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The Fundamental Work of Terzaghi. When the question of calculating the magnitude of the expansion and compression of the soil in Svir } was taken into consideration in 1929, there existed only the observations made by Terzaghi on the expansion and compression of clay and sand confined in lateral direction, and his formulas for these processes.

According to Terzaghi the expansion curve has almost the form of a logarithmic curve with the equation

$$\epsilon = -\frac{1}{A} \ln (p_s + p_i) + c_1 \quad (1)$$

where  $\epsilon$  is the coefficient of porosity,  $p_s$  the load per unit of area and  $A$ ,  $p_i$  and  $c_1$  are constants.

For the main branch of the compression curve he gives the equation

$$\epsilon = -\alpha \ln (p_s + p_0) - \beta (p_s + p_0) + c_2 \quad (2)$$

where  $\epsilon$  and  $p_s$  have the same meaning as above and  $\alpha$ ,  $p_0$ ,  $\beta$  and  $c_2$  are constants.

The expansion and compression of sand and clay is principally due to the increasing or decreasing of the pores, or voids, between the particles. Thus by lateral confinement the linear expansion and compression is also expressed by the above formula.

The coefficient of porosity  $\epsilon$  being the quotient between the volume of the voids and the volume of the particles, the volume of the particles per unit volume of the material in unloaded state can be expressed by

$$\lambda = \frac{1}{1 + \epsilon_0}$$

if  $\epsilon_0$  denotes the coefficient of porosity in unloaded state.

In a state of loading, when the coefficient of porosity is  $\epsilon$ , the same particles will occupy a volume

$$\lambda (1 + \epsilon) = \frac{1 + \epsilon}{1 + \epsilon_0}$$

and the original volume has therefore decreased by

$$1 - \frac{1 + \epsilon}{1 + \epsilon_0} = \frac{\epsilon_0 - \epsilon}{1 + \epsilon_0}$$

As in the case under consideration the change of volume occurs in one direction only, the compression  $e$ , as referred to the unit length of the material in unloaded state, will be

$$e = \frac{\epsilon_0 - \epsilon}{1 + \epsilon_0} \quad (3)$$

Inserting in the above equation the expression for  $\epsilon$  according to eq. (1) and (2) we obtain for the expansion curve

$$e = \frac{1}{A} \cdot \frac{1}{1 + \epsilon_0} \cdot \ln \left( \frac{p_s + p_i}{p_i} \right)$$

or

$$e = a \ln \left( \frac{p_s}{p_i} + 1 \right) \quad (4)$$

where  $a$  and  $p_i$  are constants.

For the main branch of the compression curve we obtain in the same way

$$e = \frac{1}{1 + \epsilon_0} \left\{ \alpha \ln \left( \frac{p_s + p_0}{p_0} \right) + \beta p_s \right\}$$

or

$$e = b \ln \left( \frac{p_s}{p_0} + 1 \right) + c p_s \quad (5)$$

where  $b$ ,  $p_0$  and  $c$  are constants. According to Terzaghi  $\beta$  and consequently also  $c$  should, as a rule, be rather small.

In order to make these formulas useful for the treatment of the problem in question, the constants must be determined experimentally. In this respect a great difficulty was encountered. As mentioned above, the Devonian deposit is not homogeneous but consists of detached strata with different properties. The experimental determination of the expansion and compression curves takes months or years due to the considerable time effect caused by the slow squeezing out of the water in the pores. For this reason it was impossible to test such great number of samples out of the deposit as would have been required to obtain mean values which could be considered to apply to the deposit as a whole. As a matter of fact the calculations had to be based on 4 samples only and these had to be taken from the upper part of the deposit, as it was not possible at the time to obtain undisturbed samples from deep layers.

The first tests were started in December 1929 at the Swedish Government Testing Institute in Stockholm and were made in a ring apparatus according to the test procedure described by Terzaghi in his "Erdbaumechanik". From two tests in this apparatus the figures given in table 1 and 2 were obtained.

