SECTION I

THEORIES, HYPOTHESES, CONSIDERATIONS OF GENERAL CHARACTER

GENERAL REPORT

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A. INTRODUCTION.

The congress programme does not set any definite questions, as usual in other international engineering congresses, but leaves the participants perfectly free in the choice of their themes and as to how they will present them. Thus the section dealing with theoretical papers and designated I, comprises the whole extensive branch of soil mechanics, from natural-historical considerations in the borderland of geology, through the physical-chemical investigation of soil properties and the descriptive and analytical treatment of definite soil mechanics phenomenon, up to the solution of concrete problems of engineering science.

In order to obtain a certain degree of system in classifying the wide range of section I, the papers received have been divided by the congress secretariat into sub-sections, ranging from papers on general subjects to papers on special subjects.

No wide-reaching conclusions concerning the topical interest of the different problems of soil mechanics can be drawn from the repartition of the 59 contributions of the section I among the sub-sections. For, the individual moment and especially the literary productivity of the separate investigators both play an essential part. Nevertheless it is possible to ascertain the tendency towards a better contact of soil mechanics with neighbouring sciences, and a better investigation of the fundamental physical phenomena underlying soil properties.

The great abundance of the material received (ca. 600 pages apart from the figures) makes it impossible for the reporter, within the scope of his report to go deeply into the separate papers or to take up any critical position regarding them. He therefore confines himself to give a short resume of the papers without passing any judgement on their value, and it must be left to the participants in the congress to extract what is interesting for them, and if desired, to ask for further explanations during the discussion.

In the following survey the papers of section I are treated in groups, in accordance with the classification mentioned above.

B. REVIEW OF THE SEPARATE PAPERS.

a. GENERAL CONSIDERATIONS.

Historical.

The papers placed in section I are introduced by an historical study by Jacob Feld (Ia 1). In a copious list, the publications dealing with the field of soil mechanics, some of which are over 100 years old and occasionally very difficult to obtain - are assembled and discussed. In the second half of the 17th century, investigators and engineers were interested above all in soil classification and identification, lateral earth-pressure theories, problems relating to pressure on bin sides and bottom, pile foundations and soil physics. The paper contains a list of 50 of the oldest publications from the field of soil mechanics accessible to the author; they begin with an article dated 1679 and end with one dated 1835

Mathematics.

As in other branches of technique, soil mechanics makes use also of mathematics as an assistant in solving theoretical and practical problems. Since the classic analytical methods are often too clumsy or prove absolutely useless the relaxation method has recently been introduced into technique. In his paper (Ib 1) G. de Allen describes the principal of these methods, which in a short time gives approximate solutions to the desired degree of accuracy for any particular case.

b) GEOLOGY AND SOIL MECHANICS.

In this section come papers which deal with the relations between geology and soil mechanics or their supporters, the geologist and the engineer, and also with papers on soil mechanics problems and the working methods from the geological standpoint.

A. von Moos and L. Bjerrum (Ib 2) give, along with numerous literary references, a short survey of the development of soil mechanics in the last 10 years. They show that this was for a long time characterized by the existence of two tendencies almost independent from each other, one purely theoretical and the other experimental and practical. Furthermore that, in recent times the need for close collaboration between the engineer and the geologist is always becoming more operative, which also finds expression in technical publications.

W.J. Turnbull and H.N. Fisk (Ib 3) describe how cooperation between the geologist and the engineer may be organised so as to be fruitful in practice. This is based on their experience gained on the Mississippi River Commission in the lower Mississippi Valley Division Corps of Engineers. The way to success here lays in mutual understanding between two special branches, in the clear delimination which led to a full understanding of the delineation of the responsibility involved, the work being solely done on the basis of common-sense judgement.

As the result of a journey through seven continental countries Hugh Q. Golder (Ib 5) deals with the methods of working adopted in soil mechanics. He comes to the conclusion, that a surprising variety is to be sought for in the variety of the problems to be solved, which in their turn are influenced by geological conditions. The working methods adopted by the different laboratories can, therefore, only be correctly judged when geological conditions are known.

While the engineer considers the properties of soils as given, Chas B. Hunt (Ib 4) endeavours to explain them by the history of their geological development. He regards above all five factors as controlling soil formation, namely the climate, the kind of parent material, the topographic position, the plant and animal life, and the length of time each of these four factors has been operative.

Similar reasoning is followed also by Hans F. Winterkorn (Ib 1), who endeavours to call the attention of engineers to the methods of pedology. This science has been developed from agriculture, which is interested above all in the uppermost layers of soil over extensive surfaces. The author states that with suitable adjustments the pedology is also useful for the engineer, particularly when designing and carrying out extensive works such as railways, roads and aeroplane run-ways. The pedological problems in engineering are described as follows:
Characterization of soil systems: classification of soil systems; mapping of soil areas.

c. PHYSICO-CHEMICAL PROPERTIES OF SOILS.

The need for collaboration between soil mechanics and natural sciences is shown not only in the border territory of geology treated under , but above all in the investigation and explanation of certain properties of the soil. These should be attributed to as few physico-chemical fundamental phenomena as possible. As shown in the papers mentioned below, physics, chemistry or mechanics then come into consideration as auxiliary sciences, according to the type of problem involved.

Hans F. Winterkorn (Ic 3) deals with the physico-chemical phenomena governing actual soil properties and with collaboration between engineer and physical chemist. The latter has a theoretical and a practical task, namely to explain the soil properties, which are found, and to furnish the engineer with methods for improving them if necessary.

In a further paper (Ic 7) the same author deals with the behaviour of soils under the seasonal rhythmic influences of heat and moisture, which are of great importance in extensive constructions with flat foundations, such as roads and airport runways. The cause of these processes is theoretically investigated; in conclusion it is pointed out, that a knowledge of them is of great practical importance for planning the constructions.

Ralph E. Grim (Ic 4) describes the fundamental factors which influence the properties of soil materials, namely the content of aluminous minerals and of organic admixtures, as well as of adsorption water. Thus it is shown that even small quantities of aluminous minerals may influence the properties of an aggregate very greatly. The organic admixtures are considered in connection with their influence on water adsorption, plastic behaviour and other properties. The manner of the adsorptive binding of the soil water is investigated in conjunction with the plastic properties of the soil.

R. Spronck (Ic 6) starts from the consideration that the soil qualities are dependent on permanent characters such as physico-chemical composition, shape of particles, etc. and on non-permanent characters, such as structure and moisture content. He suggests a method which allows the non-permanent characters to be neglected when comparing the properties of different soils.

Miles S. Kersten (Ic 5) publishes the results of thermal conductivity tests which were made on 14 different soils. They give information regarding the effect on thermal conductivity of mean temperature, moisture content, freezing, density, particle size and shape,and mineral composition.

R. Haefeli and G. Amberg (Ic 1) deal with the problem of the shrinkage of clay which is important both with respect to soil mechanics and to technology. The shrinking process is described and represented mathematically in the form of a few fundamental equations. It is shown that the theory agrees to a satisfactory extent with the results of the tests. The authors' investigation led to the conclusion that the degree of three-dimensional shrinking at a certain moment in the shrinking process depends not only on the water content still present at that moment, but also on the preparing-water content.

R.K. Schofield and H.L. Penman (Ii 1) investigate how the soil moisture content varies with time. The concept of soil moisture deficit introduced by the authors is of particular interest hydrologically and in agriculture.

d. CONSOLIDATION.

With the subsection "consolidation" we enter the real field of actual soil mechanics. The problem of consolidation, which has attracted the attention of a number of authors, consists in analysing the compacting process of soils in the course of time, and discriminating between the separate phases, conditioned by quite different physical phenomena.

E.N. Fox (Id 3) deals with the early stage of hydrodynamic settlements of layers of soil from a purely mathematical standpoint. He shows that the form of Fourier solution for calculating settlements - the form mostly adopted on the basis of the law of porous flow formulated by Terzaghi - does not converge well in the early stage. He suggests an alternative form of the solution which is better suited for practical work. The limits of its adaptability are determined by convergence arguments.

When evaluating consolidation tests, a certain amount of difficulty will be found in determining from the time-settlement curve the exact point, at which the primary consolidation ends, which conforms with the hydrodynamic law of Terzaghi and the point, at which secondary consolidation begins. This problem, which is important for determining the coefficient of consolidation, has been treated in quite different ways in two papers.

A.H. Naylor and I. Doran (Id 2) treat the problem purely mathematically and develop an analytical method, according to which the theoretically correct initial and final dial readings can be determined from the consolidation tests, through which the primary consolidation is defined. The method, which is one of trial and error, is compared with the known methods of Casagrande and Taylor and is fully demonstrated by examples.

R. Haefeli and W. Schaad (Id 6) deal with the problem from the soil mechanics side. After a short description of the three phases of the consolidating process short settlement, hydrodynamic settlement and post settlement-reference is made to the difficulties, incurred in making a sharp division in the transition from main to post settlement. Based on tests carried out for three years, the respective run of the settlement curve for different loads is investigated. Further it is shown that, with the suitable choice of methods, the hydrodynamic settlement can be decided, even with thick clay, by means of short-time tests lasting only a few hours.

T. Edelman (Id 1) gives a computation of compression during the hydrodynamic period; the same formula is used for the compression during the secular period, assuming the cause of the secular effect to be a hydrodynamic process of the adsorbed water.

The formula for the settlement of the clay cylinder, out of which the water can issue at the top and the base, is obtained by approximately assuming that the resulting flow through the top and the bottom surfaces may be regarded as super-position of two opposing flows from the semi-infinite space towards these surfaces. Nomograms are given for using the formula.

A.W. Koppejan (Id 8) takes up the problem of expressing the secular settlement in the hydrodynamic and in the secondary phase by one single mathematical formula, i.e. by combining the Terzaghi and the Buisman formulae. The author shows, that with suitable choice of the zero of time and by assuming a logarithmic time scale, all time settlement-curves become straight lines, which are asymptotes of the Buisman curve. A straight line corresponds to each pressure, all straight lines passing through one common pole; in this representation the pressure becomes parameter. If the pressure is plotted as abscissae in logarithmic scale, the load-settlement lines also become straight when time (1 day, 10 days) is chosen as parameter. Based on these facts the author has succeeded in combining the time and the pressure effect in one single analytical expression. The utility of the formula has been confirmed by numerous laboratory tests. Its practical adoption is shown in an example of calculating a settlement.

Quite new paths are struck out in the theoretical treatment of the consolidation process by E.C.W.A. Geuze and C.M.A. deBruyn (Id 7) in that they consider the problem from the standpoint of energy. The conclusion drawn from theoretical considerations, i.e. that the work done by the external forces during the consolidation process must be greater in cases where the increase in load is slow than in case where the load is increased quickly, was confirmed by experiment.

R. Haefeli (Id 4) investigates in a further paper the influence of preconsolidation on the compressibility of a soil. First of all with the help of an ideal model, the logarithmic construction of the pressure consolidating law is determined analytically. Then information is given regarding the results of tests with two different clays. Accordingly, the secondary coefficient of compression, designated $\Delta e'$, is with constant relieving stress practically independent of the magnitude of the preconsolidation load; on the other hand, with given preconsolidation load the material is all the more compressible, the more it is relieved before reloading. Since it is impossible when removing a sample from the natural layer to avoid a certain relieving, the author advises great precaution when evaluating oedometer tests. As a complement of laboratory tests, the compressibility of the soil should therefore be determined, also in the field, by means of direct loading tests.

J. Mandel (Id 9) deals with deformations of the soil occurring in consequence of external loads, taking into consideration the fact that the settlement process is not reversible. In addition to the two material constants E (Young's Modulus) and σ (Poisson's Ratio) hitherto used, still two other constants for the swelling are introduced, namely E^1 and σ^1. The mathematical theory developed under this assumption is supplemented by examples.

e. <u>SHEARING STRENGTH AND EQUILIBRIUM OF SOILS</u>.

1. <u>Shearing strength</u>.

Besides the compressibility, shearing strength is one of the most important fundamental phenomena of soil mechanics. It is closely connected with the mineral constitution, the particle size and shape and with the hydrodynamic condition of the loaded soil; it is in close relation to the plasticity.

In a purely theoretical paper P.A. Coenen (Ie 21) gives fundamental equations for the equilibrium limit of earth masses.

Takeo Mogami (Ie 1) deals in his paper with the flow phenomena taking place during a shearing test and with the pressure and volume changes connected therewith in soils without cohesion. Based on theoretical considerations of strength he has succeeded in explaining the alterations in the angle of friction observed during the shearing process in sand and in deriving the criterion which decides whether extensive flow regions or a sharply defined sliding surface will form. He comes to the conclusion that Coulomb's law is valid only in the case of steady motion of sand and with the formation of a definite sliding surface.

To supplement the shearing theory R. Haefely (Ie 12) has investigated the behaviour of coherent soils over the whole range between the dry and the saturated states. He found that in the unsaturated state, shearing strength is dependent on the water content and has for coherent soils at a certain value a maximum, from which it rapidly falls away when the water content is further increased until saturation is reached The water-content shearing-strength diagram shows a hysteresis loop, which means that also the direction of the altering of the water content influences the strength. In the second half of the paper the shearing strength in the saturated state is investigated. The constant decrease in the water content found during the shearing process is justified theoretically.

Ch. Schaerer, W. Schaad and R. Haefeli (Ie 17) discuss a general theory of shear for soil saturated with water, which has already been briefly mentioned in paper No. Ie 12. It was found that on the one hand the angle of the true internal friction (φ_r) and on the other hand the angle of the apparent internal friction (φ_s) are practically constant for relaxed pore water. Both the cohesion and also the shearing strength for constant water content are conditioned in the first place by the equivalent compacting pressure or by the preloading of the soil, whereby the water content appears as parameter. The authors show that their theory also agrees with the results obtained by other investigators, in so far as their tests can be suitably evaluated. From the experimental point of view the fields of application of the ring shearing apparatus and of the tri-axial apparatus are limited with respect to each other.

The field of plastic deformation and of plastic strength of soils, which has hitherto, been little investigated, is dealt with theoretically by Kano Hoshino (Ie 10). The author bases his investigations on the assumption that the strain energy is proportional to the mean normal stress and not, as shown by the theory of elasticity, to the square of the stress. He deduces fundamental relations between initial and main stresses and strain energy, and further between stress and strain; he gives a general plastic condition for breakage. Finally the form of the Mohr envelopes for plane and three-dimensional states of stress is mathematically justified. Not always straight lines are obtained, but also ellipses, parabolas or hyperbolas, depending on the conditions.

The paper submitted by C. Torre (Ie 16) is in a certain sense supplementary to the foregoing paper. The author investigates the state of stresses in a semi-infinite mass, whereby he assumes a bent envelope instead of the Rankine boundary condition $\tau = \pm \mu \sigma$ (for the Mohr envelope passing through the origin, in this case a straight line). Based on this assumption he finds, with a parabolic envelope, for the shearing lines at a great depth, a set of straight lines inclined at 45° and standing at right angles to each other, which are bent upwards in such a way that in the active state they are at right angles to the soil surface, and in the passive state tangential to it.

József Jáky (Ie 8) shows also that the envelope curves of the Mohr circle often deviate greatly from straight lines. He too gives a law for the state of breakage, which takes these conditions into account. The validity of Coulomb's law of stability is submitted to a critical examination. The form of the function $t = f(n)$ (relation between shearing and normal stress) is greatly influenced by the shape of the sliding surface and by the sliding movement itself.

