b - c ; these are nearly parallel to each other and the moisture content appears as a parameter. (Figs. 1 and 11).

As a first approximation the shearing resistance of a soil is given by two index values: the angle φ_s of apparent internal friction and the angle φ_r of real internal friction.
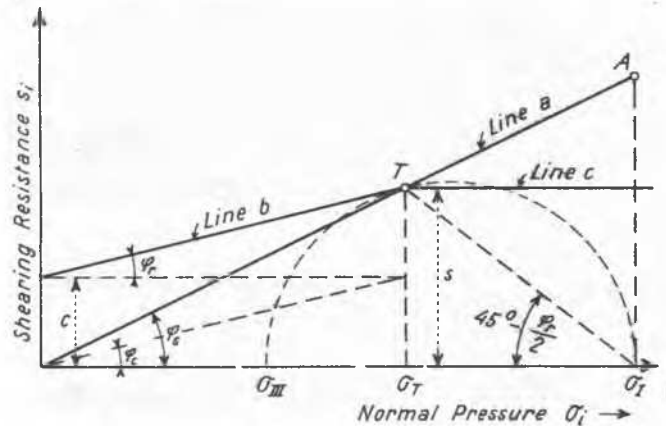
Complete Shearing Diagram for the open and the Closed system

FIG. 1

From Fig. 1 the following expressions can be given for the shearing resistance in relation to the normal pressure:

Line a : s = σ_i tan φ_s for the open system with variable moisture content and "natural" pore water,

Line b : s = c + σ_i tan φ_r for the closed system with constant moisture content. (Zone on the left of T)

where

c = σ_T (tan φ_s - tan φ_r)

- $\sigma_T \cdot$ tan φ_c

Line c : s = σ_T tan φ_s

where

$$\sigma_T = \frac{\sigma_I}{1 + \tan \varphi_s \tan(45^\circ + \frac{\varphi_r}{2})}$$

for the closed system with constant moisture content, and pore water under pressure (Zone on the right of T).

In order to understand the contribution to the theory of shearing developed below, it is most important to recognise that the moisture content and the state of cohesion are both primarily determined by the first main stress which compacts the material.

It must be considered that during the shearing test carried out in the ring shearing apparatus in order to determine line a, a first principal stress σ_I occurs in the material which is higher than the preconsolidation pressure σ_T in Fig. 1.

The normal pressure σ_T was applied to precompact the sample and remained unchanged during the test which was carried out so slowly that the material was compacted by the effect of the first main stress pore water flowing off. The open pore water included in the sample corresponds to the natural pore water which is only subject to its own hydrostatic pressure.

The first principal stress σ_I is inclined at an angle $\alpha = 45° - \dfrac{\varphi_r}{2}$ to the sliding surface. Its value can easily be calculated by means of the MOHR stress diagram.

Fig. 2 shows the results of shearing tests carried out on the open system (line a) by J. HVORSLEV with Wiener Tegel V Clay 2) which indicate especially the relationship between shearing resistance, normal pressure and moisture content before and after failure.
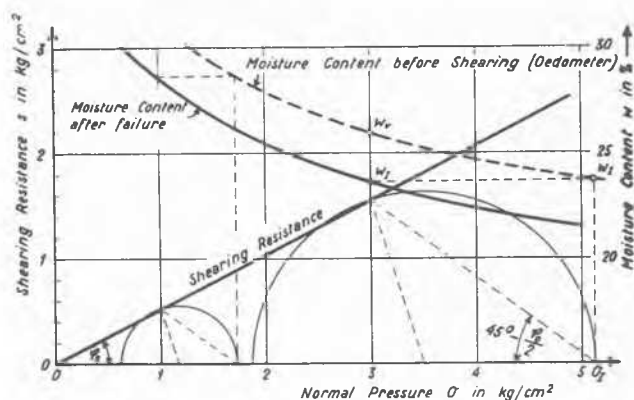
Relation between shearing resistance s, first principal stress σ_i and moisture content w. Constructed from the shearing tests carried out in the open system with Wiener tegel V clay by M. J. HVORSLEV and the developped theory.
$\sigma_v = 3$ Kg/cm^2 Precompacting normal pressure which was applied before shearing and to which corresponds the moisture content w_v.
$\sigma_I = 5.12$ Kg/cm^2 First principal stress occuring at failure and acting as a higher compacting stress. The resulting lower moisture content w_I is reported to the pressure $\sigma_v = 3$ Kg/cm^2 at which test was carried out.