Table I

Clayey Devon (Sample SP No. 82524)		
Point	Load	Compression
No	kg per cm <sup>2</sup>	mm per m
1	0.50	8.8
2	2.30	27.6
3	10.00	58.3
4	1.00	31.4
5	0.50	18.2

Table II

Sandy Devon (Sample SP No. 82527)		
Point	Load	Compression
No	kg per cm <sup>2</sup>	mm per m
1	0.50	1.5
2	2.30	5.8
3	10.00	15.3
4	2.30	11.0
5	1.00	7.3
6	0.50	2.9

During these tests it proved very difficult to form a judgement of the time required for the load to act in order to decrease the still remaining time effect to such a small figure that it could be neglected. By the repeated removal of the sample from the apparatus losses of material occurred, as particles of Devon stuck to the filter papers, and it could also be observed that a certain dissolution of the sample occurred since the surface of the water in the vessel became covered by a skin. For these reasons the test results obtained in these apparatuses were considered not quite reliable.

To obtain more reliable results later tests were made in an Oedometer designed, in principle, in accordance with the design of Terzaghi described in "Handbuch der physikalischen und technischen Mechanik, Band IV" (Page 555). The diameter of the sample was 70 mm and the height 50 mm.

One of the tests was started in November 1930 and was kept going without interruption until June 1932. At every change of load it was necessary to wait for about 3 weeks in order to decrease the still remaining time effect to a negligible quantity. The results obtained by this test are given in table 3 below and plotted by small circles on Fig. 1.