2. Equilibrium of soils.

The shearing strength of soils displays itself practically in a most striking manner in the maintaining of the equilibrium of earth masses. Problems of the equilibrium of soils are treated in the papers mentioned below.

A.W. Skempton (Ie 6) submits the $\varphi = 0$ analysis to critical consideration. This method, often adopted when calculating earthwork, is based on the fact that the angle of friction becomes zero when the pore water is subject to hydraulic over-pressure. The author shows that the $\varphi = 0$ analysis gives correct results when made immediately after the construction of earthwork, i.e. when the hydrodynamic equilization of stresses has not yet started, although with compression tests, from the position of the shear planes it is found that also in a saturated state of clays the friction does not always quite become zero. Because of this residual friction, the method gives the position of the shear surfaces in the limiting state of equilibrium incorrectly. It is also not suitable for cases where the clay soils adopted for building-purposes are not saturated with water, since the angle of friction is then not zero. Finally the method is illustrated by practical examples.

The improved process described by H.R. Raedschelders (Ie 5) for calculating the equilibrium of earthwork slopes depends on the fact that the masses of earth bounded by the surface of the slope and by the circular-shaped shear surfaces, and kept in equilibrium by the shearing strength, are no longer divided into separate parts but are considered as a whole. The adoption of the method is demonstrated by means of an example.

The objective of the paper of D.P. Krynine (Ie 22) is to give a general qualitative picture of the systems of shearing stresses that develop in a disturbed earth mass, without using the theory of elasticity. As disturbance are considered the unsupported vertical slope and the circular tunnel without lining.

L. Bendel (Ie 15) treats analytically the problem of protracted sliding surfaces such as occur particularly in landslides. He first of all deduces a slide-equation of general validity, and then deals with a great number of special cases for which the conditions of equilibrium have been given.

M. Frontard (Ie 19) contributes an extensive theoretical paper concerning the shape of the sliding surfaces and calculations of the stability of earth masses, which far exceeds the usual scope of congress contributions and which could not be studied by the reporter in the short time available.

The author deals with the following chapters:
Vertical stress p exerted on an element of the surface.
General equations of sliding curves.
Simplified sliding curves and profiles of slopes
Possibility of integration of the simplified equations in some special cases.
Study of some modified profiles.

Another stability problem, which differs essentially from the foregoing in the questions set, has been dealt with by R. Haefeli (Ie 3) He derives a general equation for the equilibrium of slopes influenced by parallel seepage. A plain solution of this equation is reached by expressing the critical angle of the slope with the aid of the hydraulic gradient along a line perpendicular to the slope. The practical adoption of the method is explained with the help of a diagram.

3. Bearing capacity.

A further adoption of the shearing theory is in calculating the bearing capacity of the soil. This problem is dealt with in the following papers.

E. de Beer and M. Wallays (Ie 4) make a comparison between the assumed known formulae of Prandtl, Buisman and Andersen for the calculation of the bearing capacity of building ground. They show that Andersen's formula, which assumes circular shaped surfaces of sliding, holds good only for foundations in which the depths are small in comparison to the width; on the other hand, Buisman's formula based on the assumption of symmetrical sliding surfaces consisting of straight lines and logarithmic spirals, would hold good only for deep and comparatively narrow foundations. In the critical region lying between these, both formulae give values which are too low. The weight of the soil itself is considered in the formulae of Buisman and Andersen.

L. Marivoet (Ie 7) starts by pointing out that two conditions have to be considered in calculating the bearing capacity of foundations, namely that no breakage of the ground (shear failure) occurs and that no dangerous settlements take place. He shows that both the ultimate bearing capacity and also the settlement depend not only on mechanical properties of the soil, but also on the size of the loaded area, its shape and its location with reference to the surface of the soil. In order to enable the two problems (settlement and stability) to be solved in as short a time as possible, the author develops diagrams for some particular cases of loading, thus considerably simplifying the adoption of the formulae.

T. Mizuno (Ie 13) calculates the bearing capacity of lines of foundations under the assumption of symmetrical sliding surfaces, which consist in the usual manner of straight lines and of curves connecting them, the weight of the mass of earth taking part in the sliding being also considered. A diagram is given for the practical application of the derived formulae.

Takeo Mogami (Ie 2) deals with the bearing capacity of clay foundations under the assumption that the soil is in a visco-elastic state of stress. This is characterized by the stress being dependent on the strain and the strain velocity.

Lujo Suklje (Ie 14) shows that the form of the sliding surfaces can be determined on the basis of two different extreme assumptions, namely that the gliding begins in an element of the surface where the shearing stress is a minimum, or in the direction in which the difference between the shearing stress and the shearing resistance shows the minimum. With the help of the law of the straight-line propagation of force in the plane semi-infinite, the extreme positions of the sliding surfaces are calculated from these assumptions and by utilizing the Coulomb law. In the case of elastic soil they suit well to the logarithmic spirals.

In the paper by F. Szelagowski (Ig 2) known publications for calculating the depth of foundations are assembled and discussed. Based on Michell's formula for determining the stress under a line foundation, and taking into account the Rankine and Coulomb conditions, simple formulae for calculating the depth of foundations are derived for coherent and granular soils.

József Jáky (Ie 11) investigates the shape of the sliding surface in the neighbourhood of the points of loaded piles. He finds for this a bulb, and comes to the result that the bearing capacity is greater than according to the theory of Prandtl regarding resistance to punching.The author also derives expressions for the minimum depth to which the pile has to be driven in order to develop a complete bearing bulb and determines the minimum distance necessary between the piles, so that the sliding surfaces do not intersect.

Based on work published previously, A.Caquot and J. Kerisel (Ie 20) develop a general theory regarding the bearing capacity of piles.

f. EARTH PRESSURE.

The earth pressure problem has been dealt with particularly by József Jáky (If 3, If 4, If 1)

In his first paper (If 3) the author states the analytical principle of the minimum value of the earth pressure based on Terzaghi's investigations. In addition to the wellknown Coulomb principle, according to which the sliding plane sets itself at the angle at which the earth pressure becomes a maximum, a further condition has still to be included, i.e. that with the sliding plane thus given, the angle of direction of the earth pressure sets itself so that it becomes a minimum. This combination of the maximum and minimum principles finds its graphic expression in a saddle shaped surface. A simple graphic solution is given for determining the earth pressure given by this theory and also its direction of action.

In a further paper (If 4), the author calculates the distribution of the specific pressure on a retaining wall which follows a power parabola, and also the exact point of attack of the pressure. The new theory confirms that the earth pressure of trench walls on the strut timbering does not continue to increase linearly from above to below, but attains a maximum at a certain depth.

The third paper (If 1) of the same author deals with the silo problem. Under the assumption of plane sliding surfaces an exponential law is found for the normal and tangential stresses acting on the silo walls. It is pointed out that the pressure depends not only on the depth but also on the diameter of the silo. Finally particulars are also given regarding the angle of internal friction and the wall friction for different agricultural products.

Curt F. Kollbrunner (If 2) investigates the stressing of the sheet pile walls in cohesive soils, basing the calculation on different assumptions respecting the run of the active and the passive earth pressure. He comes to the conclusion that the usual theory of sheet pile walls developed by Jakoby requires correction. The new method of calculation proposed by the author allows the cohesion to be taken into account mathematically or graphically.

g) STRESS DISTRIBUTION SETTLEMENT

The problem of the distribution of stress in building ground and of the calculation of settlement has greatly gained in practical importance in recent years, in consequence of the adoption of rational methods in planning roads and particularly in preparing the surfaces of airport runways.

József Jáky investigates the question as to whether the Terzaghi logarithmic law of compression and the Coulomb law of internal friction and cohesion are valid within the range of great pressures as they are at great depths of soil layers. Based on tests, he comes to the conclusion that this is not the case, and that the compression and also the shearing phenomena are better expressed by an exponential law. Further he calculates the increase in specific gravity of the soil caused by the pressure at great depths.

G.A. Oosterholt (Ig 1) shows in his paper that in consequence of the increase in the modulus of elasticity of the soil caused by pressure, the concentration factor also increases with increase in load.

K. Hruban (Ig 4) deals with the influence of the variableness of the modulus of elasticity on settlements. Basing on mathematical deduction, he comes to the conclusion that, with uniformly loaded circular surfaces, the modulus of elasticity does not increase with the depth linearly but according to an exponential function, the exponent being a fraction.

For various conditions of loading, the author develops formulae for calculating pressure distribution and settlements. He proposes, in calculating regularly stratified foundations, to replace these by a semi-infinite solid with a continually varying modulus of elasticity, thus making it more amenable to calculation.

In a further paper (Ig 5) the same author deals with the errors which occur in calculating settlements if the influence of horizontal pressure is neglected. These methods of calculation accordingly always fail if the horizontal stresses in nature differ from those in the oedometer test.

The problem touched on by the foregoing author concerning the influence of horizontal pressure on the settlement of foundations, is also dealt with by Donald M. Burmister (Id 5). He proposes a method according to which the settlements measured during laboratory compression tests are compared with those obtained by calculation of theoretical settlement under the assumption of power distribution on all sides. The ratio of these two values is introduced and designated "Natural Restraint Coefficient". Based on this method it is possible, from the results of the compression tests in the triaxial apparatus, to determine what settlements are to be expected in nature without having to carry out comparative loading tests in the field.

E.N. Fox (Ig 6) extends the solution given by Mindlin for calculating settlements occurring in consequence of a single load acting in the interior of the semi-infinite elastic medium to the case of deep rectangular foundations, loaded uniformly. The expression found is evaluated numerically and the result is illustrated in a family of curves which show how the mean displacement changes with the shape factor of the rectangle, and with the depth of the loaded area beneath the surface.

A.H.A. Hogg (Ig 7) develops an approximation method for calculating pavement slabs on non-rigid foundations. The method is based on the actual deformable soil being replaced in the calculation by an equivalent elastic layer on a rigid bed, which experiences the same deformations at the surface as the natural soil. Tests have been carried out, which show that the calculated settlements are not accurate, but are sufficient for practical purposes.

In the paper by Ralph E. Fadum (Ig 9) the problem of estimating normal stresses in quasi-elastic foundations caused by building loads is dealt with. First of all a brief review is given of contributions from the theory of elasticity. Then the influence values for the vertical stresses in the soil are assembled, both for the Boussinesq and also for the Westergaard formulae. These are based on cases of loading which often occur in practice; namely load applied at a point, load applied on a line of finite length, and uniformly distributed load on rectangular and circular areas. The influence values given, partly in tables and partly in diagrams are intended for practical use.

E. de Beer (Ig 3) deals with an approximate method for computation of beams resting on soil. This is based on the principle of successive approximations, in that the arbitrarily assumed sub-soil reactions in the foundation surfaces are altered until the settlements of the soil and the bendings of the foundation agree with each other. The method is adopted for the computation of rectangular beams subjected to a single central load.

In a work which extends far beyond the usual scope of congress papers, Henry Marcus (Id 10) deals with the following two problems with the help of the adoption of the theory of elasticity, namely the axial-symmetrical stress changes, produced by the sagging of a soft soil during consolidation, and the analysis of the strains required to induce, through skin-action, a part of the pile load into the surrounding soil and to develop the full resistance at the base of the pile.

h) VIBRATION AND MATHEMATICAL PROBLEMS

The problem of the dynamic stressing of building ground has been raised in one single paper, namely in that submitted by J.H. Crockett and R.E.R. Hammond (Ih 2) The two authors show that each ground has its natural frequency of oscillation, which may come into resonance with the vibrations from machines. They consider the oscillating system, composed of the foundations and of the so-called bulb of pressure lying under them, which acts as a yielding bed with a given weight of its own.

A more mathematical problem from the theory of potential flow is dealt with by Rob. Ruckli (Ic 2). For a circular shaped, and for an infinitely long rectangular cold store situated on ground level, the author derives simple formulae according to which the distribution of the heat flow from the sub-soil to the foundation-surface, and also the insulation or the auxiliary heating system can be calculated.

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SUB-SECTION I b

GEOLOGY AND SOIL MECHANICS

I b 6

WRITTEN DISCUSSION ON SUBSECTION Ib

J. ABOLLADE Y ARIBAU (Spain)

I go to write as brief as possible of some points suggested to me by the reading of papers presented at this Congress.

The first is connected with the relationship between Geology and Soils Mecanics and from this standpoint I think it may be interesting to give a brief history of the development of the geologic and foundation services of Spain and their actual organisation.

The amount of true loose grounds on Spain and detremped soils is on this country so little that the characteristic problems of Soil Mechanics are not very common, except on the harbours.

Among the more pressing questions, those of the impounding water with large dams and the rocky grounds are the most commons.

As the resistance of these is sufficient but impermeability very dubious the first organisation was a geological one, the Commission for Geological Studies of Hydraulics Works of the Public Works Ministery. This Commission was formed by Civil Engineers, Mining Engineers and University Doctors on Geological Sciences. At their orders were the sounding services, which worked upon their indication. This was the geological stade and it is to say that the complex commission does not work too smoothly.

It was suppressed but the service of soundings and foundation was conserved as a special service of the Public Works and its name changed. It was adjoined the words of geological reports, it is to say the complete name was Soundings Foundations on Geological Reports, which is its actual name. This organisation is the link with the last and actual stade.

At the moment by the side of this executive office there is a consulting one, the so-called Asesoria Geologica, whose mission is the theoric standpoint but it is not formed by more personal than Civil Engineer specialised on the engineering geology or on the construction problems. Notwithstanding it is considered the possibility of the appointment of some specialist of other class, universitary likely, on the cases of special theoretical difficulty.

This board not commands directly the service of foundations but indicates to the ministry the most convenient.

There is also other question on which the geological Board has no intervention and on which the Service of Soundings and Foundations makes its works directly and studies the problems of the soils with its proper resources. Generally it does so on bridges, buildings and Miscellaneous Work.

It only rests to present the actual staff of the two organisations to see at the best the relation between them.

The "Jefatura de Sondeos Cimentaciones e Informes geologicos" has a chief engineer for engineers in charge and eleven adjoint engineers.

The "Asesoria Geologica de Obras Publicas" has one chairman or President on the categorie of General Inspector of Civil Engineers and one Vice-President and a Secretary as permanent committee and nine Members. Five of these are also engineers of the "Jefatura" car at the two years of practice on the last they pass to form part of the Asesoria. The chief engineer of that is also member of the Asesoria if he has the two years of service on Soundings.

There are also in Spain other offices working on soils Mecanics and Geology, but isolately and on their proper duty. The School of Civil Engineers of Madrid has an outstanding development on investigation field, the Geological Institute on mining geology and on the geological map of Spain.

It is clear, in consequence, that the problem of the relationship of Soil Mechanics and Geology is an ancient one in Spain and at the moment the adopted solution is on the engineering side. The geological studies on the Public Works are made by specialised civil engineers only.

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SUB-SECTION I d

STRESS-STRAIN RELATIONS; CONSOLIDATION

I d 10

DISCUSSION ON PAPER Id 8

A.W. KOPPEJAN (Netherlands)

My intention was to consider consolidation quite extensively, but by lack of time I will confine myself. I hope Mr. Rückli will allow me to make only two remarks about his report of paper Id 8 (page 2 of the general report, last par.) He says there, that I took up the problem of expressing "the secular settlement in the hydrodynamic and in the secondary phase by one single mathematical formula".