FIG. 2

The shearing test to determine a point on line b is carried out on "overcompacted" samples in the closed system, i.e. the sample is compacted at a normal pressure σ_v (Fig. 3), the pressure reduced to a lower value σ_i and then the sample is sheared under this load. It is presumed that no change of moisture content and consequently no change of volume should occur. The cohesion as a measure of the state of consolidation does not change and, like the moisture content, corresponds to the equivalent compaction pressure σ_v.

If the cohesion c resulting from the precompacting load σ_v is to become effective then the first main stress σ_I occurring during the shearing test should not increase above the value of the preconsolidation pressure σ_v. If
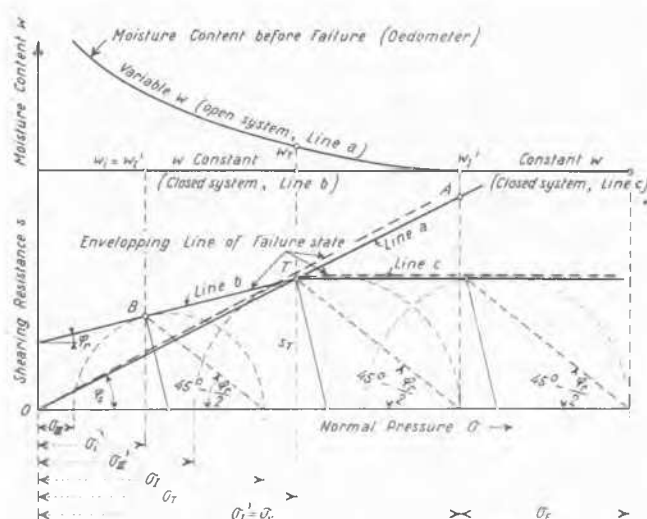

Shearing diagram
Relation between shearing resistance, normal pressure σ and moistuer content w in the open system (line a) and the closed system (lines b and c)

FIG. 3

this does occur σ_I will determine the cohesion and shearing resistance.

Carrying out the shearing test on the open system with a normal pressure σ_i lying between the preconsolidation stress σ_v and the lower limit stress σ_T, the first main stress occuring during the test exceeds the preconsolidation stress σ_v. The sample thus undergoes an additional compaction which corresponds to the line a on which the corresponding shearing resistance effectively lies. We only have a closed system when we reduce the normal pressure σ_i under the limit value σ_T. The mecanical rules for the relation between normal pressure and shearing resistance formulated up to now are thereby extended and broadened further.

If we compact a material of low perviosity by a preconsolidation pressure σ_v and Suddenly increase the normal pressure, the moisture content at first remains constant since, as a consequence of the low permeability, pore water does not flow off. In this case the water bears practically the whole additional load σ_E (Fig. 3). As no further consolidation of the sample occurs, shearing resistance also does not increase. The line c which corresponds to this state of stress consequently runs nearly parallel to the pressure axis. Considering the increase of normal pressure in the the solid phase, shearing resistance will strictly speaking increase a little. This part of the additional load, however, is so small, as shown by the following calculation, that it may be neglected over the usual range of pressures.
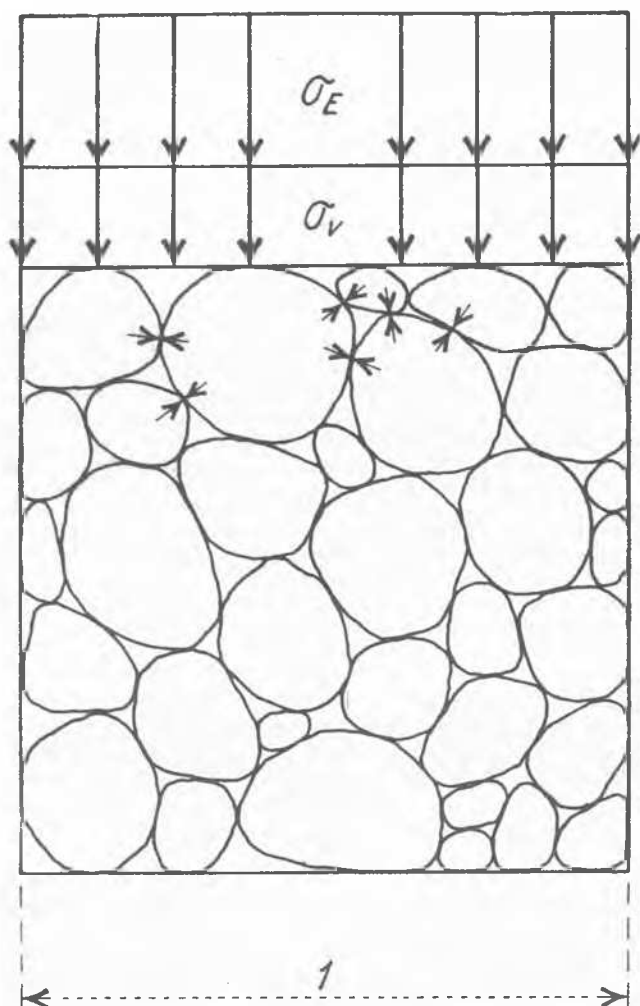
Distribution of on additional external stress
on the solid and liquid phases in a closed
system without air.