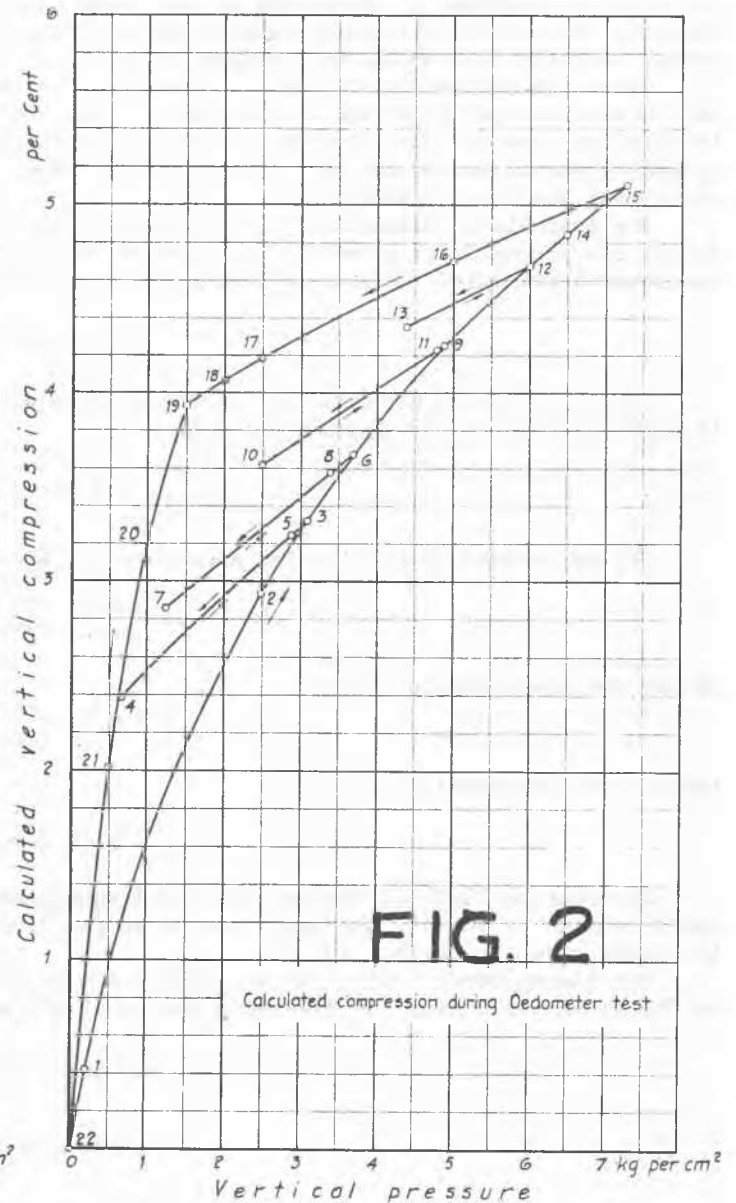
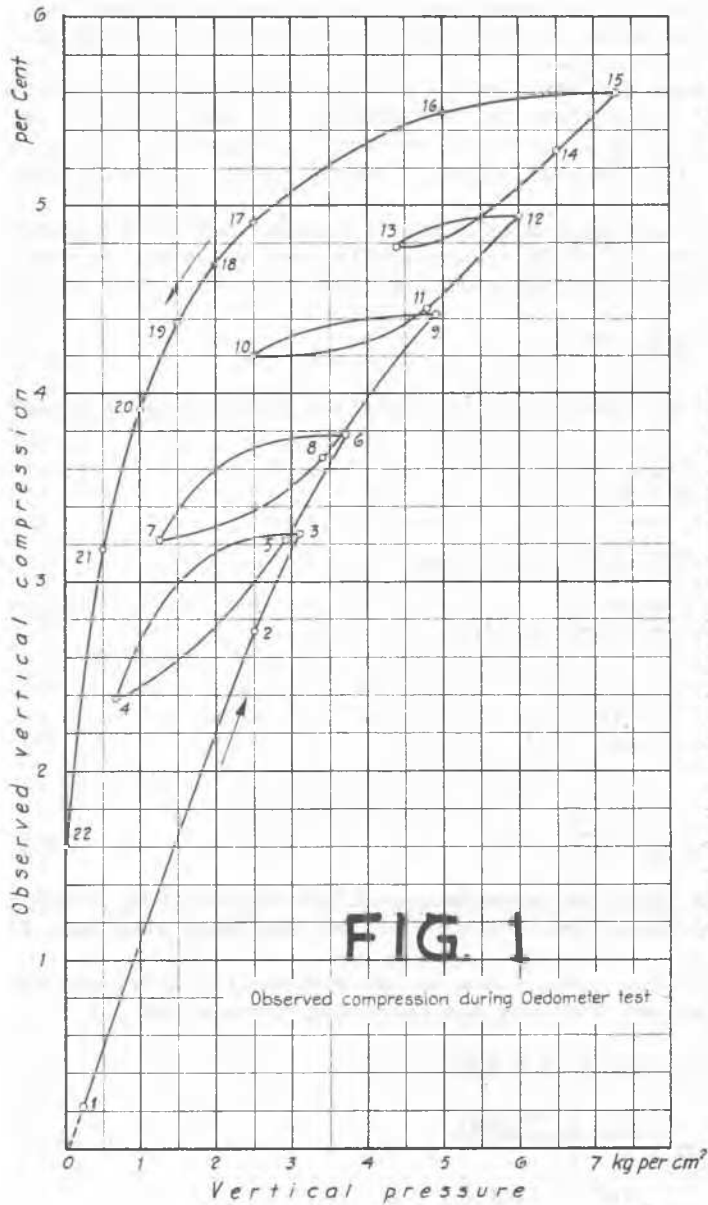
Table III

Clayey Devon (Sample SP No. 89957)

Point	Load	Compression
No	kg per cm <sup>2</sup>	mm per m
0	0.00	0.0
1	0.20	2.2
2	2.50	27.4
3	3.10	32.5
4	0.65	23.8
5	2.90	32.1
6	3.70	37.7
7	1.25	32.2
8	3.40	36.6
9	4.90	44.2
10	2.50	42.0
11	4.80	44.5
12	6.00	49.4
13	4.40	47.8
14	6.50	52.9
15	7.30	55.9
16	5.00	54.9
17	2.50	49.1
18	2.00	46.8
19	1.50	43.8
20	1.00	39.2
21	0.50	31.7
22	0	16.0

The Working Hypothesis of 1931. It became very soon evident that the test results obtained could not directly be used for calculating the swelling and settling of the ground. The question how the test results should be applied for such calculations was taken into serious consideration in the beginning of 1931.

