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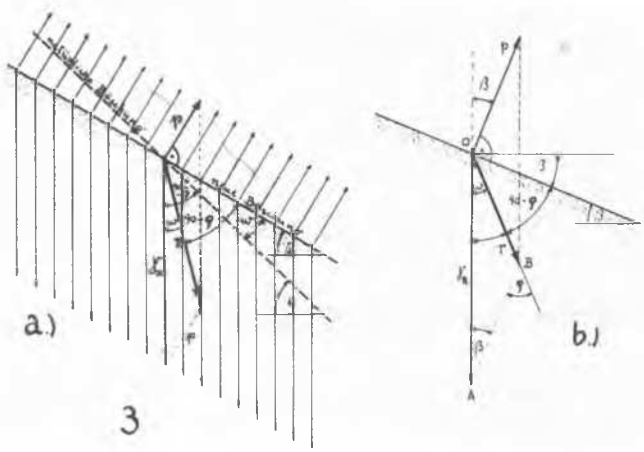

FIG. 3

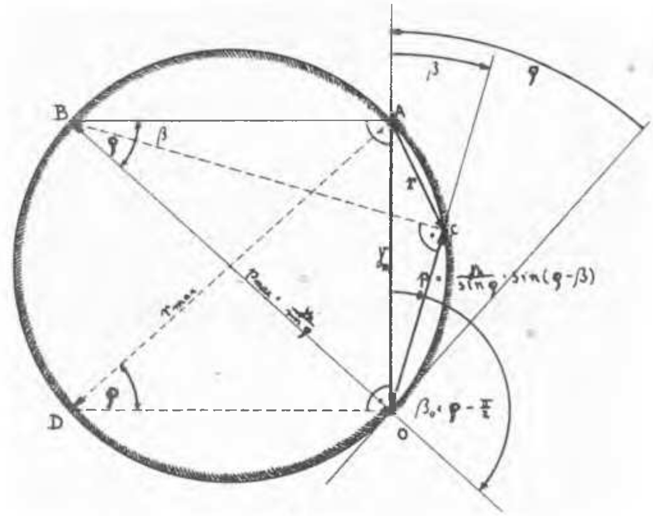

FIG. 4

is within the sphere of the capillary rise. The slope circle derived from equation (1) is represented by fig. 4. Section A -C-0 is for groundwater leaving the slope, and section 0 - D - B - A for such entering it.

The flow pressure parallel to the slope being

$$p_p = \gamma \cdot i$$

whereby γ is the specific weight of water = 1, and i the hydraulic gradient, it follows in connection with equation (2) that:

$$Tg\ \beta_{max} = tg\ \rho \cdot \frac{\gamma R}{\gamma R + \gamma} \qquad (3)$$

The equation (3) is independent of the depth of the point under consideration below the surface, and, as any existing cohesion may be locked upon as an additional load or increased depth of the point under consideration below the surface, the equation applies also to coherent material.

It may be said therefore of slopes through which groundwater passes parallel to its surface, i.e. whereever the pores are saturated with water held at a certain level by capillary force and no matter whether the material is sand or clay, that they are stabile in all cases when their inclination β to the horizontal does not exceed the value indicated by the equation (3).

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SLIDING SURFACES AND STABILITY CALCULATIONS FOR SOIL MASSES OF CULVILINEAR PROFILE

I e 19

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SUMMARY OF THE FRENCH REPORT

In continuation of mathematical studies already published, the author now tackles in all its aspects the problem of the sliding of a heavy mass whereof the free surface follows any curve, in respect of Rankine's condition of equilibrium.

Adopting the following notation:

- C = cohesion of the material composing the mass,
- φ = angle of friction,
- = specific weight,
- y = vertical, downward ordinate in relation to an axis Ox, of arbitrary direction and origin, at any point P of the profile X of the mass,
- y' = ordinate of point M of the slide curve under consideration G, situated on the same vertical,
- ρ = radius of curve at M,
- q = horizontal thrust at M,
- η = gradient of the tangent at P with the horizontal,
- α = angle made by the directions of the two tangents at P and at M; the author shows that y and y' are related by the following equation:

$$y'-y = \frac{C \cos\varphi}{\Delta} \cdot \frac{\cos(2\alpha+\varphi)}{m \cos\eta\left[\sin\eta - \sin\varphi\cos(2\alpha-\eta+\varphi)\right]} \qquad (1)$$

where m indicates a variable coefficient given by the formula

$$m = 1 - \frac{\int_P^M q\, dy'}{\Delta\rho(y' - y)\cos^3\eta} \qquad (2)$$

The coefficient m, less than unity for convex profiles and greater than unity for concave profiles, can be likened to the latter in a first approximation. The author gives the name of '<u>simplified slide curves</u>' to the slide curves <u>obtained by this approximation</u>, for a given embankment profile, and that of '<u>simplified embankment profiles</u>' to the profiles <u>determined in the</u> same manner for a given slide.