Now the question is firstly that instead of "the secular settlement" it would be right to say "the consolidation process", which is subdivided in possibly an initial consolidation, in a hydrodynamic or primary consolidation, and in a secular or secondary consolidation. But even then I must say that I have not tried to express the hydrodynamic settlement and the secular settlement in one single formula, for this seems to be impossible, due to the fact that probably secular effects complicate the hydrodynamic consolidation starting from the very beginning, namely in any way according to the increase of the effective pressures. What I have done is in reality the extension of Prof. Terzaghi's load-settlement formula in order to include the late Prof. Keverling Buisman's logarithmic secular time-settlement formula, namely in those cases where no final settlement due to any finite load can be observed. Applying this new formula, an approximation of the hydrodynamic retarding effect during the first phase (that is a function of layer thickness) must, if so required, be given separately.

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SUB-SECTION I e

SHEARING STRENGTH AND EQUILIBRIUM OF SOILS

I e 21

DISCUSSION ON PAPER Ie 11

A.J. COSTA NUNES (Brasil)

Some experiments, (very few indeed) of other authors on static formulas, Ferrandon and Mayer, for instance, seem to indicate that the pressure developed around the point of a pile does influence the value and the distribution of the pressures on the lateral surface of the pile.

It is not easy to admit that, in spite of the existance of bulb pressures involving an important length of pile, as shown in figures 3 and 4 of the paper on discussion, the pressure of the earth on the lateral surface of the pile continues to maintain the linear growth with the depth.

In the case of long piles for which the author adopts the Prandtl Caquot's formula (formula 1a) it does not seem very clear either, how the rupture surfaces of fig. 1 stop at a depth h, without propagating themselves until they meet a zone of discontinuity.

In the considerations regarding the economic length of piles, pg. 102, the author seems to ignore the skin friction, which, in a homogeneous soil, may constitute a substantial part of the total resistance.

Finally, the author's conclusion, that the base resistance on sand is several times larger that that on clay, is probably limited in its application to very soft clays. We should ask ourselves if, on account of the eventual rapidity of the growth of the load, it might not be permissible to suppose that the experiences that give for a pile driven into the clay a bearing capacity 25 times less than that of the same pile driven into the sand, may be interpreted considering only the effective pressure when computing the contribution of the internal friction to the same bearing capacity.

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I e 22

DISCUSSION

J. MANDEL (France)

On reading the Proceedings of the Congress (sections Ie, f, g) it appeared to me, that the mathematical works of the French school on the subject of the equilibrium limit of soils were insufficiently known. For several years there has existed a mathematical solution, almost complete, of the problem of the plane equilibrium limit of a pulverulent or coherent body, when the intrinsic curve of the material is taken to be independent of the deformations it undergoes, which allows the conception of an equilibrium in which, at all points, Mohr's circle is tangential to a well-defined intrinsic curve. The only theoretical difficulties are those presented by the problems of where the displacements are imposed on the contours of the body and the problems of mixed equilibrium, superabundant in certain zones, restricted in others.

The fundamental property dominating the question of equilibrium limits is that the characteristic lines of the equations of partial derivatives which define these equilibria are slip lines. This property has extremely important consequences. It had already been indicated by Kötter for the rectilinear intrinsic curve usually accepted for soils. I rediscovered it and published it at the beginning of 1938, without knowing Kötter's work and moreover, for absolutely any intrinsic curve. Later Mr. Charreau extended it to the case of a non-homogeneous isotropic medium. Mr. Bonneau for his part, referred to it the "Annales des Ponts et Chaussées" for 1938 for the rectilinear intrinsic curve of soils.

One of the consequences of this property is the existence of a differential relationship, along a slip line, between the two parameters which define the stress: the angle θ of the greatest principal restraining force with a fixed direction and the parameter which fixes Mohr's circle, for instance, the abscissa n of the centre of the circle. We have:

$$\cos\varphi\, dn - 2\,R\, d\theta = U\cos\varphi\, ds_u$$ along a line u, inclined $\theta - \left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$

$$\cos\varphi\, dn + 2\,R\, d\theta = V\cos\varphi\, ds_v$$ along a line v, inclined $\theta + \left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$

U,V designating the oblique components of the mass force per unit of volume following the 2 characteristic directions, R the radius of Mohr's circle, ds_u, ds_v the elementary arcs of the lines u and v.

When these relationships integrate, without it being necessary to know the form of the

slip lines, it is possible to find the exact solution of a limit equilibrium problem from the restraining forces on the contours. This happens when there are no mass forces, and also for the forces deriving from a function of the forces W when
$\varphi = o$. In the first case we have:

(1) $\int \frac{\cos\varphi}{2R} dn + \varepsilon^{\theta} = C^{te}$ on the line

of inclination $\theta + \varepsilon\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$ ($\varepsilon = \pm 1$)

taking $R\, e^{\varepsilon 2\theta\, tg} = C^{te}$

if the intrinsic curve is rectilinear.
In the second case we have:

(2) $n - W + \varepsilon 2\, R.\theta = C^{te}$

In 1938 I showed how the network of slip lines could be determined. We therefore know how to solve exactly the following problems:
distributions of restraining forces in zones small enough for the gravity to be ignored;
distributions of restraining forces in ponderable coherent terrain, but devoid of internal friction. In this limited case the following problems: tunnels, retaining walls, banks, hoppers, deep foundations, are capable of exact solution.
We are left with the case of ponderable terrains with $\varphi \neq o$. This case is not exactly integrable and we must have recourse to successive approximations. Résal, Mr. Caquot, and Mr. Bonneau have pointed out the way to be followed. In his work "Poussée des terres", Mr. Ravizé has given us the results of his long and precious calculations, which complete Résal's and indicate to us the coefficients of thrust.
Although all these works relate to the ideal case of a soil whose characteristics are in all respects the same and not modified by slips which originate locally, they should serve as a guide to the engineer in the field of strong stresses, just as the theory of elasticity guides him in the case of weak stresses.
Is the foregoing theory absolutely inaccessible to engineers not broken to the rigours of mathematical anslysis? Let us judge by the two following examples:

1. Stamping - a slip line runs from P to M, at M, θ and n are known. At P, θ is known from the factor of the angle of friction on the tube. Hence the pressure at P by the relationship (1). If the rectilinear intrinsic curve is taken to be:

$$R_M = (p + H)\frac{\sin\varphi}{1 - \sin\varphi} \qquad \theta_M = \frac{\pi}{2}$$

with $H = C \cot g\, \varphi$, where C is the cohesion.

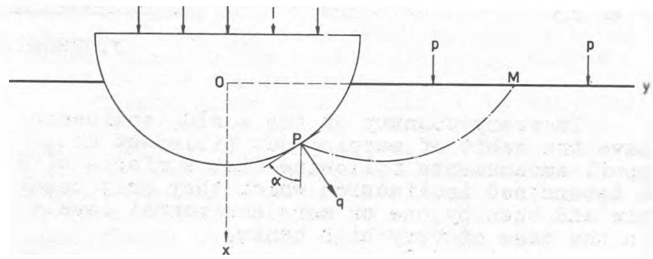

FIG. 1

If we take the wall without friction

$$\theta_p = \frac{\pi}{2} - \alpha$$

whence: $R_p = (p + H)\frac{\sin\varphi}{1 - \sin\varphi} e^{2\alpha tg\varphi}$

and the pressure q on the wall is given by

$$q + H = R_p \frac{1 + \sin\varphi}{\sin\varphi} = (p + H)\frac{1 + \sin\varphi}{1 - \sin\varphi} e^{2\alpha tg\varphi}$$

For $\alpha = \pi/2$ we have Mr. Caquot's formula for foundations.

2. Dangerous height for a flat bank with a slope of i for a terrain devoid of internal friction ($\varphi = o$).
At the two extremities P,M, of the slip line (line v), θ and n are known. The relationship (2) where $W = \Delta x$ (Δ specific gravity) fixes the value of the dangerous height h.

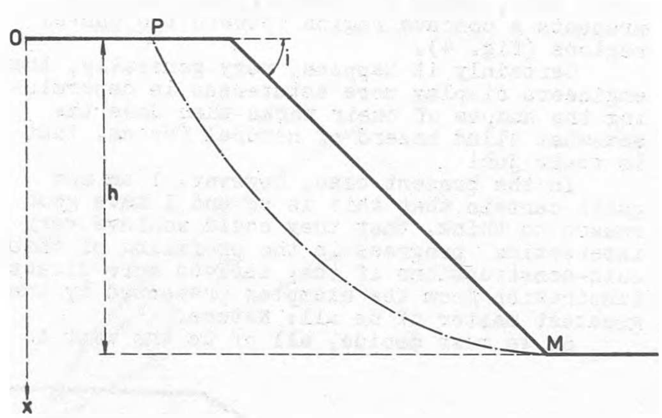

FIG. 2

at P we have: $n = - R_o$ $\quad \theta = o$ $\quad x = o$
at M we have: $n = + R_o$ $\quad \theta = \frac{\pi}{2} - i$ $\quad x = h$

whence: $h = \frac{R_o}{\Delta}(2 + \pi - 2i)$

This is the formula Mr. Frontard gives for the case where $\varphi = o$.

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I e 23

DISCUSSION ON SLIDING SURFACES

J. FRONTARD (France)

In every country of the world, engineers have the habit of mapping out filled-up or spoil embankments following flat surfaces with a determined inclination which they only break now and then by one or more horizontal levels in the case of very high banks.

Figs. 1 and 2.

This conception is legitimate for embankments made in sandy and gravel soils. When one thinks of it, it should appear as quite surprising for those made in clayey or clay-sandy soils, for in that case it is in direct opposition to all we have observed in nature. I do not know whether any of the engineers, all of them specialists in the study of soils, who are attending this conference, have ever seen hills of a geological clayey or clay-sandy deformation, which have been given by nature slopes whose inclination, however small, affect the form of plane surfaces. I only know that I have never seen any. Actually, we see everywhere hills with a similar structure which at their bases at least have shapes resembling whether those of a button head, the profile of which I have shown here for two cases which are almost equally frequently encountered. In the first of these two cases (fig. 3) the profile is convex everywhere and only offers a variable radius of curvature which generally runs transversely from the foot of the hill to its summit. In the second case the profile, convex, at all points of the lower part, bends not far from the top and presents a concave region between two convex regions (fig. 4).

Certainly it happens, very generally, that engineers display more astuteness in determining the shapes of their works than does the somewhat blind hazard of natural forces. That is their job!

In the present case, however, I am not quite certain that this is so and I have good reason to think, that they could achieve very interesting progress in the profiling of their soil-constructions if they derived more direct inspiration from the examples presented by the greatest master of us all: Nature.

So we must decide, all of us who want to master the technique of soil works, to take up unreservedly the study of the stability of banks with curvilinear profiles. This study is also necessary, and even more necessary than that of plane banks, because the latter are basically only a specific case which has the merit of a certain simplicity, but no other.

Such is the conception which has guided me in the mathematical investigations which I have presented to this Congress. Believe me, I am the first to deplore their apparent complication. It may be added that in effect the mathematical problems involved were extremely difficult. I certainly do not pretend to have solved them all and obviously much remains to be done by my colleagues and our successors to develop and improve upon these initial investigations. At least I have applied to them methods, which have already been proven in the special case of plane banks and these enabled me finally to extend them to any shape of bank with very saticfactory approximation and for these, I succeeded in determining a maximum limit, the equilibrium equations given by Résal 40 years ago for plane banks. It will be remembered that Résal's past experiments were directed in principle towards bodies with an inferior state of equilibrium, in conformity with Rankine's Law, i.e. fulfilling the condition that the restraining forces undergone by every plane element parallel to the bank should be vertical in direction, or inversely, that the restraining forces undergone by every plane vertical element (perpendicular to the profile under consideration) should be parallel to the bank.

I do not believe that Rankine himself ever intended generalization of this "Law", which bears his name and whose principal merit it is, that it simplifies many calculations, for the case of curvilinear banks. The principal fundamental innovation which I may have made as a starting point for my calculations, was to transpose it to the case of curvilinear

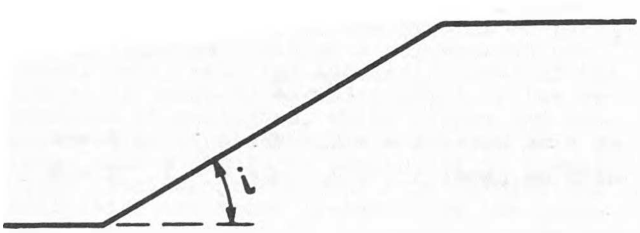

FIG. 1

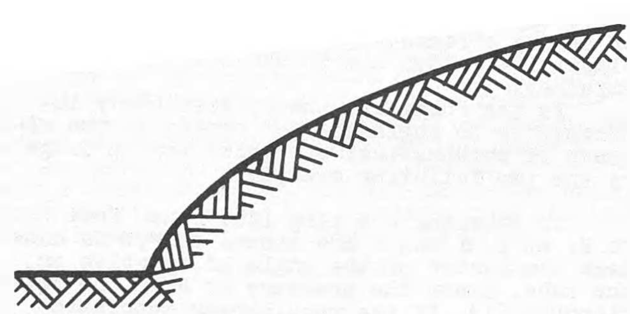

FIG. 3

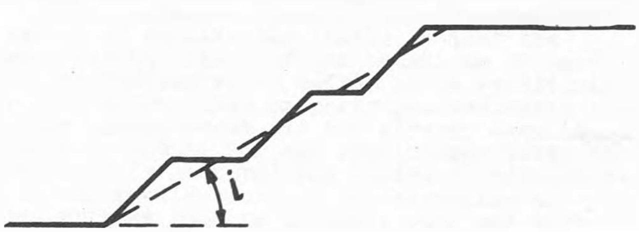

FIG. 2

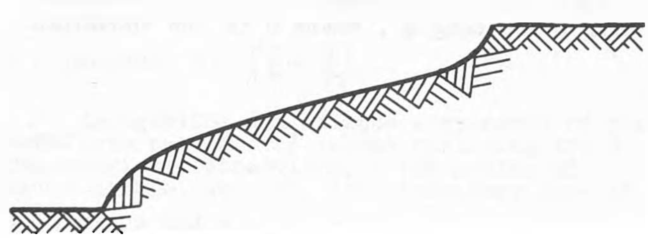

FIG. 4

embankments by translating it by the condition that the plane conjugated elements of the same vertical at the basis of the mass are all parallel to the free surface elements, of the bank traversed by the same vertical. This amounts to admitting as the basis of the calculations that we are concerned with a state of equilibrium in which the stress surfaces of the mass in question are identical in form to the free surface and can be deduced from that free surface by a simple vertical, descending translation. It is a hypothesis like any other and particularly for banks with a radius curvature, however slight it should not be considered any more condemnable here than in the case of plane banks.

That, quite naturally, is the basis of my calculations and it has led to differential equations which appear to be hardly more complicated than their elder sisters, already given for plane banks. Unfortunately in almost every case it is a question of appearance. In actual fact, with the exception of some particular, simple cases, they would lead to inextricable calculations, striking in their sterility in practical application. In this general case I have happily chosen the word "unfortunately" for the general case. But I am now going to say, that on the contrary they happily lead to very simple results in some special cases and these albeit few in number, appear by good fortune the most interesting conceivable.