FIG.4

σ_v = Preconsolidation stress (equivalent
compaction stress)

σ_E = Additional external stress.

σ_k = Additional stress in the solid phase

σ_w = Additional stress in the liquid phase

M_E = Coefficient of compression at σ_v

E_w = Coefficient of elasticity of water.

All the stresses refer to the whole sec-
tion of the sample.
The equilibrium condition is given by :
$$\sigma_E = \sigma_k + \sigma_w.$$
As the compression of both phases must be
equal, we may write :

$$\frac{\sigma_k}{M_E} = \frac{\sigma_w}{E_w} \quad ; \quad \sigma_w = \sigma_k \cdot \frac{E_w}{M_E} \qquad (2)$$

$$\sigma_E = \sigma_k \left[1 + \frac{E_w}{M_E} \right] \qquad (1)$$

$$\frac{\sigma_k}{\sigma_E} = \frac{1}{1 + \frac{E_w}{M_E}}$$

e.g.

$$\sigma_v = 1 \text{ Kg/cm}^2 \quad ; \quad M_E = 20 \text{ Kg/cm}^2 \quad ;$$
$$E_w = 2.10^4 \text{ Kg/cm}^2 \quad ;$$

$$\frac{\sigma_k}{\sigma_E} = \frac{1}{1 + \frac{2 \cdot 10^4}{20}} = \frac{1}{1000}$$

i.e. the water will carry 99.9 % of the addi-
tional external stress.

As shown in Fig. 3, the Mohr circle which
passes through the point T constitutes the
limit between the tests made in order to de-
termine the three lines a, b and c. This point
is given by the condition that all the three
tests should yield to the same principal stress
$\sigma_I = \sigma_i^1 = \sigma_v$. Considering that the line
b is a limit curve for the state of failure,
this Mohr circle must touch it as shown by
TERZAGHI.But this circle also represents the
stress circle for the state of failure in the
open system at point T.(line a) and it corres-
ponds further to a point on the line c which
characterises the closed system with compressed
pore water. As a consequence,we have, for a sin-
gle line a inclined at an angle φ_s and charac-
teristic for each material an infinite number
of lines b - c.Their point of intersection T
corresponds to a given moisture content measured
on the conclusion of the shearing test. This
will appear as a parameter for the tests carried
on the closed system. (Fig 11)

For coarse non-cohesive soils there is no
difference between φ_r and φ_s and as a conse-
quence of the high permeability no compressed
pore water will occur.

The condition for failure established by
KREY-TIEDEMANN, 1) proceeds from the point of
view that the cohesion c only depends on the
maximum normal pressure, and that it remains
constant whilst the load is being reduced.This
assumption has been confirmed by the tests
made to determine line b. As shown in Fig. 2,
the highest stress occurring either before or
during the test on the open system determines
the moisture content and the state of cohesion.
It does not matter whether this is the normal
pressure or the first main stress occurring
during the test.

J. HVORSLEV stated the general conditions
of failure as follows: 2)

$$s = \mu_o \cdot p + \varpi \cdot p_e$$

where μ_o is the coefficient of effective in-
ternal friction and ϖ the coefficient of effec-
tive cohesion, p the normal pressure and p_e the
equivalent compacting pressure. On the basis
of his investigations Hvorslev finds that the
shearing resistance of a cohesive soil is a
function both of the normal stress at the sli-
ding surface at the instant of failure and of
the void ratio, where this function is indep-
endent of the former state of stress.

Hvorslev purposely carried out his tests
on overcompacted samples without maintaining
constant the moisture content which was attain-
ed under preloading. When taking off the load
he always waited until the sample has ended
its swelling and soaking process although this
sometimes took about seven days.

We have assumed on the contrary that the
shearing tests with overcompacted samples,the
load on which has been reduced, have to be
carried out without changing the moisture con-
tent attained under the precompaction load: On
this assumption the cohesion considered as a
measure of the compaction attained also remains
constant.