Now the infinitesimal triangles MNM', PR'R and MN'M' give the following ratios:

$$dy' - dy = \frac{\cos\omega \sin\alpha}{\cos\eta \cos(\alpha-\eta)}\, dx \qquad (3)$$

$$dy = \frac{\sin(\omega-\eta)}{\cos\eta}\, dx \qquad (4)$$

$$dy' = \frac{\sin(\alpha+\omega-\eta)}{\cos(\alpha-\eta)}\, dx \qquad (5)$$

By differentiating the equation (1) and eliminating $dy' - dy$ between the equation thus obtained and equation (3) the author arrives at the following equation:

$$dx = dx' = \frac{C\cos\varphi}{\Delta\cos\omega} \cdot \frac{\cos(\alpha-\eta)}{\sin\alpha\cos\eta\left[\sin\eta - \sin\varphi\cos(2\alpha-\eta+\varphi)\right]} \cdot$$

$$\cdot\left\{\cos(2\alpha+\varphi)\left[\cos 2\eta - \sin\varphi\sin(2\alpha-2\eta+\varphi)\right]d\eta + \sin 2\eta\left[\sin(2\alpha+\varphi) - \sin\varphi\right]d\alpha\right\} \qquad (6)$$

Equations (4),(5),(6) completely define both the simplified system of slide lines corresponding to a given embankment profile and the profile group of simplified embankment profiles corresponding to a given slide line.

Their integration with some chance of success can only be undertaken if α and η are made to stay together by a linear equation of the form

$$A\alpha + B\eta + D = 0$$

and on condition moreover of given particularly simple values to the numerical coefficients A,B and D.

The author has studied four particular cases:

1) A = 0. This is the case of <u>free plane surface masses</u> whereof the slide <u>curves-(oblique cycloids</u> if $\eta > \varphi$; logoids if $\eta < \varphi$ and semi cubic paraboles if $\eta = \varphi$. 1)

2) B = 0. This is the case of free curvilinear surface masses admitting (among others) a sliding surface such that the respective tangents of the two surfaces at points P and M of the latter situated on the same vertical shall make a constant angle. It does not appear to offer much practical interest.

3) 2A +B = 0: If the x axis is directed in such a manner that is equal to - D/B the connection indicated between the angles α and η will be defined by the formula

$$\alpha - 2\eta + 2\omega = 0$$

which results in the condition that the respective directions of the tangents at P and M are constantly bissected by the fixed direction Ox.

The embankment profiles thus obtained are of considerable interest at least when ω happens to be equal to φ . The author has named them '<u>simplified isolisthenic profiles</u>' 2).

In the special case $\omega = \varphi = 0$ the integration is particularly easy, one obtains:

$$x = x' = \frac{C}{2\Delta}\cdot\left(\frac{\cos\eta - 4\sin^2 2\eta}{2\sin^2\eta} - \log_e \cos\eta\sin^7\eta\right) + Ct.$$

$$y = -y' = -\frac{C}{\Delta}\cdot\frac{\cos 4\eta}{\sin 2\eta} \qquad (7)$$

4) A+B = 0. This is the case of <u>free-curvilinear masses allowing a plane sliding surface</u> whereof the author has already determined the profile 3). He has given them the name of '<u>simplified ortholisthenic sections</u>' 4).

More particularly should be noted, as being subject to many applications, those whereof the sliding plane is inclined to the horizontal at an angle equal to φ . The author has named them '<u>maximum simplified ortholisthenic profiles</u>'. Their equations, in relation to two axes

Ox directed according to the slide line

Oy vertical upward,

are as follows:

$$\left.\begin{aligned} x &= \frac{C}{2\Delta\cos\varphi}\cdot\left(z^2 - 1 + 2\log_e z\right) \\ y &= \frac{C}{\Delta\cos\varphi}\cdot\frac{z^2-1}{z} \end{aligned}\right\} \qquad (8)$$

The auxiliary variable corresponds to the expression

$$z = \frac{\cos(\alpha+\varphi)}{\sin\alpha} \qquad (9)$$

The author has shown that the stability of slopes thus drawn in profile remains perfectly assured, however far their ridge A may be placed on the curve OX. Here then are curves presenting at every point a slope greater than φ, and nevertheless presenting no limitation in respect of dangerous height.