To take the main feature of these, let us now return for a moment to the special case, the most special of all, that of the plane bank.

In what way do plane banks collapse? We are now beginning to know something about it, or at least to have less deeply erroneous ideas about it then formerly. We are no longer at the stage of the first research workers who based their calculations on the wholly theoretical determination of an inclined plane by which they wished to see the top part of the mass slide on to its lower part which had remained intact. We all know that on the contrary it is along a curved line that the slide takes place, the only point left at issue being that of the mathematical line which represents as generally no possible this curved surface. Is it a circle as first postulated by Mr. Fellenius, or an elliptical curve as thought by Mr. Caqout, a logarithmic spiral according to Mr. Rendulic's ideas. Undoubtedly I am forgetting some, but the engineers who are victims of my omissions, will pardon me, since I have forgotten still other curves and to which I bear the love of a father for his children. At all events the idea which guided me in the first place in the investigation of curved profiles was the following.

"Since plane banks collapse by slipping along a curved surface, let us take the inverse problem and let us see which curved banks collapse by slipping along a plane surface." This enabled me to arrive at the end of my calculations, first in 1928 by a fairly approximate solution and then for this very Congress by a collection of solutions which are notably more complete and of a verified mathematical exactness. To banks of such profile I have given the generic name of Ortholistenic banks, derived form the Greek words "ορθος ", which means straight and" ὀλισθαινειν ", which means "to slide". The most interesting of these are those, whose plane of sliding is inclined to the horizontal at an angle equal to the internal angle of friction of the soil (fig. 6). In the second place I had the gooa luck to find that the latter can attain heights considerably above plane banks without the risk of slipping.

Furthermore the famous vlot empirical rule "Never two without three" was again proven right. A third piece of good fortune showed me in addition that in certain cases of thrust they are capable of acting against relatively high thrust forces exercised on their top part, which may enable their height to be increased practically without the risk of slipping, by adding to their top part a plane bank suitably arranged (fig. 7).

From that moment this property was given special attention, or the capacity of the Ortholisthenic massifs was tested specially.

There is an interesting story attached to

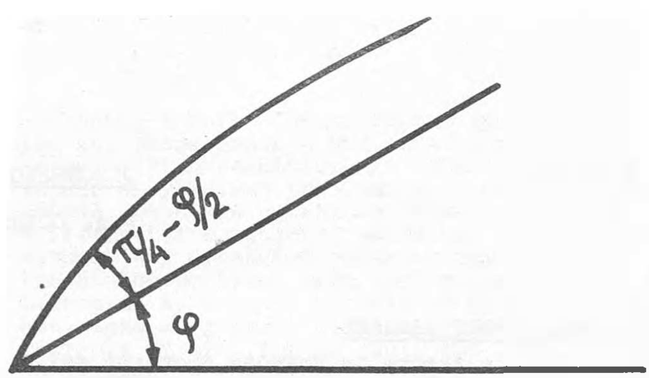

FIG. 6

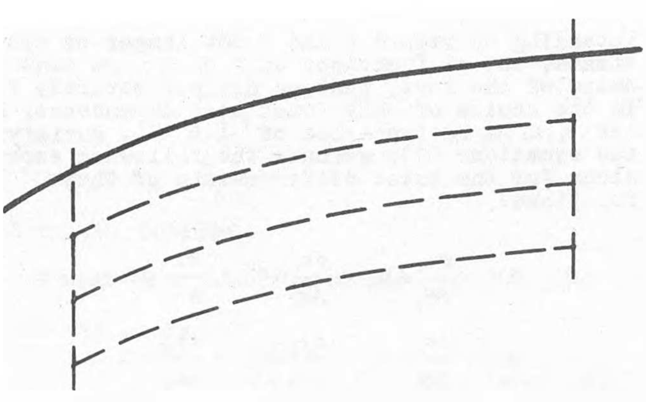

FIG. 5

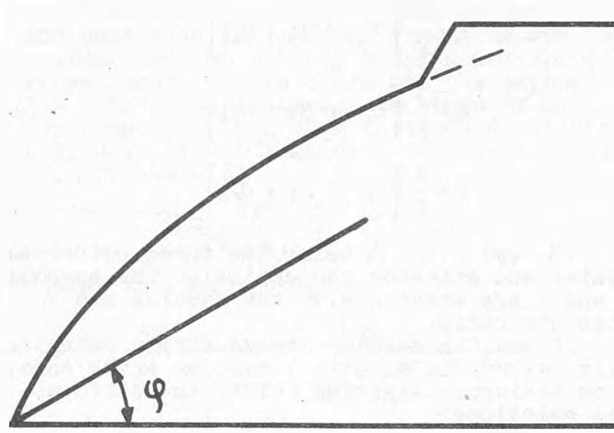

FIG. 7

this which I am going to tell you. In Burgundy there is an earth dike 17 metres high, constructed about 1900 as a feeding reservoir for the Burgundy canal. Its banks were built up of a succession of three inclined planes, the top one 1.5 base per unit of height, the middle one 2 base per unit of height and the lowest 2.5 d^o per 1 - d. Each of them was separated from the next by a berm. It was conceived and constructed by one of the best French engineers of the time, Mr. Galliot, and it was admired by all his colleagues.

Like others, I admired it at the time I began, but at a point of capital importance, the most important I would say, exactly the top part of his dike, it was not admirable at all and about 1920, the entire top portion of the bank which was 10 metres in height experienced a considerable slip, which caused the subsidence of several metres of its crown.

From that time I was impressed with the really low dimensions of this stone bedding and being too young and uninfluential to have my advice followed to double or triple the volume of these ripraps, I at least communicated my apprehensions to the then Chief Engineer of the Burgundy canal. I advised him always to maintain the water level in the neighbourhood of his maximum bank.

This well-meant advice was carefully observed by all engineers during the successive 25 years in the Burgundy canal service, not only because I gave the advice, but because having forced the water level about 5 metres above its maximum about 1925, the engineers discovered the opening near the top part of the dike, one which formed a prelude to the big landslide of 1920. They then issued formal orders to the weir-keeper, never to let the water level fall below 4 metres.

This situation offered serious inconveniences for water supplies to the Burgundy canal and in the course of the last few years, which have been particularly dry, it was decided to rectify matters. This was when, in my function of Inspector General I simply proposed to increase the capacity of thrust, hitherto very inadequate, of the riprap bed, by giving it the general shape of an ortholisthenic massif. Naturally I shall not give you the calculations which have enabled me to fix with certainty and with a margin of safety of the order of magnitude of 1.35 the dimensions of this massif. I shall only give you its general appearance which is nearly four times as voluminous as the poor massif which preceded it.

The work is just finishing this year for the feeding of the Burgundy canal and likewise for the tranquillity of the engineers of the canal, who were sometimes prevented to sleep by the disquieting situation of the bank.

You will likewise find in my memoirs presented to this Congress indications of other profiles of curved banks, which also, be it less than the ortholistenic profiles, show some advantages upon plane slopes. I finish my survey in expressing the wish that my investigations will draw others and that progresses will result from it in the art of profiling high slopes, which has no doubt remained somewhat practised up till now.

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I e 24

DISCUSSION ON PAPER Ie 10

P.A. COENEN (Netherlands)

1. PRELIMINARY REMARKS.

Elastic theory is founded upon the well-known combined Hooke-Poisson stress-strain relations:

$$\begin{aligned} \varepsilon_1 &= \frac{1}{E}\left\{\sigma_1 - \nu\,(\sigma_2+\sigma_3)\right\} \\ \varepsilon_2 &= \frac{1}{E}\left\{\sigma_2 - \nu\,(\sigma_3+\sigma_1)\right\} \qquad (1) \\ \varepsilon_3 &= \frac{1}{E}\left\{\sigma_3 - \nu\,(\sigma_1+\sigma_2)\right\} \end{aligned}$$

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\sigma_1, \sigma_2, \sigma_3$ being the three principal strains and stresses respectively. The symbols E and ν are constants, E the modulus and ν Poisson's ratio.

If now, in seeking stress-strain relations valid beyond the elastic limit, we write down, as an analogous starting point, the differential relations:

$$\begin{aligned} d\varepsilon_1 &= \frac{1}{E}\left\{d\sigma_1 - \nu(d\sigma_2 + d\sigma_3)\right\} \\ d\varepsilon_2 &= \frac{1}{E}\left\{d\sigma_2 - \nu(d\sigma_3 + d\sigma_1)\right\} \qquad (2) \\ d\varepsilon_3 &= \frac{1}{E}\left\{d\sigma_3 - \nu(d\sigma_1 + d\sigma_2)\right\} \end{aligned}$$

intending to regard E and ν not longer as constants, but as functions of $\sigma_1, \sigma_2, \sigma_3$, we must be aware of the fact, that we are not entirely free in the choice of this functional dependence. For, let $\varepsilon_1, \varepsilon_2, \varepsilon_3$ be functions of $\sigma_1, \sigma_2, \sigma_3$, satisfying the equations (2), we have the following expressions for the total differentials of these functions:

$$\begin{aligned} d\varepsilon_1 &= \frac{\partial\varepsilon_1}{\partial\sigma_1}d\sigma_1 + \frac{\partial\varepsilon_1}{\partial\sigma_2}d\sigma_2 + \frac{\partial\varepsilon_1}{\partial\sigma_3}d\sigma_3 \\ d\varepsilon_2 &= \frac{\partial\varepsilon_2}{\partial\sigma_1}d\sigma_1 + \frac{\partial\varepsilon_2}{\partial\sigma_2}d\sigma_2 + \frac{\partial\varepsilon_2}{\partial\sigma_3}d\sigma_3 \\ d\varepsilon_3 &= \frac{\partial\varepsilon_3}{\partial\sigma_1}d\sigma_1 + \frac{\partial\varepsilon_3}{\partial\sigma_2}d\sigma_2 + \frac{\partial\varepsilon_3}{\partial\sigma_3}d\sigma_3 \end{aligned}$$

which obviously must be identical with (2). Equating coefficients, we find:

$$\frac{\partial \varepsilon_1}{\partial \sigma_1} = \frac{1}{E} \quad \frac{\partial \varepsilon_1}{\partial \sigma_2} = -\frac{\nu}{E} \quad \frac{\partial \varepsilon_1}{\partial \sigma_3} = -\frac{\nu}{E}$$

$$\frac{\partial \varepsilon_2}{\partial \sigma_1} = -\frac{\nu}{E} \quad \frac{\partial \varepsilon_2}{\partial \sigma_2} = \frac{1}{E} \quad \frac{\partial \varepsilon_2}{\partial \sigma_3} = -\frac{\nu}{E}$$

$$\frac{\partial \varepsilon_3}{\partial \sigma_1} = -\frac{\nu}{E} \quad \frac{\partial \varepsilon_3}{\partial \sigma_2} = -\frac{\nu}{E} \quad \frac{\partial \varepsilon_3}{\partial \sigma_3} = \frac{1}{E}$$

relations, which must be fulfilled as conditions of integrability of the equations (2). Taking into consideration the first and the second columns of this scheme, we can derive by computing second partial derivatives in two ways, equations for E and ν as functions of σ_1 and σ_2 :

$$\frac{\partial^2 \varepsilon_1}{\partial \sigma_1 \partial \sigma_2} = -\frac{1}{E^2}\frac{\partial E}{\partial \sigma_2} = -\frac{E\frac{\partial \nu}{\partial \sigma_1} - \nu\frac{\partial E}{\partial \sigma_1}}{E^2} \longrightarrow \nu\frac{\partial E}{\partial \sigma_1} + \frac{\partial E}{\partial \sigma_2} = E\frac{\partial \nu}{\partial \sigma_1} \quad (3)$$

$$\frac{\partial^2 \varepsilon_2}{\partial \sigma_1 \partial \sigma_2} = -\frac{E\frac{\partial \nu}{\partial \sigma_2} - \nu\frac{\partial E}{\partial \sigma_2}}{E^2} = -\frac{1}{E^2}\frac{\partial E}{\partial \sigma_1} \longrightarrow \frac{\partial E}{\partial \sigma_1} + \nu\frac{\partial E}{\partial \sigma_2} = E\frac{\partial \nu}{\partial \sigma_2} \quad (4)$$

$$\frac{\partial^2 \varepsilon_3}{\partial \sigma_1 \partial \sigma_2} = -\frac{E\frac{\partial \nu}{\partial \sigma_2} - \nu\frac{\partial E}{\partial \sigma_2}}{E^2} = -\frac{E\frac{\partial \nu}{\partial \sigma_1} - \nu\frac{\partial E}{\partial \sigma_1}}{E^2} \longrightarrow \nu\left[\frac{\partial E}{\partial \sigma_1} - \frac{\partial E}{\partial \sigma_2}\right] = E\left[\frac{\partial \nu}{\partial \sigma_1} - \frac{\partial \nu}{\partial \sigma_2}\right] \quad (5)$$

Subtracting (3) and (4) we further find:

$$(\nu - 1)\left[\frac{\partial E}{\partial \sigma_1} - \frac{\partial E}{\partial \sigma_2}\right] = E\left[\frac{\partial \nu}{\partial \sigma_1} - \frac{\partial \nu}{\partial \sigma_2}\right] \quad (6)$$

From (5) and (6) we obtain easily:

$$\frac{\partial E}{\partial \sigma_1} = \frac{\partial E}{\partial \sigma_2} \quad \text{and} \quad \frac{\partial \nu}{\partial \sigma_1} = \frac{\partial \nu}{\partial \sigma_2}$$

Substituting this in (3) we find:

$$(\nu + 1)\frac{\partial E}{\partial \sigma_1} = E\frac{\partial \nu}{\partial \sigma_1} \quad \text{or} \quad \frac{\partial \lg E}{\partial \sigma_1} = \frac{\partial \lg(1+\nu)}{\partial \sigma_1}$$

It is clear that these relations can be extended to all three stresses $\sigma_1, \sigma_2, \sigma_3$ by taking into consideration other couples of columns, so that we have:

$$\frac{\partial E}{\partial \sigma_1} = \frac{\partial E}{\partial \sigma_2} = \frac{\partial E}{\partial \sigma_3} \quad (7)$$

and

$$\frac{\partial \lg E}{\partial \sigma_1} = \frac{\partial \lg(1+\nu)}{\partial \sigma_1}$$

$$\frac{\partial \lg E}{\partial \sigma_2} = \frac{\partial \lg(1+\nu)}{\partial \sigma_2} \quad (8)$$

$$\frac{\partial \lg E}{\partial \sigma_3} = \frac{\partial \lg(1+\nu)}{\partial \sigma_3}$$

From (7) follows:

E must be a function of $\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ (9)

and (8) gives

$$\nu = \alpha E - 1 \quad (\alpha = \text{constant}) \quad (10)$$

Substituting this in (2), the equations can really be integrated:

$$d\varepsilon_1 = \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_2 + d\sigma_3) \longrightarrow \varepsilon_1 = 3\int\frac{d\sigma_m}{E(\sigma_m)} - \alpha(\sigma_2 + \sigma_3)$$

$$d\varepsilon_2 = \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_3 + d\sigma_1) \longrightarrow \varepsilon_2 = 3\int\frac{d\sigma_m}{E(\sigma_m)} - \alpha(\sigma_3 + \sigma_1)$$

$$d\varepsilon_3 = \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_1 + d\sigma_2) \longrightarrow \varepsilon_3 = 3\int\frac{d\sigma_m}{E(\sigma_m)} - \alpha(\sigma_1 + \sigma_2)$$

This represents the only case of integrability.