The Devonian deposit is very old and has for this reason been subject to much varying loads during



long geological periods. During the last glacial period, for instance, it was subject to the pressure of an ice sheet several hundred meters thick. On account of the great and changing loads every permanent compression of which it is capable should already have taken place. The deformations which might be observed in the tests should therefore be of a reversible character. The fact that the sample at complete unloading did not resume its original length was considered due to errors of testing and, above all, to the impossibility to obtain, at the starting of the test, a perfect fit between the sample and the apparatus and also to the difficulty of reducing the load to zero without effecting a disintegration of the sample. That a great increase in volume actually takes place at complete unloading was observed in the cuts. Pieces of Devon in contact with water were loosened and gradually transformed into pulp.

Except the very uppermost layer, the ground under the structures will never be fully unloaded. The expected swelling during the excavation and the subsequent settling during the construction of the works will be caused principally by fairly small changes of the stresses in the lower layers. By such changes it is not possible to follow the expansion curve at unloading and the main branch of the compression curve at loading, since the former curve as a rule has a different tangent as compared with the latter. Thus if these curves were applied, small repeated changes of the stresses would lead to a permanent deformation, which is in contradiction to our assumption. On the contrary, such small changes should not cause any deformation.

The curves given by eq. (1) and (2), and by eq. (4) and (5), respectively, should therefore be considered to give the relation between pressure and coefficient of porosity, and the linear expansions and compressions, by always increasing or always decreasing stresses, only. When the sample is alternatively loaded and unloaded the transition from the expansion to the compression curve will have to be given by a transition curve the shape of which would depend on the stresses which the sample has already suffered.

At any attempt to build up a trial function which would give the deformation as a function of the stresses under the above-mentioned conditions it should be borne in mind that the deformation is principally due to changes in the volume of the pores and that it is therefore of the nature of volumetric changes. This is also indicated by Terzaghi's formulas which directly give  $\epsilon$  and thus the volume of the pores, and only indirectly the changes in length of the sample.

Since the volumetric changes in materials following Hooke's law are a linear function of the sum of the three principal stresses it was close at hand to assume that the two formulas (4) and (5) could be brought into one by inserting instead of  $p$  the sum  $s_1 + s_2 + s_3$  of the principal stresses  $s_1, s_2, s_3$  and by making convenient and different assumptions regarding the two lateral stresses  $s_2$  and  $s_3$  during the prevailing loading and unloading.

The most simple assumption in this respect which suggested itself was to assume that in the sample during the prevailing increase of the load an active condition is created, which can be characterized by the lateral principal stresses  $s_2$  and  $s_3$  being a certain fixed fraction  $\mu$  of the vertical stress  $s_1$ ; thus

$$s_2 = s_3 = \mu \cdot s_1$$

Similarly it was assumed that during the prevailing decrease of the vertical load a passive condition is created which can be characterized by

$$s_2 = s_3 = \frac{1}{\mu} s_1$$

If, as assumed above, the two equations (4) and (5) are put into one

$$e = a \ln \left( \frac{s_1 + s_2 + s_3}{p} + 1 \right), \quad (6)$$

we get for the expansion curve

$$e = a \ln \left( \frac{s_1 \left( 1 + \frac{2}{\mu} \right)}{p} + 1 \right) \quad (6a)$$

and for the compression curve

$$e = a \ln \left( \frac{s_1 \left( 1 + 2\mu \right)}{p} + 1 \right) \quad (6b)$$

In order to find out whether the above equations, by a suitable choice of the constants  $\mu$ ,  $a$  and  $p$ , could be made to satisfy the test results so far obtained a determination of the constants were made for the tests given in table 1 and 2.