Fig. 5 shows the tests carried out by Hvorslev with overcompacted samples of Wiener Tegel Clay V. The measured values of moisture content before and after failure make it possible to determine from the present theory the equivalent compaction pressure for each range of loading investigated. The construction is as follows: From the point 5 with a shearing resistance s_5, for instance, draw the line b inclined at an angle $\varphi_r = \varphi_o = 17^\circ 30'$ as determined by Hvorslev. Its point of intersection T_5 with the line a inclined at an angle $\varphi_s = 25^\circ 40'$ determines the Mohr limit stress circle for the failure state in the open system and consequently the equivalent compaction pressure σ_{e_5}. As can be seen from Fig. 5, the moisture content of the sample corresponding to this stress σ_{e_5} on the curve of moisture content before shearing against pressure is practically the same as the moisture content measured by Hvorslev after failure.
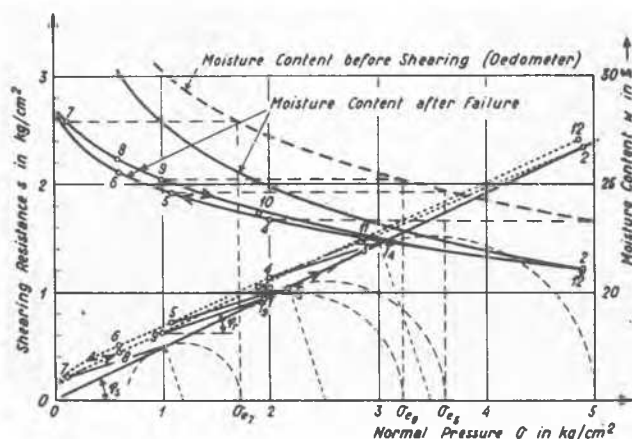
Relationship between shearing resistance, equivalent compaction stress and moisture content as it may be interpreted from the shearing tests carried out with overcompacted samples of Wiener tiegel V clay by M.J. HVORSLEV and the developed theory.

FIG. 5

From the theory developed here the same shearing resistance as determined by Hvorslev in test 5 should be obtained if the Clay sample is preloaded at a pressure $\sigma_{e_5} = \sigma_v = 3{,}62$ Kg/cm^2, the pressure reduced to $\sigma_1 = 1.0$ Kg/cm^2 and the shearing test carried out maintaining the moisture content unchanged at the value reached with $\sigma_{e_5} = \sigma_v$

It appears that the increase of moisture content which occurs during the soaking process corresponds to a definite reduction in the equivalent compaction stress. Further investigations are required to clarify the relation between the equivalent compaction stress, the magnitude of the range of pressure reductions, and the shearing resistance of overcompacted samples.

3. EXPERIMENTAL BASES

Today two, in principle different, types of apparatus are used mainly for determining the shearing diagram experimentally. The first one, the ring shearing apparatus, is a shearing machine proper and the second one is the triaxial apparatus 7). In the former failure is obtained by increasing the shearing stress while the normal pressure remains unchanged; in the second it is realised by increasing the difference between both the principal stresses σ_I and σ_{III}. The vertical stress is produced by the piston and the horizontal stress by the pressure of a liquid which is effective on all sides. In principle all failure states of the shearing diagram can be attained by both of these apparati.

The question of which equipment is the best looses its importance when we realise that the ring shearing apparatus is especially well suited to carrying out shearing tests on the open system (line a), and the triaxial apparatus for tests on the closed system with constant moisture content (lines b and c), and furthermore determining the precompaction load on undisturbed samples. The ring-shaped sample of small thickness which is used in the former apparatus can be drained easily, so that the moisture content can adapt itself to the state of compaction, as assumed for tests on the open system when carried out sufficiently slowly (line a). The triaxial apparatus is normally used in testing samples of cylindrical shape enclosed in a rubber membrane. It thus constitutes a closed system, the water content of which, as a rule, remains unchanged (lines b and c) if no special measures are taken for draining. Parallel use of both types of apparatus furthermore offers the basic advantage that certain test results obtained with one can be checked with the aid of the other, and so results which are independent of the special test arrangements may be obtained.

In our laboratory we normally carry out the following tests to determine the complete shearing diagram:

1. Determination of φ_s , (line a).

A 2 cm thick removed sample with an initial moisture content as near as possible to the liquid limit of Atterberg is placed in a ring shearing apparatus with a surface area of 250 cm^2. It is slowly preconsolidated by applying a normal pressure σ_v in steps of 0.25 , 0.5 , 1.0 Kg/cm^2. After settlement has occurred the shearing test is carried out under water for a period of 350 – 450 minutes. The measured shearing resistance is called $s_{1/1}$ to show that the test has been made with the same normal pressure $\sigma_i = 1$ Kg/cm^2 under which the sample has been precompacted. For purposes of checking a similar test is made with $\sigma_v = \sigma_i = 2$ Kg/cm^2.