2. KANO HOSHINO'S FUNDAMENTAL EQUATIONS.

Seeking for new stress-strain relations Kano Hoshino arrives in the above mentioned paper at the following fundamental equations:

$$d\varepsilon_1 = \frac{d(\tau_1 - \tau_3)}{2\mu^2 V} + \frac{d(\sigma_1 + \sigma_2 + \sigma_3)}{3V}$$

$$d\varepsilon_2 = \frac{d(\tau_2 - \tau_1)}{2\mu^2 V} + \frac{d(\sigma_1 + \sigma_2 + \sigma_3)}{3V} \quad \text{(formulae 14)}$$

$$d\varepsilon_3 = \frac{d(\tau_3 - \tau_2)}{2\mu^2 V} + \frac{d(\sigma_1 + \sigma_2 + \sigma_3)}{3V}$$

as being valid on his assumptions beyond the elastic limit. The symbols used are:

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_2 = \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_3 = \frac{\sigma_3 - \sigma_1}{2}$$

$$\mu^2 = \frac{3(1-2\nu)}{4(1+\nu)} \quad \text{(formulae 15)}$$

$$V = \frac{V_o}{\sigma_o}\sqrt{\sigma_m^2 - \frac{1}{\mu^2}\tau_m^2} \quad \text{(formula 13)}$$

$$\tau_m^2 = \frac{1}{3}(\tau_1^2 + \tau_2^2 + \tau_3^2)$$

V_o being a bulk modulus of the soil body, holding its shape under a hydrostatic pressure σ_o as an initial condition. It is a pity, that the author only solved problems with one variable stress, when his equations reduce to simple differential equations, which can be integrated without any essential restriction. If he had tried also problems with two or three variable stresses, he perhaps himself would have found out, that in general his equations are not integrable at all, for he could not have succeeded in finding solutions.

To prove this statement, we remark, that he started from equations (2), as we did in the foregoing paragraph following his example, and that this fundamental equations are merely another mode of writing these equations (2). If we insert in his equations the expressions for $\tau_1 \tau_2 \tau_3$ and replace ν in stead of μ^2, we can transform them back into our equations (2), using for E Hoshino's expression given by his formula (15)

$$E = \frac{6\mu^2}{3 + 2\mu^2}V = (1 - 2\nu)V \quad (11)$$

We find for instance:

$$d\varepsilon_1 = \frac{d(2\sigma_1 - \sigma_2 - \sigma_3)}{4\mu^2 V} + \frac{d(\sigma_1 + \sigma_2 + \sigma_3)}{3V} =$$

$$= \frac{1}{V}\left\{\left(\frac{1}{2\mu^2} + \frac{1}{3}\right)d\sigma_1 - \left(\frac{1}{4\mu^2} - \frac{1}{3}\right)(d\sigma_2 + d\sigma_3)\right\}.$$

$$= \frac{1}{V}\left\{\left(\frac{2(1+\nu)}{3(1-2\nu)}+\frac{1}{3}\right)d\sigma_1-\left(\frac{1+\nu}{3(1-2\nu)}-\frac{1}{3}\right)(d\sigma_2+d\sigma_3)\right\}=$$

$$= \frac{1}{V}\left\{\frac{d\sigma_1}{1-2\nu}-\frac{\nu(d\sigma_2+d\sigma_3)}{1-2\nu}\right\}=\frac{1}{E}\left\{d\sigma_1-\nu(d\sigma_2+d\sigma_3)\right\}$$

Now Hoshino considers ν as a constant, which entails in virtue of (10) that E must be also a constant, in contradiction with (11), V being a function of $\sigma_1, \sigma_2, \sigma_3$. Therefore Hoshino's fundamental equations are in general not integrable. His investigations rest on a mistake, which he made at the very beginning. He overlooked the existence of integrability conditions and made assumptions which are in contradiction with them.

3. PHYSICAL MEANING OF THE INTEGRABILITY CONDITIONS.

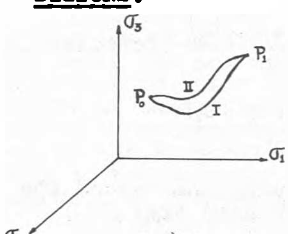

Fig. 1

Each set of values $\sigma_1, \sigma_2, \sigma_3$ defines a state of stress of the considered soil body, the principal directions remaining the same. Therefore the states of stress can be represented by the points in a 3-dimensional σ-space (fig. 1). If P_0 is considered as an initial state, the body can be brought in another state P_1 along an infinity of paths P_0P_1, each representing a continuous succession of intermediate states, i.e. the history passed through by the body since the state P_0. Now let be given the differential expression.

$$A_1\,d\sigma_1 + A_2\,d\sigma_2 + A_3\,d\sigma_3 \qquad (12)$$

The A's being functions of the σ's. If the path I is given in the parametric form

$$\sigma_1 = \varphi_1(t)$$
$$\sigma_2 = \varphi_2(t)$$
$$\sigma_3 = \varphi_3(t)$$

the points P_0 and P_1 corresponding to t_0 and t_1 respectively, the expression (12) reduces along this path to

$$(A_1\dot{\varphi}_1 + A_2\dot{\varphi}_2 + A_3\dot{\varphi}_3)\,dt \qquad (13)$$

a dot denoting differentiation with regard to the parameter t. This expression readily can be integrated, giving a certain function ε_I of t

$$\varepsilon_I = \int_{t_0}^{t_1}(A_1\dot{\varphi}_1 + A_2\dot{\varphi}_2 + A_3\dot{\varphi}_3\,dt) + \varepsilon^\circ \qquad (14)$$

which for $t = t_0$, that is in P_0, takes the value ε°. Another neighbouring path II, may be given by

$$\sigma_1 = \varphi_1(t) + \lambda\psi_1(t)$$
$$\sigma_2 = \varphi_2(t) + \lambda\psi_2(t)$$
$$\sigma_3 = \varphi_3(t) + \lambda\psi_3(t)$$

λ being a small amount, so that its powers can be neglected. As this new path also must go through P_0 and P_1, we must have

$$\psi_1(t_0) = \psi(t_1) = 0$$
$$\psi_2(t_0) = \psi(t_1) = 0 \qquad (15)$$
$$\psi_3(t_0) = \psi(t_1) = 0$$

Then we get another function ε_{II} of t

$$\varepsilon_{II} = \int_{0}^{t}\Bigg[\left\{A_1 + \lambda\left(\psi_1\frac{\partial A_1}{\partial\sigma_1} + \psi_2\frac{\partial A_1}{\partial\sigma_2} + \psi_3\frac{\partial A_1}{\partial\sigma_3}\right)\right\}(\dot{\varphi}_1 + \lambda\dot{\psi}_1) +$$
$$+\left\{A_2 + \lambda\left(\psi_1\frac{\partial A_2}{\partial\sigma_1} + \psi_2\frac{\partial A_2}{\partial\sigma_2} + \psi_3\frac{\partial A_2}{\partial\sigma_3}\right)\right\}(\dot{\varphi}_2 + \lambda\dot{\psi}_2) +$$
$$+\left\{A_3 + \lambda\left(\psi_1\frac{\partial A_3}{\partial\sigma_1} + \psi_2\frac{\partial A_3}{\partial\sigma_2} + \psi_3\frac{\partial A_3}{\partial\sigma_3}\right)\right\}(\dot{\varphi}_3 + \lambda\dot{\psi}_3)\Bigg]dt + \varepsilon^\circ$$

which also may take the value ε° in P_0. Integrating partially with respect to the ψ's, we can write this, in virtue of (14) and (15), as follows:

$$\varepsilon_{II} = \varepsilon_I + A_1\psi_1 + A_2\psi_2 + A_3\psi_3 + \lambda\int_{t_0}^{t_1}\Bigg[\psi_1\left\{\dot{\varphi}_2\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)-\dot{\varphi}_3\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)\right\}+$$
$$+\psi_2\left\{\dot{\varphi}_3\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)-\dot{\varphi}_1\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)\right\}+$$
$$+\psi_3\left\{\dot{\varphi}_1\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)-\dot{\varphi}_2\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)\right\}\Bigg]dt$$

Now the soil body comes along both paths in the state of stress P_1. Then we have, substituting in the last expression $t = t_1$:

$$\varepsilon'_{II} = \varepsilon'_I + \lambda\int_{t_0}^{t_1}\Bigg[\psi_1\left\{\dot{\varphi}_2\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)-\dot{\varphi}_3\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)\right\}+$$
$$+\psi_2\left\{\dot{\varphi}_3\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)-\dot{\varphi}_1\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)\right\}+ \qquad (16)$$
$$+\psi_3\left\{\dot{\varphi}_1\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)-\dot{\varphi}_2\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)\right\}\Bigg]dt$$

Let ε be connected with the state of strain of the body, then we see from this, that in general it reaches at P_1, in a different state of strain, this state notably depending on the path, i.e. on the history passed through. Therefore we can state:
If the strain is defined by giving differential expressions like

$$A_1 d\sigma_1 + A_2 d\sigma_2 + A_3 d\sigma_3$$

it generally depends on the history, which means, that an infinity of states of strain are possible in the same state each state of strain belonging to a special history. In other words: there do not exist stress-strain relations, which vould enable to calculate the strain, when the stresses are known.
When the two states P_0 and P_1 are fixed, as well as path I, ε'_{II} will be equal to ε'_I, however path II be choosen, if the coefficients of the ψ's in (16) vanish along the path I. That gives the conditions:

$$\dot{\varphi}_2\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)-\dot{\varphi}_3\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)=0$$

$$\dot{\varphi}_3\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)-\dot{\varphi}_1\left(\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}\right)=0$$

$$\dot{\varphi}_1\left(\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}\right)-\dot{\varphi}_2\left(\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}\right)=0$$

which may be written as:

$$\frac{d\sigma_1}{\frac{\partial A_3}{\partial\sigma_2}-\frac{\partial A_2}{\partial\sigma_3}}=\frac{d\sigma_2}{\frac{\partial A_1}{\partial\sigma_3}-\frac{\partial A_3}{\partial\sigma_1}}=\frac{d\sigma_3}{\frac{\partial A_2}{\partial\sigma_1}-\frac{\partial A_1}{\partial\sigma_2}} \qquad (17)$$

These relations define a twofold infinity of characteristic curves in σ-space. Hence the condition reads: only when path I coincides with a characteristic curve, ε_{I}' will be equal to ε_{II}'.

When, however, the initial state P_0 and the end state P_1 are not laying on such a curve, ε_{II}' will differ from ε_{I}', except in the case that

$$\frac{\delta A_3}{\delta\sigma_2}-\frac{\delta A_2}{\delta\sigma_3}=0 \qquad \frac{\delta A_1}{\delta\sigma_3}-\frac{\delta A_3}{\delta\sigma_1}=0 \qquad \frac{\delta A_2}{\delta\sigma_1}-\frac{\delta A_1}{\delta\sigma_2}=0 \tag{18}$$

for then (17) reduces to

$$d\sigma_1=0 \qquad d\sigma_2=0 \qquad d\sigma_3=0$$

or to

$$\sigma_1=c_1 \qquad \sigma_2=c_2 \qquad \sigma_3=c_3$$

c_1 c_2 c_3 being arbitrary constants, which means the existance of a characteristic space, occupying the whole σ-space.

Hence, when conditions (18) are fulfilled, ε_{II}' always will be equal to ε_{I}', whereever P_0 and P_1 are laying in σ-space, no matter how both paths be traced between them. To each state of stress, there corresponds only one state of strain, not depending on the history of the soil body, When the state of stress is known, the state of strain is wholly determined by one-valued stress-strain relations, the existence of which is warranted. Therefore conditions (18) are called: conditions of integrability. Their physical meaning can generally be expressed by saying, that they must be fulfilled, whenever functions of state are wanted not depending on history.

4. STRAIN-ENERGY AND THE CONTRADICTORY CHARACTER OF KANO HOSHINO'S THEORY.

Let A be the strain-energy per unit-volume of the strained state, when the soil body in the state of strain, defined by $\varepsilon_1, \varepsilon_2, \varepsilon_3$, the state of stress being given by σ_1, σ_2, σ_3. If both states vary a little, A also changes by an amount dA. Now we have

$$dA\,dx\,dy\,dz=(\sigma_1\,dy\,dz)(d\varepsilon_1 dx)+(\sigma_2\,dz\,dx)(d\varepsilon_2\,dy)+$$
$$+(\sigma_3\,dx\,dy)(d\varepsilon_3\,dz)$$

hence

$$dA=\sigma_1\,d\varepsilon_1+\sigma_2\,d\varepsilon_2+\sigma_3\,d\varepsilon_3 \tag{19}$$

as is easily seen, the volume $dx\,dy\,dz$ having its faces parallel to the principal planes. From this follows, that the integrability of the $d\varepsilon$'s depends on the integrability of the $d\varepsilon$'s, or in other words, that the non-integrability of the $d\varepsilon$'s excludes the existence of strain energy as a function of state.

Now Kano Hoshino operates with non-integrable $d\varepsilon$'s, against which in itself no objection can be made, but on the other hand he introduces the assumption of the existence of strain energy as a function of the σ's i.e. as a function of the state of stress. Though it is in seeking for stress-strain relations from the very nature of the question not recommendable to start with non-integrable $d\varepsilon$'s, the contradictory character of Kano Hoshino's theory does not lie in the first place in this choice of the starting point, but in the fact, that the notion of strain-energy as a function of the reached state of stress, is wholly irreconcilable with the notion of a state of strain, which fundamentally depends on the history of the soil body.

5. ILLUSTRATIVE EXAMPLE.

To give an illustrative example, we take the first problem treated by Kano Hoshino, the case of two dimensional pure shear. As he puts for the treatment of that case:

$$\sigma_1=\sigma_0+\bar{\sigma}$$
$$\sigma_2=\sigma_0$$
$$\sigma_3=\sigma_0-\bar{\sigma}$$

he integrates in σ-space (fig. 2) from P_0 $(\sigma_0, \sigma_0, \sigma_0)$ to P_1 $(\sigma_0+\bar{\sigma}, \sigma_0, \sigma_0-\bar{\sigma})$ along path I, [1] determined by the parametric equations.