It may be observed that the case is similar for the isolisthenic profiles more recently discovered by the author but their ordinates are hardly more than 2/3 of those of the present profiles. These appear to have a promising future in respect of very deep trenches where they seem to obtain - except for slight theoretical corrections indicated below the most economical profiles imaginable.

The author shows further that the slopes thus drawn in profile may, moreover, serve as abutment against very considerable thrusts, whereof the components of the resultant,if suitably distributed on the vertical of the ridge, are,

Parallel to Ox, $R_x = Cx$

Parallel to Oy, $R_y = Cy$

It will thus be possible, more particularly, to adopt mixed profiles, composed of an ortholisthenic curve surmounted by a rectilinear and more inclined embankment. In fact such profiles are much nearer than place embankments,to those natural profiles actually encountered during important levelling operations.

Finally the author has been able to determine, in certain cases concerning maximum ortholisthenes, an upper limit (very approximative be it said) of the error made by considering the factor m as unity. He has given to the profiles thus obtained the name of '<u>maximum rectified ortholisthenic profiles</u>'.

He shows that in a particular case the error resulting from adoption of simplified profiles is at greatest 8% and acts in favour of stability; it can thus be included in the margin of security which it is in any case essential to conserve in practical application of problems relating to the strength of materials.

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2) From the Greek ἴσος ,equal and ὀλισθαινειν , to slide.
3) Report to the Academie des Scienees, 11 June 1928 p.1597.
4) From the Greek ὀρθος, straight, ὀλισθαινειν , to slide.

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I e 20

GENERAL THEORY OF THE BEARING CAPACITY OF PILES

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SYNOPSIS OF THE FRENCH REPORT

We have already given the mathematical elements for the calculation of the bearing capacity of piles x). They are founded on an approximate value of the passive soil pressures. The complete calculation of passive pressures tables that we have just finished now makes it possible to obtain a firmer grip of the elements of the problem.

The bearing capacity of a foundation in a medium of internal friction angle φ and specific weight ω is the sum of three terms :

1) A surface term expressed by $s\,\omega\,\rho\, f(\varphi)$ proportioned to the mean radius ρ and the foundation surface S, an invariable term whatever the depth, remaining alone where penetration is nil.

The numerical calculation of the passive pressure, exerted by a soil with a free horizontal surface on a screen situated in the plane of the free surface gave us for f (φ) the following expression:

$$0{,}384\ tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\left(e^{4{,}55\ tg\varphi}-1\right)$$

For piles this term is negligible compared with those that follow, because the mean radius ρ is very small when compared with the depth h.

2) A pile point resistance term of the form $s\,\omega\,h\,g(\varphi)$ proportional to depth h.

3) A lateral friction term of the form

$P.\frac{1}{2}\,\omega h^2\ K\ tg\,\varphi$ proportional to the square of depth h, P being the perimeter of a section and K the normal component of the soil passive pressure acting obliquely on a vertical screen from a mass with a free horizontal surface.

The numerical calculation of the passive pressure table has shown us that K can be represented by the expression :

$$K = e^{19/30\ tg\varphi}\left(4 + tg\,\varphi^{\frac{2}{3}}\right)$$

from which one deduces for $g(\varphi)$ the expression

$$\left(I + 0{,}32\ tg^2\varphi\right) tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) e^{\pi tg\varphi}$$

It differs from the classical expression

$$tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) e^{\pi tg\varphi}$$

in the multiplicator $\left(I + 0{,}32\ tg^2\varphi\right)$ xa)

This coefficient takes into account the fact that on the horizontal plane of the foundation, and continuing on either side of it, the stress exerted, owing to the soil above, is not vertical.

But, if the pile is driven into a soil of angle φ , topped by a soil of angle $\varphi' < \varphi$ this multiplying coefficient can be brought down to unity for $\varphi'=0$. So we therefore suggest the following general formula for the bearing capacity of piles:

Point resistance (1)

$$\omega\,h\left(I + 0{,}32\ tg^2\varphi'\right) tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) e^{\pi tg\varphi}$$

Lateral resistance (2)

$$P.\tfrac{1}{2}h^2\ tg\varphi'\, e^{19/30\ tg\varphi'}\left(4 + tg\,\varphi'^{\frac{2}{3}}\right)$$

φ' and φ being the angles of friction of the soil situated respectively above and below the foundation plane.

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x) A. Caquot, Equilibre des massifs à frottement interne, (Equilibrium of large bodies with internal friction)- Paris, Gauthier-Villars 1934.
J. Lehuerou-Kerisel, La force portante des pieux (The bearing capacity of piles) Annales des Ponts et Chaussées 1938 No 21.

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xa) The value of this coefficient is 1,32 for $\varphi = 45°$ Here is the explanation of the DELFT laboratory experimental coefficient 1.3