2. Determination of φ_r (line b).

The test is usually carried out in the ring shearing apparatus on removed samples, like test 1. The preconsolidation stress normally amounts to $\sigma_v = 4$ Kg/cm^2, because the higher the stress the smaller is the influence of the dispersion of test results on the value of φ_r. After having waited for a sufficient time for settlement to take place, the pressure on the sample is reduced to $\sigma_i = 1$ Kg/cm^2. The shearing test is carried out immediately after for a period of 200-300 minutes, without absorption of water. If the perviosity of the soil is very low, the moisture content will not change even when the test is carried out under water, under which conditions no active capillary stresses can occur. It is not recommended to reduce the normal pressure below

Failure of a clay sample obtained by increasing the vertical pressure.

(Preconsolidation stress σ_v = 2.0 kg/cm^2)
(Compressive resistance β_d = 1.0 ")
(Failure occured at $\{ \sigma_I$ = 1.0 ")
(and $\{ \sigma_{III}$ = 0 ")

FIG. 6 a

σ_i = 1 Kg/cm^2 because the dispersion of test results will increase. The measured shearing resistance is called $s_{4/1}$. (see point B, Fig.8) to show that the test has been carried out with a normal pressure of σ_i = 1 Kg/cm^2 and that the sample has been precompacted at σ_v = 4 Kg/cm^2.

This test can, of course, also be carried out with the triaxial apparatus. However, the time required for the sample to settle under the preconsolidation stress may be long, especially when testing soils of low perviousity. In this case, we may provoke failure of the sample under reduced normal load in two ways, as shown in figures 6a and 6b. In order to determine line b, the Mohr circle of figure 3 can be obtained once by reducing the pressure σ_I produced on the sample by the piston and diminishing the lateral pressure σ_{III} of the liquid until failure occurs (Fig. 6a). On the other hand we can reduce the pressure σ_{III} on the sample produced by the piston and reincrease the lateral pressure of the liquid until failure occurs at the value σ_I (Fig. 6b) The latter method has the advantage that it produces no ambiguous criterion of failure

Failure of a clay sample obtained by increasing the lateral pressure.

(Preconsolidation stress σ_v = 2.0 kg/cm^2)
(Failure occured at $\{ \sigma_I$ = 2.95 k ")
(and $\{ \sigma_{III}$ = 2.0 ")

FIG. 6 b

since failure will be accelerated as a consequence of the deformation of the sample.
It was not possible to obtain a simple mathematical relationship between σ_v, σ_i and σ_r.
This is shown graphically in Fig. 7. These curves were calculated from the equation below which follows from Fig. 1.

$$\tan\varphi_s = \frac{\cos\varphi_r \cdot \left[s_{v/i} + \tan\varphi_r (\sigma_v - \sigma_i) \right]}{\left[\sigma_i (1+\sin\varphi_r) - s_{v/i} \cdot \cos\varphi_r \right] \cdot \tan\varphi_r - s_{v/i} + \sigma_v \cdot \cos\varphi_r}$$

σ_v = precompaction stress (σ_v = 4 Kg/cm^2 in the example)

σ_i = value to which the normal pressure is reduced and at which the shearing test is carried out
(σ_i = 1 Kg/cm^2 in the example)

φ_s = angle of apparent internal friction

φ_r = angle of real internal friction

$s_{v/i}$ = shearing resistance obtained by carrying out the test on a sample precompacted with a pressure σ_v

(= 4 Kg/cm² in the example) and sheared at the reduced pressure σ_i (= 1 Kg/cm² in the example).