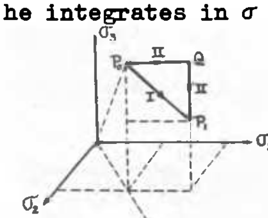

Fig.2

$$\begin{aligned}
&\sigma_1=\sigma_0+\bar{\sigma}t=\sigma_0(1+kt) && d\sigma_1=k\sigma_0\,dt\\
&\sigma_2=\sigma_0 && d\sigma_2=0\\
&\sigma_3=\sigma_0-\bar{\sigma}t=\sigma_0(1-kt) && d\sigma_3=-k\sigma_0\,dt\\
&\bar{\sigma}=k\sigma_0 \qquad 0<t\leq 1
\end{aligned} \tag{20}$$

We find $\sigma_m=\sigma_0$, $\tau_m=\frac{1}{2}\sigma_0^2k^2t^2$, and

$$E=(1-2\nu)\,v_0\sqrt{1-\frac{k^2}{2\mu^2}t^2}$$

Substituting this in

$$d\varepsilon_1=\frac{1}{E}\left[d\sigma_1-\nu\,(d\sigma_2+d\sigma_3)\right]$$

we get:

$$d\varepsilon_1=\frac{k\sigma_0(1+\nu)}{(1-2\nu)\,v_0}\cdot\frac{dt}{\sqrt{1-\frac{k^2}{2\mu^2}t^2}}$$

which gives

$$(\varepsilon_1)_I'=\frac{k\sigma_0(1+\nu)}{(1-2\nu)v_0}\int_0^1\frac{dt}{\sqrt{1-\frac{k^2}{2\mu^2}t^2}}=\frac{3}{2\mu\sqrt{2}}\;\frac{\sigma_0}{v_0}\sin^{-1}\frac{k}{\mu\sqrt{2}} \tag{21}$$

this being just Kano Hoshino's formula (17).

Let us now take another path II, leading via the state of stress represented by point $Q\,(\sigma_0+\bar{\sigma}, \sigma_0, \sigma_0)$. This path consists of two parts, in parametric form given by

$$\begin{aligned}
&\sigma_1=\sigma_0(1+kt) \quad d\sigma_1=\sigma_0k\,dt && && \sigma_1=\sigma_0(1+k) \quad d\sigma_1=0\\
&\sigma_2=\sigma_0 \quad d\sigma_2=0 && \text{and} && \sigma_2=\sigma_0 \quad d\sigma_2=0\\
&\sigma_3=\sigma_0 \quad d\sigma_3=0 && && \sigma_3=\sigma_0(1-kt) \quad d\sigma_3=-\sigma_0k\,dt
\end{aligned} \tag{22}$$

We now find

$$\sigma_m=\sigma_0\left(1+\frac{1}{3}kt\right) \qquad \text{and} \qquad \sigma_m=\sigma_0\left\{\left(1+\frac{1}{3}k\right)-\frac{1}{3}kt\right\}$$

$$\tau_m^2=\frac{1}{6}\sigma_0^2k^2t^2 \qquad\qquad \tau_m^2=\frac{1}{6}\sigma_0^2k^2(1+t+t^2)$$

$$E'=(1-2\nu)\,v_0\sqrt{1+\frac{2}{3}kt-\left(\frac{1}{6\mu^2}-\frac{1}{9}\right)k^2t^2} \qquad \text{and}$$

$$E''=(1-2\nu)\,v_0\sqrt{\left(1+\frac{1}{3}k\right)^2-\frac{k}{6\mu^2}-\left[\frac{2}{3}k\left(1+\frac{1}{3}k\right)+\frac{k}{6\mu^2}\right]t-\left(\frac{1}{6\mu^2}-\frac{1}{9}\right)k^2t^2}$$

Substituting this in

$$d\varepsilon_1 = \frac{1}{E}\left\{d\sigma_1 - \nu\,(d\sigma_2 + d\,\sigma_3)\right\}$$

we get

$$d\varepsilon_1' = \frac{\sigma_0 k\,dt}{E'} \quad \text{and} \quad d\varepsilon_1'' = \frac{\nu\sigma_0 k\,dt}{E''}$$

which gives

$$(\varepsilon_1)'_{II} = \sigma_0\,k\int_0^1\frac{dt}{E'} + \nu\,\sigma_0\,k\int_0^1\frac{dt}{E''}$$

Further on we calculate with $\frac{1}{\mu^2} = 2 \quad \left(\nu = \frac{1}{8}\right)$ and $k = 1$, avoiding in this way too cumbersome formulae.
Then we get

$$E' = \frac{1}{4}V_0\sqrt{9 + 6t - 2t^2} \quad \text{and} \quad E'' = \frac{1}{4}V_0\sqrt{13 - 11t - 2t^2}$$

Thus

$$(\varepsilon_1)'_{II} = \sigma_0\int_0^1\frac{dt}{\frac{1}{4}V_0\sqrt{9+6t-2t^2}} + \frac{1}{8}\sigma_0\int_0^1\frac{dt}{\frac{1}{4}V_0\sqrt{13-11t-2t^2}}$$

which gives after integration:

$$(\varepsilon_1)'_{II} = \frac{1}{4}\,2\left\{8\left(\sin^{-1}\frac{1}{3} - \sin^{-1}\frac{1}{3\sqrt{3}}\right) + \left(\sin^{-1}1 - \sin^{-1}\frac{11}{15}\right)\right\}\frac{\sigma_0}{V_0} = 1{,}20\frac{\sigma_0}{V_0}$$

With the same numerical values (21) gives

$$(\varepsilon_1)'_I = 2{,}35\,\frac{\sigma_0}{V_0}$$

From this we see, that if we simultaneously increase σ_1 and decrease σ_3, till the endstate of stress

$$\sigma_1 = 2\,\sigma_0\ ,\quad \sigma_2 = \sigma_0\ ,\quad \sigma_3 = 0$$

is reached, the component of strain ε_1 takes the value $2{,}35\,\frac{\sigma_0}{V_0}$; if, however, we first increase σ_1, the other stresses remaining the same, and afterwards decrease only σ_3 till we finally reach the same endstate of stress, ε_I takes the different value $1{,}20\,\frac{\sigma_0}{V_0}$

6. <u>STRAIN-ENERGY AND BREAKAGE</u>.

According to Kano Hoshino the strain energy in the assumed initial state $P_0(\sigma_0, \sigma_0, \sigma_0)$ is given by

$$A_0 = \frac{3\,\sigma_0^2}{V_0}$$

Further he assumes, that in the endstate $P_1(\sigma_1, \sigma_2, \sigma_3)$ the strain energy is

$$A_1 = \frac{3\,\sigma_0}{V_0}\sqrt{\sigma_m^2 - \frac{1}{\mu^2}\tau_m^2} \tag{23}$$

This however, cannot be true, for the energy stored must depend on the mode of bringing the soil body in the endstate, as Kano Hoshino operates with non-integrable $d\varepsilon$'s . It is still worse, as will now be shown. From (23) we can compute the change dA along the path choosen

$$dA = \frac{3\sigma_0}{2V_0}\,\frac{\frac{d}{dt}\left(\sigma_m^2 - \frac{1}{\mu^2}\tau_m^2\right)}{\sqrt{\sigma_m^2 - \frac{1}{\mu^2}\tau_m^2}}\,dt$$

With $\sqrt{\sigma_m^2 - \frac{1}{\mu^2}\tau_m^2} = \frac{\sigma_0 E}{V_0(1-2\nu)}$ and $\frac{1}{\mu^2} = \frac{4(1+\nu)}{3(1-2\nu)}$

this can be reduced to

$$\begin{aligned} dA = \frac{1}{3E}\Bigg[&\left\{(2-\nu)(\sigma_2+\sigma_3) - (1+4\nu)\sigma_1\right\}\frac{d\sigma_1}{dt} + \\ &+\left\{(2-\nu)(\sigma_3+\sigma_1) - (1+4\nu)\sigma_2\right\}\frac{d\sigma_2}{dt} + \\ &+\left\{(2-\nu)(\sigma_1+\sigma_2) - (1+4\nu)\sigma_3\right\}\frac{d\sigma_3}{dt}\Bigg]dt \end{aligned} \tag{24}$$

On the other hand we have according to (19)

$$\begin{aligned} dA = \frac{1}{E}\Bigg[&\left\{\sigma_1 - \nu(\sigma_2+\sigma_3)\right\}\frac{d\sigma_1}{dt} + \left\{\sigma_2 - \nu(\sigma_3+\sigma_1)\right\}\frac{d\sigma_2}{dt} + \\ &+\left\{\sigma_3 - \nu(\sigma_1+\sigma_2)\right\}\frac{d\sigma_3}{dt}\Bigg]dt \end{aligned} \tag{25}$$

Only when these two expressions are identical along the path between P_0 and P_1, Kano Hoshino's energy will be found in the endstate. That gives a condition for the course of the path:

$$\begin{aligned} &\left\{(2-\nu)(\sigma_2+\sigma_3) - (1+4\nu)\sigma_1\right\}\frac{d\sigma_1}{dt} + \left\{(2-\nu)(\sigma_3+\sigma_1) - (1+4\nu)\sigma_2\right\}\frac{d\sigma_2}{dt} + \\ &+\left\{(2-\nu)(\sigma_1+\sigma_2) - (1+4\nu)\sigma_3\right\}\frac{d\sigma_3}{dt} = 3\Bigg[\left\{\sigma_1 - \nu(\sigma_2+\sigma_3)\right\}\frac{d\sigma_1}{dt} + \\ &+\left\{\sigma_2 - \nu(\sigma_3+\sigma_1)\right\}\frac{d\sigma_2}{dt} + \left\{\sigma_3 - \nu(\sigma_1+\sigma_2)\right\}\frac{d\sigma_3}{dt}\Bigg] \end{aligned}$$

which reduces to:

$$(\sigma_2 + \sigma_3 - 2\sigma_1)\frac{d\sigma_1}{dt} + (\sigma_3 + \sigma_1 - 2\sigma_2)\frac{d\sigma_2}{dt} + (\sigma_1 + \sigma_2 - 2\sigma_3)\frac{d\sigma_3}{dt} = 0$$

But this can be brought in a remarkable form, if we write it as follows:

$$(\sigma_1+\sigma_2+\sigma_3-3\sigma_1)\frac{d\sigma_1}{dt} + (\sigma_1+\sigma_2+\sigma_3-3\sigma_2)\frac{d\sigma_2}{dt} + (\sigma_1+\sigma_2+\sigma_3-3\sigma_3)\frac{d\sigma_3}{dt} = 0$$

for then we have

$$(\sigma_1+\sigma_2+\sigma_3)\frac{d}{dt}(\sigma_1+\sigma_2+\sigma_3) - 3\left(\sigma_1\frac{d\sigma_1}{dt} + \sigma_2\frac{d\sigma_2}{dt} + \sigma_3\frac{d\sigma_3}{dt}\right) = 0$$

or, after integration

$$\frac{d}{dt}\left\{(\sigma_1+\sigma_2+\sigma_3)^2 - 3(\sigma_1^2+\sigma_2^2+\sigma_3^2)\right\} = 0$$

written otherwise, we have

$$\frac{d}{dt}\left\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right\} = 0$$

or finally

$$\frac{d\tau_m^2}{dt} = 0 \tag{26}$$

This means, that the path must run on a surface of constant mean shearing stress. Only in that case the strained soil will possess Kano Hoshino's energy in the endstate.
The surfaces of constant mean shearing stress

are given by

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = c^2$$

They are cylinders of revolution, the axis of revolution being $\sigma_1 = \sigma_2 = \sigma_3$, fig. 3.

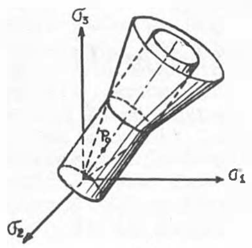

Fig.3

Two arbitrary points are not laying on such a cylinder, i.e. in general there is not a change in energy between two states of the soil body, which is determined by Kano Hoshino's energy function. Especially this is not the case between P_o and any arbitrary state P_1, except, when P_1 lies on the axis of revolution.

From this we conclude:

As it is impossible to start otherwise than from P_o, this being the assumed initial state it follows, that only by changing the hydrostatic pressure σ_o, thus going along the axis of revolution (c=o), Kano Hoshino's energy formula will be valid in P_1, and no point P_1, laying elsewhere in σ -space, will have the energy, which Kano Hoshino supposed to have contributed to it.

This fact has a fatal consequence for his theory as a theory of breakage, for Kano Hoshino's condition of breakage reads

$$A = 0 \quad \text{or} \quad \tau_m^2 = \mu^2 \sigma_m^2$$

so that the breaking points lie on the surface

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \frac{4}{3}\mu^2(\sigma_1 + \sigma_2 + \sigma_3)^2$$

this being a cone of revolution, with its vertex in the origin, the axis of revolution coinciding with that of the foregoing cylinder. Starting from P_o of course the points of this cone are attainable along a multitude of paths, but it follows from the above argument, that then A will not be zero. Here the contradictory character of the theory becomes apparent: breaking point is reached, the strain energy being however not zero, in contradiction with the fundamental assumption. Therefore Kano Hoshino's theory of breakage must be abandoned. To illustrate, we remark, that we have in the case of two dimensional pure shear, along the path used by Kano Hoshino,

$$dA = \frac{(\nu+1)\left[\sigma_o(1+kt)k\sigma_o\,dt - \sigma_o(1-kt)k\sigma_o\,dt\right]}{(1-2\nu)\,V_o\sqrt{1-\frac{k^2}{2\mu^2}t^2}} =$$

$$= -\frac{3\sigma_o^2}{2V_o}\,\frac{d\left(1-\frac{k^2}{2\mu^2}t^2\right)}{\sqrt{1-\frac{k^2}{2\mu^2}t^2}}$$

Integration gives

$$A_t = \frac{3\sigma_o^2}{V_o}\left[1-\sqrt{1-\frac{k^2}{2\mu^2}t^2}\right] + \frac{3\sigma_o^2}{V_o}$$

A being $\frac{3\sigma_o^2}{V_o}$

At the breaking point $1 - \frac{k^2}{2\mu^2}t^2 = 0$, so that we find for the strain energy at breakage

$$A = \frac{6\sigma_o^2}{V_o} = 2A_o \tag{27}$$

in stead of zero.

7. STRAIN ENERGY AS A FUNCTION OF STATE.

In this paragraph we will make some concluding remarks.

We saw that the notion of strain energy as a function of state only can be reconciled with integrable $d\varepsilon$'s.

Operating with these we arrived in the first paragraph at the expressions:

$$\begin{aligned} d\varepsilon_1 &= \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_2 + d\sigma_3) \\ d\varepsilon_2 &= \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_3 + d\sigma_1) \\ d\varepsilon_3 &= \frac{3\,d\sigma_m}{E(\sigma_m)} - \alpha(d\sigma_1 + d\sigma_2) \end{aligned} \tag{28}$$

which are the direct consequences of the assumed integrability. Substitution in (19) gives the equally integrable differential of the strain energy

$$dA = \frac{9}{2}\frac{d\sigma_m^2}{E(\sigma_m)} - \alpha\,d(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

In virtue of $\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = 3\sigma_m^2 - 2\tau_m^2$

we may write this as follows:

$$dA = \left(\frac{9}{2E(\sigma_m)} - 3\alpha\right)d\sigma_m^2 + 2\alpha\,d\tau_m^2 \tag{29}$$

which gives after integration the strain energy as a function of state:

$$A = \int\left(\frac{9}{2E(\sigma_m)} - 3\alpha\right)d\sigma_m^2 + 2\alpha\tau_m^2$$

If we restrict ourselves to quadratic forms, we see that $E(\sigma_m)$ must be constant, Inserting (10)

$$\alpha = \frac{1+\nu}{E}$$

we then find:

$$A = \frac{3(1-2\nu)}{2E}\sigma_m^2 + \frac{2(1+\nu)}{E}\tau_m^2 \qquad 0 \le \nu \le \frac{1}{2} \tag{30}$$

This is the wellknown expression for the strain energy in elastic theory. It is a positive definite quadratic form, not leaving room for the construction of a condition of breakage with vanishing strain energy as a physical basis.