It should be noted that the curves of Fig. 7 may only be utilised for tests carried out as described.
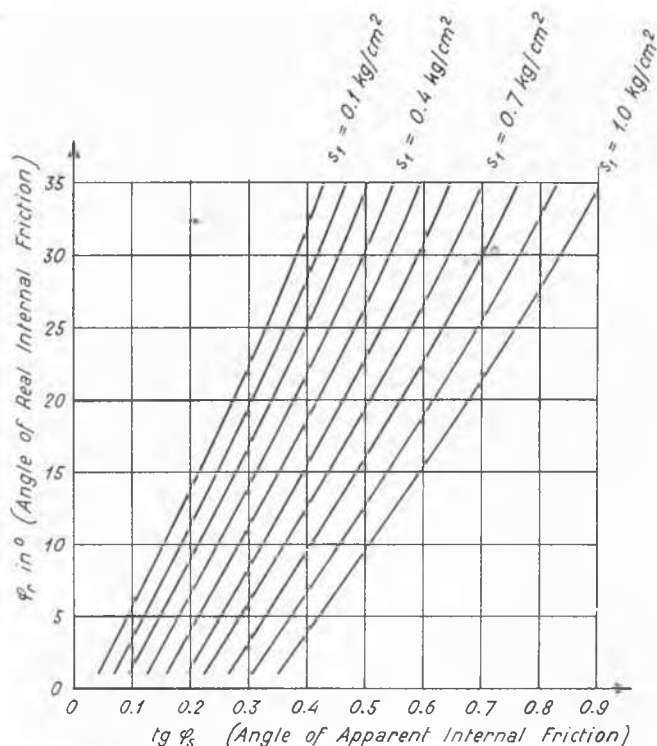
Nomogramm to determine φ_r
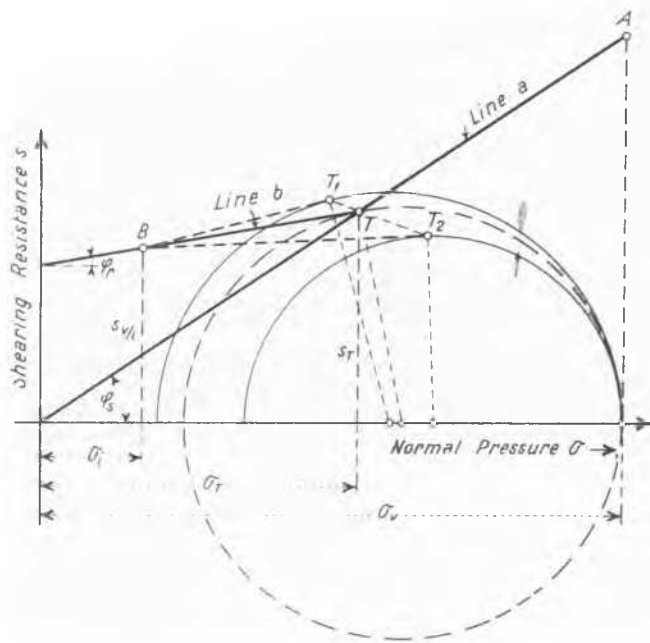
σ_v = 4.0 Kg/cm²   Preconsolidation stress

σ_i = 1.0 Kg/cm²   Reduced normal pressure at which the test is carried out

s_i in Kg/cm²   Shearing resistance at the reduced pressure σ_i = 1.0

FIG. 7

An approximate graphical solution, shown in Fig. 8, has also been developed in order to determine the angle φ_r when φ_s and $s_{v/i}$ are known. A first circle is drawn passing through the point σ_v just cutting the line a and having its centre on the σ axis, and a second one passing below line a. Draw BT_1 and BT_2 tangents to the two circles. The point of intersection T of the straight lines T_1 and T_2 and line a is approximately the actual point of intersection of lines a and b, corresponding to the preconsolidation stress σ_v. Consequently the values of φ_r, s_t and σ_t are known.

3. Determination of the preconsolidation stress (line c).

This line, which is assumed to have been determined with disturbed samples according to 1 and 2, is fixed for a given moisture content by the intersection T of the lines a and b. It is of special interest to carry out a test of this kind in order to determine the preconsolidation stress σ_v of an undisturbed sample.

In this case the shearing test is made in the triaxial apparatus over a range of stresses which ensures that failures will occur in the zone of compressed pore water. (Fig. 8, right

Approximata graphical solution to determine the angle φ_r

FIG. 8

hand circle). As s is determined, we find the point T by tracing line c parallel to the σ axis. The preconsolidation stress σ_v is given by the Mohr limit circle passing through point T and tangential to line b, where the latter is parallel to line b_d passing through T.

4. Further relationships.

Proceeding from our theory, it is possible to formulate the relationships between the preconsolidation stress σ_v, the angles φ_s and φ_r of apparent and real internal friction and the cohesion c_i, or the compression resistance β_d, or the tensile resistance β_z, where failure must occur as a shearing failure.

As shown in Fig. 10, we can write :

$$c_i = \sigma_v \frac{\tan\varphi_s - \tan\varphi_r}{1 + \tan\varphi_s \cdot \tan(45° + \frac{\varphi_r}{2})}$$

$$\beta_d = \frac{2c_i}{\tan(45° - \frac{\varphi_r}{2})} = 2c_i \cdot \tan(45° + \frac{\varphi_r}{2})$$

$$\beta_z = 2c_i \tan(45° - \frac{\varphi_r}{2})$$

4. CONCLUSION

A saturated soil, constituted by a solid and a liquid phase, subjected to an increasing consolidation stress in an open system undergoes regularly increasing compaction. The mechanical coefficients which characterise the solid bodies, and consequently also the shearing resistance, increase, while the moisture content decreases. As shown elsewhere 7) 8), the initial moisture content, the anisotropy and the range of compression stress may restrict the theory developed here to a relationship as a first approximation between shearing resistance and normal pressure.
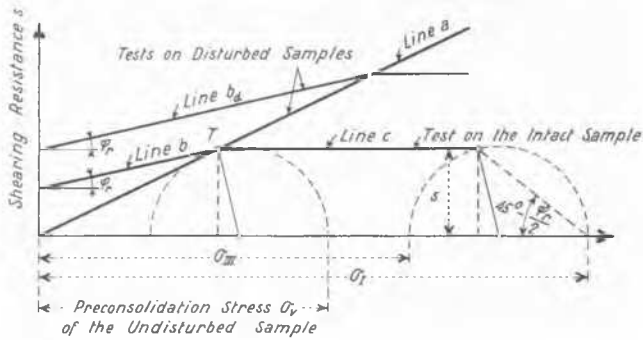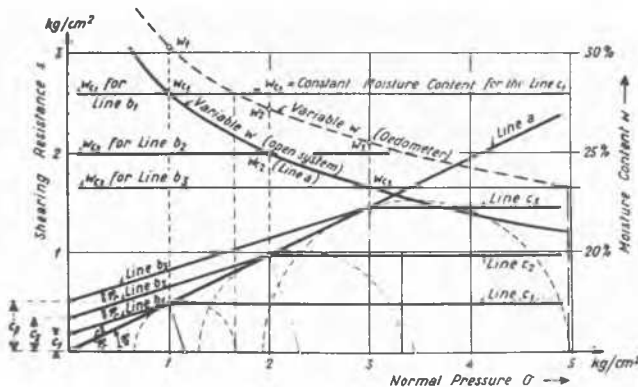
Triaxial test carried out to determine the pre-
consolidation stress of intact samples.
The lines a and b have to be determined former-
ly an disturbed samples of the same material.

FIG. 9
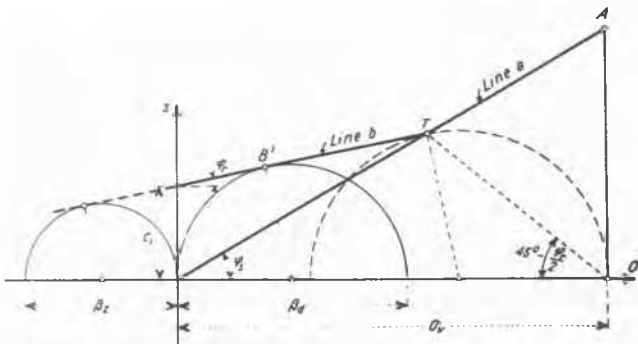
Relationship between the preconsolidation stress
σ_v, the angles φ_s and φ_r and cohesion c_i or
compression resistance β_d or tensile resis-
tance β_z

FIG. 10

Shearing diagram of wiener tegel V clay, construc-
ted from the investigations of M.J.Hvorslev and
the developped shearing theory.

FIG. 11

In the open system with natural pore wa-
ter this relation is represented by the prac-
tically straight line a inclined at the angle
of apparent internal friction φ_s and includes
the effects of friction and cohesion. In the
closed system with constant moisture content
the relationship mentioned above is represent-

ed by the infinite number of broken lines b
and c. The horizontal lines c characterise the
zone in which an overpressure occurs in the
pore water and the lines b inclined at the
angle of real internal friction φ_r the zone in
which an underpressure occurs. Moisture con-
tent at failure is a parameter. The points of
intersection T of the pairs of lines b and c
lie on the line a and are directly related to
the equivalent compaction pressure or precon-
solidation stress. Either former or the maxi-
mum principal stress in the solid phase deter-
mine the cohesion and it makes no difference
whether this stress occurs in the material be-
fore or only during the shearing process.

Figure 11 shows the shearing diagram of
the Wiener Tegel Clay V as it may be construct-
ed from the published investigations of
Hvorslev and the theory developed above.

The ring shearing apparatus and the triax-
ial apparatus for obtaining the shearing dia-
gram experimentally are remarkably complement-
ary to each other. The former is especially
suitable for carrying out shearing tests on the
open system with varying moisture content,
(line a); the latter is best for tests on the
closed system with constant moisture content
(lines b and c). When making triaxial tests by
increasing the lateral pressure and maintain-
ing the vertical stress constant, it is poss-
ible to work with fairly short cylindrical spe-
cimens without disturbing effects on the boun-
dary layers, and to obtain a direct criterion
for the actual instant of failure.

In calculating all problems where know-
ledge of the shearing resistance is required,
it is consequently possible to utilise its ef-
fective value at the time provided that the
state of tension can be determined exactly. It
is especially necessary to consider both the
zones of the closed pore water system having
regard to the fact that the moisture content
will in most cases change with time. It is thus
necessary to know not only the mechanical pro-
perties of the soils concerned, but also the
thickness of the stratum and the perviosity
and homogeneity, so that the possibility of
draining and soaking up the water can be taken
into account 9). The developped theory is not
a new method allowing to calculate the actual
criterion of stability. It tries to give a bet-
ter interpretation and understanding of the
shearing phenomenon of satured soils.