As our analysis, operating with integrable $d\varepsilon$'s to avoid contradictions, leads back to elastic theory, without bringing any new formula, Kano Hoshino's starting point turns out to be unsuitable for the purpose aimed at by him, the foundation of a theory of plastic deformation and breakage of soil.

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I e 25

DISCUSSION

J. BRINCH HANSEN (Denmark)

In his papers Ie 6, IId 3 and IVd 2 Skempton draws attention to the fact that, whereas fully saturated clay, under conditions of no water content change, exhibits a shearing strength independent of the exterior total pressure, the inclination of the failure planes against the principal planes is not 45° but 45° + $\frac{\varphi}{2}$, where φ is the true angle of internal friction.

Whether there is question of a slope analysis or of an earth pressure computation the present state of affairs seems to be that the usual calculations lead to an incorrect position of the sliding surface but give the correct safety factor or earth pressure value, if the shearing strength is put at half the unconfined compression strength. This discrepancy is, at least from a theoretical point of view, unsatisfactory, but I think, however that it should be possible to eliminate it.

Let us consider a simple case of passive earth pressure on clay of the kind mentioned above.

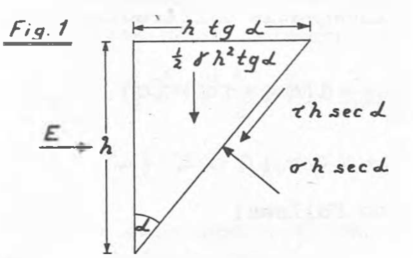

FIG. 1

By projecting on the sliding plane as usual we get:

$$E = \tfrac{1}{2}\gamma h^2 + \frac{2h\tau}{\sin 2\alpha}$$

As τ is constant the usual minimum condition gives $\alpha = 45°$ and:

$$E = \tfrac{1}{2}\gamma h^2 + 2h\tau$$

It is clear, however, that this sliding plane cannot be the correct one, as this must intersect the unloaded surface at an angle of $45° - \varphi/2$, as Mohr's circle shows. Let us, therefore, consider the sliding plane that does so.

At the surface, where the minor total principal stress is zero, the total normal stress σ_o on the sliding plane may be found by means of Mohr's circle:

$$\sigma_o = \tau\, ctg\left(45° + \frac{\varphi}{2}\right)$$

The remaining part of σ is simply the hydrostatic pressure, and the mean value of the total pressure is, therefore:

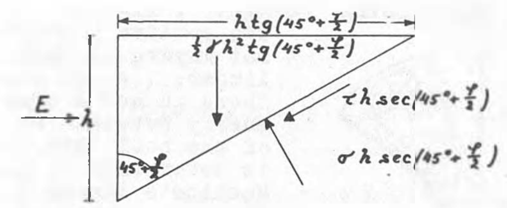

FIG. 2

$$\sigma = \sigma_o + \frac{1}{2}\gamma h^2 - \tau\, ctg\left(45° + \frac{\varphi}{2}\right) + \frac{1}{2}\gamma h^2$$

By projecting on a vertical line it will be found that equilibrium exists, and by projecting on a horizontal line the earth pressure is found:

$$E = \frac{1}{2}\gamma h^2 + 2h\tau \sec\varphi$$

If τ is the direct shearing strength, the Mohr's circle shows that $\tau \sec\varphi$ is half the unconfined compression strength. Therefore, equation (2) and the corresponding $\alpha = 45° + \varphi/2$ is in accordance with test, which equation (1) and $\alpha = 45°$ is not.

Of course, formula (1), too, may give the correct value of the earth pressure, but only when τ is put at half the unconfined compression strength. However, to get a correct result by inserting a wrong quantity in a wrong equation, corresponding to a wrong sliding surface, seems hardly a desirable kind of accordance between theory and practice.

What I have meant to show is, that a computation, based on a single equation of equilibrium and a maximum condition, cannot be relied upon, if the other equations of equilibrium are not known to be satisfied. On the other hand, if all these equations are satisfied, a maximum condition is not needed at all.

The same principle can probably be applied to the sliding surfaces in slope analysis. Jaky did something of the kind in the Proceedings of the First Soil Mechanics Conference, as he satisfied the conditions of equilibrium at the end point of the sliding circle, but he did not investigate the equilibrium of the entire soil mass above the circle, which in my opinion is more important.

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I e 26 ELEMENTARY STUDY OF THE EQUILIBRIUM OF AN IMMERSED BANK IN CASE OF SUDDEN EMPTYING OF THE BASIN

A. COUARD (France)

We know from experience, that the equilibrium bank of pulverulent soils is to a considerable extent, the same as the bank, whether immersed or not, in other words, that the angle φ is the same in both cases. On the other hand, we also know from experience, that sudden emptying of a reservoir or lock, or tidal basin, often entails landslides and that the banks of basins subjected to the tide are invariably softer than the angle of friction of the sands constituting them would often suggest.

It is obviously the influence of the flow of the water impregnating these soils which is responsible for this casement of the slope.

It would seem possible, to calculate approximately the equilibrium bank in an elementary manner.

The entrainment of soil is a phenomenon, which in these cases appears to be superficial at the outset, extending in depth due to continuous erosion. It all seems to happen as if the elements, half solid, half liquid, shift until the moment when, for a sufficiently gentle slope, dissociation commences.

It would seem, that the porosoty of the soil, which conditions the speed of resurgence of the water enclosed in solid soil and the granular structure of the soil which conditions the minimum speed of water capable of entraining elementary particles, should play a considerable part, which has moreover been proved by the stabilizing influence of superficial revetments made up of large materials.

But it would appear quite difficult to figure out these influences quantitatively and somewhat illusive even to wish to calculate them. At most it is legitimate to look for an order of magnitude for shifting sands.

Let us consider a unit of volume at the surface of such a bank. It contains a percentage of spaces ν and a solid volume of $1-\nu$ of an absolute density d and an angle of friction φ; the space is full of water of a density 1.

This volume behaves as one body from the point of view of the weight subject to the effect of gravity, but only its solid part, lightened by Archimedes thrust, exercises a stabilizing friction.

The weight tending to rush down the bank is therefore $\nu + (1-\nu)d$

The friction is only that corresponding to $(1-\nu)$ $(d-1)$ in these.

If α is the angle of the slope of equilibrium in these conditions, we shall have

$$\left[\nu + (1-\nu)d\right] \sin\alpha = (1-\nu)(d-1)\cos\alpha \, tg\,\varphi$$

whence

$$tg\,\alpha = \frac{(1-\nu)(d-1)}{\nu + (1-\nu)d} \, tg\,\varphi$$

In the case of quartzose sands: d = 2.65, ν = 0.32, $\varphi = 35^\circ$ and $\alpha = 20^\circ$.

It will therefore be wise to take this value into consideration, to provide the slope of banks subjected to rapid variation in the level of the water, which washes them.

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SUB-SECTION I g

STRESS DISTRIBUTION

I g 10 DISCUSSION ON PAPER Ig 3

A.J. COSTA NUNES (Brasil)

It would seem that the parabolic distribution of the contact pressures is a hypothesis which is not always verified by experience.

According to the distribution of the load, and even for the finits beam with a concentrated load, the contact pressure may depend to a large extent on the nature of the soil, on the depth of the foundations and on the rigidity of the beam.

In the classical books of Brennecke-Lohmeyer and Kögler-Scheidig - tentative assumptions are given regarding the contact pressures under a beam with a concentrated load, in the cases of sand and clay supports. Kögler's book even gives tables to compute settlements - and moments in the cases of several contact pressures, including the parabolic one.

It would seem that the equality of the deformation of the beam and the soil does not define the distribution of contact pressures, it then being necessary to assume a further condition as:

1) an arbitrary or experimental distribution of pressures as do Mr. Beer and Kögler.
2) an equation relating the load to the deformation (the hypothesis of a constant modulus of soil, for instance).
3) a determined law of pressure distribution in the soil.

The third method is really the most sustained by theory, but, unfortunately, very cumbersome to be developed by the method of finite differences (See Habel, 1937 2), or any other, and only liable to direct solution in very rare cases (Borowicka 1936 1), Habel 1937 2) 1938 3)).

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I g 11 INFLUENCE SCALE AND INFLUENCE CHART FOR THE COMPUTATION OF STRESSES DUE, RESPECTIVELY, TO SURFACE POINT LOAD AND PILE LOAD

O. GRILLO (Brasil)

INFLUENCE SCALE FOR POINT LOAD

In the determination of vertical stresses in a point in the soil, when its depth z beneath a shallow spread footing is larger than three times the width of the footing, the error involved in assuming the distributed load as a point-load is negligible.

Boussinesq's formula for the vertical stresses due to point load is

$$\sigma_z = \frac{3P}{2P}\,\frac{z^3}{(r^2+z^2)^{5/2}} = \frac{P}{z^2}\,\frac{3/2}{\left[1+(r/z)^2\right]^{5/2}} = \frac{P}{z^2}\,I_\sigma$$

A table of the influence values I_σ has been computed by Prof. G. Gilboy 1) Hooks and Gray 2) developped a nomograph for the computation of σ_z.

While the use of the nomograph permits greater speed than that of the table, it still requires the measure of the distance r, the determination of r/z and operation on the nomograph,

In the computation of stresses for settlement analysis purposes, σ_z is determined for various depths and is generally a summation of stresses thrown by a great number of spread footings. In this case, computation by any of the above two methods is very laborious.

The writer uses, in such instances, an influence scale which permits, for given values of z, much greater speed, without sacrifice of accuracy.

For a constant z

$$\sigma_z = P\frac{I_\sigma}{z^2} = \frac{P}{100}\,I'_\sigma$$

where I'_σ varies only with r. Values of I'_σ have been computed for z = 10; 12,5; 15; 17,5; 20 and 25 m, and for values of r up to 30 m. The values of I'_σ, increasing by 0,001, for each of the above six values of z have been marked in each of the six sides of a triple scale at the corresponding distances r, to a 1=50 scale. (This is the scale to which the spread footings plan is generally drawn in the metric system).

By use of this scale the values of I'_σ can be read direct from the scale just as a distance. Interpolation is hardly necessary.

INFLUENCE CHART FOR PILE LOAD

Prof. Mindlin's solution for the computation of stresses due to vertical point load in the interior of semi-infinite homogeneous and elastically isotropic space is 3)

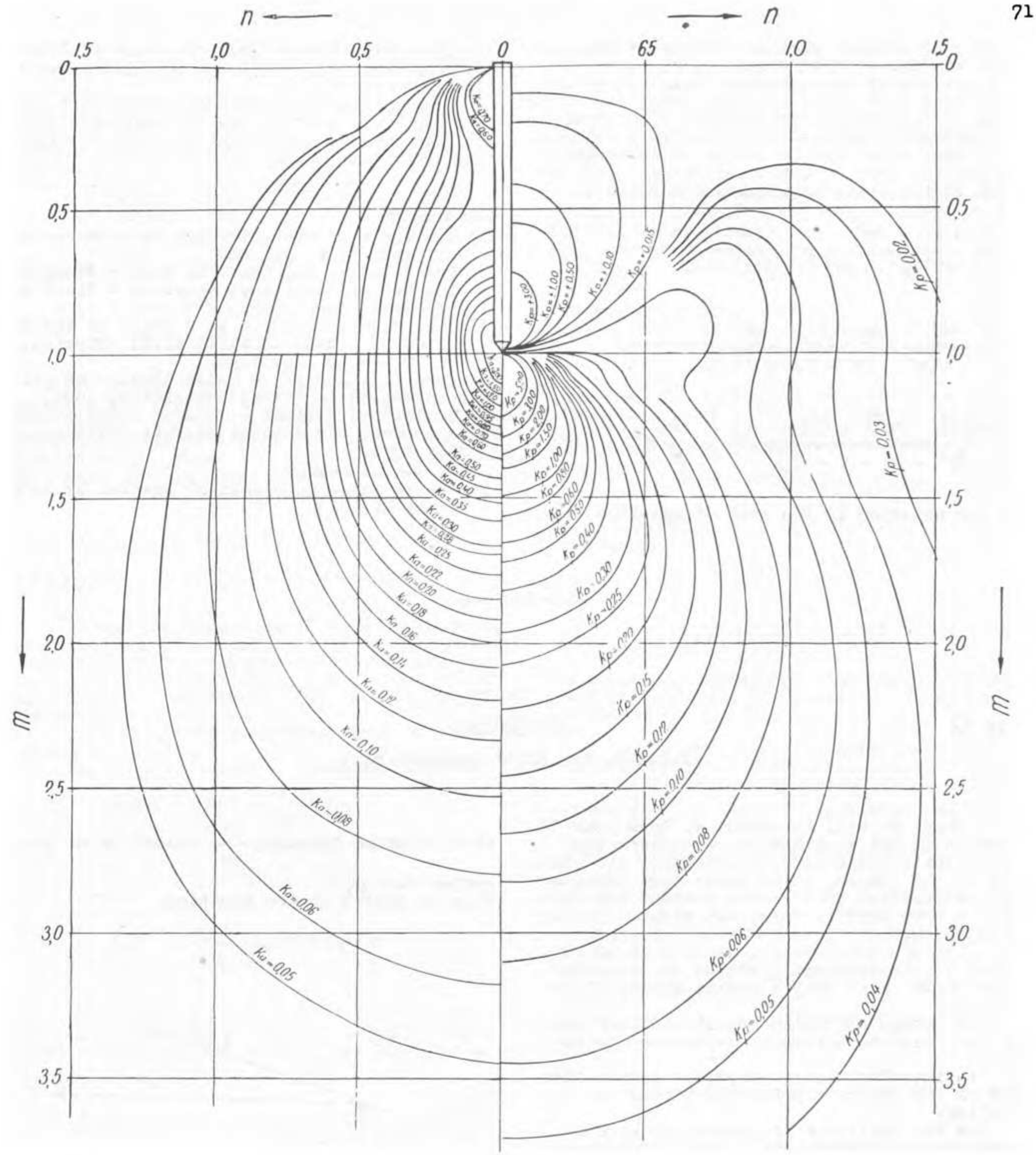
$$Z = \frac{P}{8\pi(1-\mu)} - \left[\frac{(1-2\mu)(Z-C)}{R_1^3} - \frac{(1-2\mu)(Z-C)}{R_2^3} + \right.$$
$$- \frac{3(Z-C)^2}{R_1^5} - \frac{3(3-4\mu)Z(Z+C)^2 - 3C(Z-C)(5Z+C)}{R_2^5} +$$
$$\left. - \frac{30\,C\,(Z+C)^3}{R_2^7}\right]$$

which, for Poisson's Ratio $\mu = 1/2$ can also be written:

$$\sigma_Z = \frac{P}{c^2}\,\frac{3}{4\pi}\left[-\frac{(m-1)^3}{\sqrt{(m^2-2m-1-n^2)^5}} + \right.$$
$$-\frac{m^3(m^2+20+7m)+m(24+11)}{\sqrt{(m^2+2m+1+n^2)^7}} +$$
$$\left. + \frac{mn(m^2n-2m+5mn)+1+n^2}{\sqrt{(m^2-2m-1-n^2)^7}}\right] = \frac{P}{c^2}\,K_p \qquad (1)$$

where $m = \frac{Z}{C}$, $n = \frac{x}{C}$, Z is the depth of the sought point below the surface, x is the distance between the point and the axis of the pile and C the length of the pile.