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THE EXTREME INCLINATION OF SLOPES WITH GROUND WATER PASSING THROUGH THEM 1). 2)

Ie 18

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The slope of an accumulation of cohesionless material reaches the limit of its stability as soon as its inclination to the horizontal forms an angle of ρ and an angle of $90° - \rho$ is reached with the vertical, whereby ρ represents the angle of interior friction within the aforesaid material. As soon as the slope is subjected to the in- or outflow of groundwater, the limit of its inclination is either reduced or increased. If, therefore, a new field of flow-pressure is superimposed over the field of gravitation which acts vertically, the extreme inclination of the new slope must again form an angle of $90° - \rho$ with the direction of the resulting field.   3)

Extreme inclinations of slopes formed by sand have been tested and examined by the author with the help of a contrivance which is shown in diagram by fig. 1. For the horizontal slope the corresponding flow pressures were found in accordance to Terzaghi's irruption equation. By transporting the flow pressures found from pole O according to their size and direction (see fig. 1 and 2 respectively) we find that their ends are situated on a circular line (the socalled "slope circle").

Conditions are shown by fig. 3. At a given inclination of the slope the maximal flow pressure, providing it is vertical to the slope, is calculated as follows:

$$p = \frac{\gamma_R}{\sin \rho} - \sin (\rho - \beta) \qquad (1)$$

and if parallel to the slope

$$p_p = \frac{\gamma_R}{\cos \rho} - \sin (\rho - \beta) \qquad (2)$$

whereby γ_R is the weight per unit of volume of the material under water, ρ is its friction angle, and β the inclination of the slope to the horizontal.

Flow pressure that is vertical to the slope takes effect under water in the case of outflowing groundwater, whilst if parallel to the slope, it is encountered whereever the slope
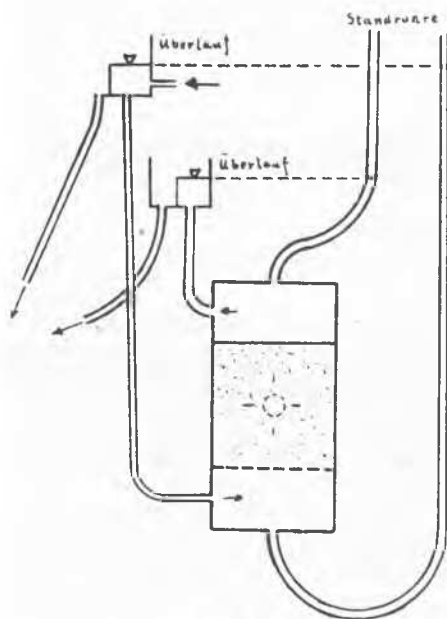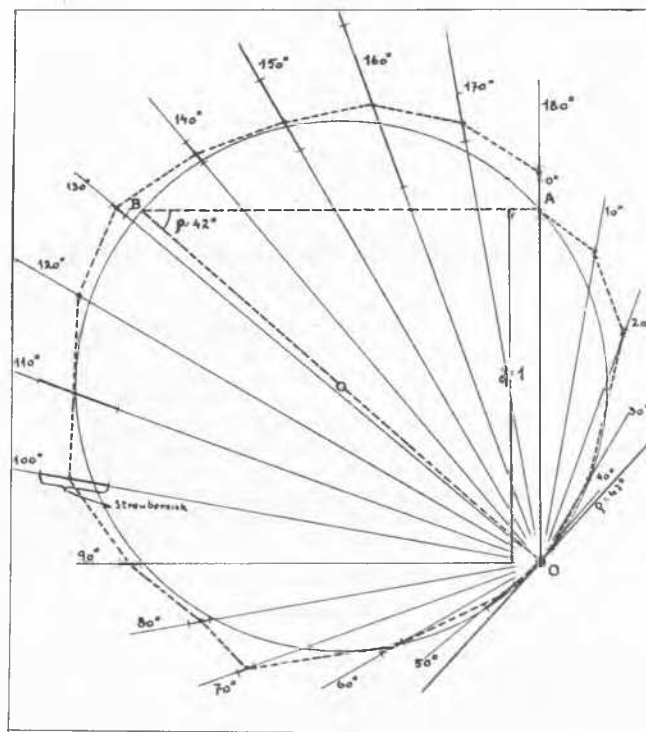

FIG. 1

FIG. 2