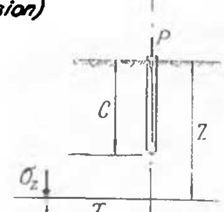
Ruderman 4) extended Mindlin's solution to the case where the load is uniformly distributed over the length C, but his solution has not been published.



Influence curves for vertical stresses due to uniform skinfriction along the pile

(All values are negative or compression)
Poisson's modulus $\mu = 1/2$

$$\sigma_z = \frac{P_s}{C^2} \times K_a$$
$$m = \frac{z}{c}$$
$$n = \frac{x}{c}$$



Influence curves for vertical stresses due to pile point-load

(K_p values preceded by (+) means tension)
Poisson' modulus $\mu = 1/2$

$$\sigma_z = \frac{P_p}{C^2} \times K_p$$
$$m = \frac{z}{c}$$
$$n = \frac{x}{c}$$

FIG. 1, 2

Mr. H. Antunes Martins (deceased) when research student at the Foundation Division of the Institute of Technological Research of São Paulo (Brazil) x) has, in 1942, under the supervision of the writer, then Chief of the Division, extended independently Mindlin's Solution 5). He arrived at the following extended solution for the case of uniform friction along the length of the pile, with Poisson's Ratio $\mu = \frac{1}{2}$

$$\sigma_z = \frac{P}{c^2}\frac{1}{4\pi}\left[\frac{2m^2}{\sqrt{(m^2-n^2)^5}} + \frac{m(6-3m)-2n^2-3}{\sqrt{(m^2-2m-1-n^2)^3}} + \right.$$

$$- \frac{m^2(5m^2+22m+32)-mn^2(7m-10)}{\sqrt{(m^2-2m-1-n^2)^5}} +$$

$$\left. + \frac{-n^2(5-2n^2)-3(6m-1)}{\sqrt{(m^2-2m-1-n^2)^5}}\right] = \frac{P}{c^2}K_a \qquad (2)$$

The notation is the same of equation (1).

The influence of the variation of Poisson's Ratio over the values of K_a is shown on the curves of fig. 1.

The necessary values of influence K, both for Mindlin's solution and the extended solution have been computed for plotting the influence chart in fig. 2.

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x) "Paula Souza Scholarship" granted by Estacas Franki Ltda.

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I g 12

DISCUSSION

J.D.M.M. ten BRINK (Netherlands)

I think it will be useful to draw your attention to the application of Hooke's law and Poisson's ratio in soil mechanics and specially to the reason, which makes that application intolerable. Thus I will present the problem in a more drastic form, but without giving the solution.

I therefore assume a line-load in the direction of the abscissa y whilst in the plane of the paper (x,z) only a radial stress σ_ρ is operating.

The normal stress in the direction ϕ and also the shearing stress τ are assumed to be zero.

In the direction y perpendicular to the plane of the paper, a principle stress σ_y is operating.

Now the section ρ is assumed to be displaced over a distance $\Delta\rho$. This displacement is caused directly (right-ahead) by the stress σ_ρ and indirectly (by side ways) by the stress σ_y.

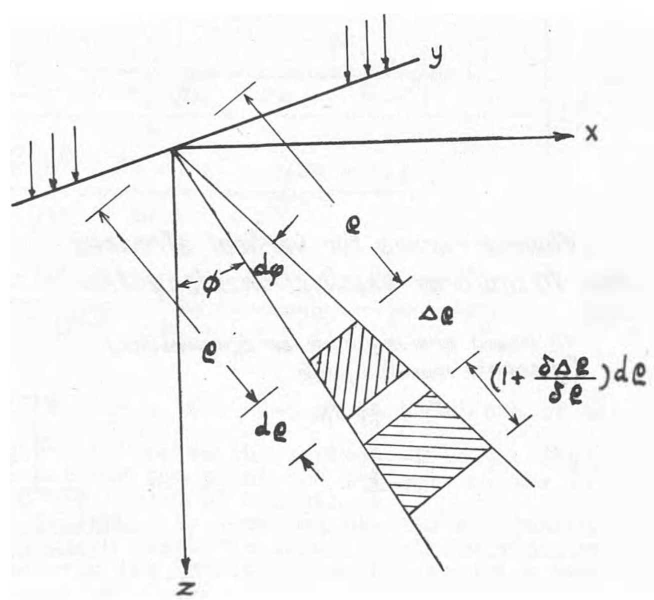
We do not know in what degree the displacement $\Delta\rho$ is influenced partially by σ_ρ and partially by σ_y . Therefore I assume two undetermined functions: $R(\sigma_\rho)$ for the right ahead influence of σ_ρ and $S(\sigma_y)$ for the side-way influence of σ_y. Whilst the section ρ is displaced from ρ to $(\rho+\Delta\rho)$, the section $(\rho+d\rho)$ is displaced to

$$\left(\rho + d\rho + \Delta\rho + \frac{\delta\Delta\rho}{\delta\rho}\,d\rho\right)$$

The length of the elementary soil particle, at first $d\rho$ increase to $\left(1+\frac{\delta\Delta\rho}{\delta\rho}\right)d\rho$, resulting in a relative increase $\frac{\delta\Delta\rho}{\delta\rho}$ caused by σ_ρ together with σ_y .

Thus we arrive at the equation:

$$R(\sigma_\rho) + S(\sigma_y) = \frac{\delta\Delta\rho}{\delta\rho} \qquad (\mathrm{I})$$



It is to be noticed, that, as no shearing stresses are assumed, the angle $d\phi$, which determines the wedge-shape of the soil particle, undergoes no change. The soil particle $\rho d\phi d\rho$ which was at first enclosed between the lines ϕ and $(\phi+d\phi)$, can thus, no matter how the displacement in ϕ -direction may be, be brought back between the lines ϕ and $(\phi+d\phi)$.

The initial breadth $\rho d\phi$ increases after the displacement to $(\rho+\Delta\rho)d\phi$. The relative increase in direction ϕ , caused by side-way influences both of σ_ρ and σ_ϕ , is $\frac{\Delta\rho}{\rho}$.

Thus a second equation is as follows:

$$S(\sigma_\rho) + S(\sigma_y) = \frac{\Delta\rho}{\rho} \qquad (\mathrm{II})$$

All planes (x,z) are in a symmetrical position with regard to the line-load.

In the direction y, subjected to the right-ahead influence of σ_y , and to the side-way influence of σ_ρ , there cannot be any displacement. This leads to the third equation:

$$S(\sigma_\rho) + R(\sigma_y) = 0 \quad (\mathrm{III})$$

After adding the three equation we find:

$$(R+2S)(\sigma_\rho)+(R+2S)(\sigma_y) = \frac{1}{\rho}\frac{\delta}{\delta\rho}(\rho\Delta\rho) \qquad (\mathrm{A})$$

In the lefthand side of this equation appears the complete influence of all the assumed stresses upon the deformation of the soil particle, each stress working right shead (R) in one direction and sideway (S) in both remaining directions.

What about the right-hand side of the last mentioned equation?

The initial volume $\rho\, d\phi\, d\rho$ has increased to

$$(\rho + \Delta\rho)\, d\phi \left(1 + \frac{\delta\Delta\rho}{\delta\rho}\right) d\rho$$

Thus

$$\frac{V+\Delta V}{V} = \left(1+\frac{\Delta\rho}{\rho}\right)\left(1+\frac{\delta\Delta\rho}{\delta\rho}\right)$$

and

$$\frac{\Delta V}{V} = \frac{1}{\rho}\frac{\delta}{\delta\rho}(\rho\Delta\rho)$$

By comparing the volume V with the density D and bearing in mind that V x D = constant, the righthand side of the equation A can be written as: minus $\frac{\Delta D}{D}$

Starting from the premisses adopted, we therefore come to the conclusion that the complete influence of all normal stresses upon the deformation of the elementary soil particle is a percentage of increasing density.

Now we may indicate exactly the difference between this conclusion and Hooke's law.

Hooke's law assumed a proportion $\frac{\Delta \ell}{\ell} = \frac{\sigma}{E}$ whereas we have concluded minus $\frac{\Delta D}{D}$ = influence of all normal stresses.

In Hooke's law the length l is nearly invariable whereas in our conclusion deformation depends upon the highly variable density D.

So much as regards Hooke's law.

Adding equation I + III and subtracting equation II we arrive at

$$R(\sigma_\rho) + R(\sigma_y) = \rho\frac{\delta}{\delta\rho}\left(\frac{\Delta\rho}{\rho}\right)$$

As according to equation II

$$\rho\frac{\delta}{\delta\rho}\left(\frac{\Delta\rho}{\rho}\right) = \rho\frac{\delta}{\delta\rho}\left[S(\sigma_\rho)+S(\sigma_y)\right]$$

the sum of the right-ahead influence both of σ_ρ and σ_y can be brought into an equation with the sum of the sideway influences of both stresses, as follows

$$R(\sigma_\rho) + R(\sigma_y) = \rho\frac{\delta}{\delta\rho}\left[S(\sigma_\rho)+S(\sigma_y)\right]$$

This condition is satisfied by the following relation .

$$R(\sigma) = \rho\frac{\delta}{\delta\rho}\left[S(\sigma)\right]$$

$$\text{or} \quad S(\sigma) = \int R(\sigma)\frac{\delta\rho}{\rho} \qquad (\mathrm{B})$$

a relation between the right-ahead influence and the side-way influence caused by one of both stresses

The equation B will make clear the contraction ratio. If for instance the right-ahead influence of a stress upon a deformation might be $R = C(\sigma)^m$, and assuming that $\sigma = \frac{f(\varphi)}{\rho}$, then alone the side-way influence of the same stress will be $S = -\frac{1}{m}R$, in that case according with the Poisson ratio.

It is noted, that the equations I, II and III are satisfied by

$$R(\sigma_\rho) = \frac{\delta\Delta\rho}{\delta\rho} \pm \rho\int\frac{\Delta\rho}{\rho^3}\,d\rho$$

$$Z(\sigma_\rho) = \frac{\Delta\rho}{\rho} \pm \rho\int\frac{\Delta\rho}{\rho^3}\,d\rho$$

$$R(\sigma_y) = -\frac{\Delta\rho}{\rho} \mp \rho\int\frac{\Delta\rho}{\rho^3}\,d\rho$$

$$Z(\sigma_y) = \qquad \mp \rho\int\frac{\Delta\rho}{\rho^3}\,d\rho$$

I trust that experiments will reveal a plausible relation $R(\sigma)$ in connection with the Density D.

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CLOSING REMARKS

R.F.X. RÜCKLI (Switzerland)

First I should like to thank Mr. Koppejan for his information and Mr. Coenen for giving the first answer to the question I put in my general report about plasticity theory. For the rest that, apart from all that has been brought into discussion, there is no need to change the conslusions of my oral report.

The French conslusions, which also in the future will play a very important part in soil mechanics, are proof of the importance of theoretical investigations.

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CLOSING DISCUSSION

Prof. K. TERZAGHI (U.S.A.)

The contributions to Section I have shown that notable advances have been made during the last decade in the field of theoretical soil mechanics. The principal subjects of the investigations were the theories of elastic deformation of soils due to load, the consolidation of clay, the secondary time effect and the conditions for the stability of slopes. However, the application of the theories to practical problems involving cohesive soils such as clay is still impaired by the uncertainties associated with the evaluation of the soil constants which appear in the equations.

Regarding the modulus of elasticity of soft, undisturbed clays, we know that it is very much greater than the modulus obtained by laboratory tests on undisturbed samples. Yet quantitative information regarding the magnitude of the difference is still very scarce. The methods for determining the compression index of heavily pre-compressed clays are still unsatisfactory. The factors which determine the secondary time effect are not yet clearly understood. However, the most disturbing gaps in our knowledge center about the shearing resistance of clay.

Some engineers assume that the shearing resistance of clay, immediately after the application of the shearing stress, is roughly equal to one half of the unconfined compression strength, $\frac{1}{2}\,q_u$, of undisturbed samples. Others determine it by means of quick shear or triaxial tests on undisturbed samples, after consolidation of the samples by a confining pressure equal to the overburden pressure. Quite recently Lyman Carlson in Sweden has constructed a device which permits the measurement of the shearing resistance of clay in situ, below the bottom of a drill hole. The testing device is described in Carlson's contribution to the Proceedings of this conference (Paper No. IIIb 3).

A comparison between the shear values obtained by means of these four procedures leads to the following conclusion: for the uppermost ten or fifteen feet the values are roughly equal. With increasing depth the value $\frac{1}{2}\,q_u$, obtained by means of the unconfined compression test, is almost independent of depth or it increases slightly with depth. By contrast the values obtained by means of any one of the other three methods increase considerably with depth. Hence the question arises, which one of the values thus obtained should be trusted?

Twelve years ago, at the First International Conference on Soil Mechanics, I described several slopes which had failed as soon as the average shearing stress on the surface of sliding became roughly equal to one-half of the unconfined compressive strength, q_u. The observations in the tunnels and cuts of the Chicago Subway, 1939 to 1941, led to similar conclusions. Finally, in one of their contributions to the 1948 Proceedings, Skempton and Golder described several slides which occurred as soon as the average shearing stress became roughly equal to $\frac{1}{2}\,q_u$ (Paper No. IVd 2).

On the other hand, a few weeks ago, while demonstrating his subsurface shearing device on the Väsby airdrome, Mr. Carlson told me about two slopes on soft clay which did not fail until the average shearing stress on the potential surface of sliding became equal to the average of the shear values obtained by means of his device. This value was very much higher than that obtained by means of unconfined compression tests. The discrepancy between Carlson's observations and those which I described before can be explained in different ways. It is conceivable that the samples which he used for his compression tests were excessively disturbed. However, it is also conceivable that there are exceptions to the rule that the average shearing resistance is as low as $\frac{1}{2}\,q_u$ and Carlson happened to encounter such exceptions. In any event, the issue continues to be controversial and the practical implications of the prevailing uncertainties are very important.

If the shearing resistance of clay never exceeds $\frac{1}{2}\,q_u$ it should not be possible to make deep cuts in soft or medium clay, even if the slopes are very gentle, and it should not be possible to make high earth dams entirely out of clay. It is probable that there are exceptions to this rule, but we are still unable to find out, in advance of construction, whether ot not the shearing resistance of the clay will be greater than $\frac{1}{2}\,q_u$. If we make our decisions on the basis of the assumption that the shearing resistance is equal to $\frac{1}{2}\,q_u$, we may be, in a few instances, overconservative. On the other hand, if we evaluate the shearing resistance of the clay consistantly on the basis of the results of consolidated quick shear tests, we are likely to experience failures. Hence, at the present state of our knowledge, the first procedure decidedly deserves the preference.

Conclusive information regarding the relative merits of the two procedures could be obtained only on the basis of numerous slide analyses. Each one of these analyses would require location of the surface of sliding, computation of the average shearing stress on this surface at the instant of failure, and accurate determination of the average shearing resistance of the clay by means of both the unconfined compression test and one or several of the other methods. In connection with soft or medium clay, Carlson's procedure appears to be much more expedient than the shear test or the triaxial test. Compared to the practical importance of such investigations, that of further refinements of our theories of the plastic equilibrium of cohesive soils is very small indeed.

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