For clayey Devon (sample SP No. 82524) the points No. 1 and 3 are on the compression curve and No. 4 on the expansion curve. To determine the constants we may thus use the following three equations

$$0.0088 = a \ln \left( \frac{0.5 \cdot (1 + 2\mu)}{p} + 1 \right)$$

$$0.0583 = a \ln \left( \frac{10 \cdot (1 + 2\mu)}{p} + 1 \right)$$

$$0.0314 = a \ln \left( \frac{1.0 \cdot (1 + \frac{2}{\mu})}{p} + 1 \right)$$

From these equations we get

$$\mu = 0.45, \quad a = 0.0267, \quad p = 2.409$$

If, by using these constants, point No. 2 on the compression curve and point No. 5 on the expansion curve are calculated we obtain for these points 27.6 and 20.2 mm per m, which is in good agreement with the measured values 27.6 and 18.2 mm per m.

For sandy Devon (sample SP No. 82527) the following values of the constants were estimated

$$\mu = 0.45, \quad a = 0.0105, \quad p = 5.73$$

With these values of the constants we obtain the following comparison between the observed and the calculated deformations.

Table IV

Point No.	Linear Compression in mm per m	
	Calculated	Observed
1	1.6	1.5
2	5.9	5.8
3	15.3	15.3
4	12.1	11.0
5	7.0	7.3
6	4.1	2.9

As can be seen the agreement is very good.

The question about the transition curve between the compression and the expansion curves was solved by assuming that the lateral stresses  $s_2$  and  $s_3$  do not change their values when  $s_1$  varies between the boundaries.

$$\left. \begin{array}{l} \mu s_2 \\ \mu s_3 \end{array} \right\} < s_1 < \left\{ \begin{array}{l} \frac{s_2}{\mu} \\ \frac{s_3}{\mu} \end{array} \right.$$

At the time (March 1931) also the first five test results given in table 3 were obtained for clayey Devon in the Oedometer.

If the deformation for the loads in question are calculated according to the above working hypothesis and by using the constants for clayey Devon determined above we obtain the figures given in table 5 below

Table V

## Clayey Devon

Point No.	Load kg per cm <sup>2</sup>	Linear compression in mm per m	
		Calculated	Observed
1	0.20	4.2	2.2
2	2.50	29.4	27.4
3	3.10	33.2	32.5
4	0.65	23.8	23.8
5	2.90	32.4	32.1

As can be seen the agreement is surprisingly good.

For another sample consisting of clayey Devon with embedded sandy layers also some new test results were at the time obtained in the ring apparatus. If the constants in equation (6) are taken at the following values

$$\mu = 0.45, \quad a = 0.0205, \quad p = 2.52,$$

the deformation for the loads in question can be calculated at the values given in table 6 below

Table VI

## Clayey Devon with sandy layers

Load kg per cm <sup>2</sup>	Linear compression in mm per m	
	Calculated	Observed
0.20	2.9	2.4
2.50	21.6	21.6
3.10	24.6	24.3
0.65	17.9	18.3
2.90	23.6	23.7

As can be seen from the above table the agreement is also in this case surprisingly good.

Later Experience. Thanks to the working hypothesis it was possible to use the obtained test results for the calculation of the expected settlements. Very extensive calculations in this respect were made during the next year and thus the expected settlements and tiltings of the different parts of the works were estimated. The methods used and the very great difficulties encountered during these calculations will not be discussed in this paper. It may only be mentioned that in the original state of the ground the lateral stresses were assumed to be equal to the vertical stresses, and that the changes in the vertical stresses were calculated according to Boussinesq's equations, whereby also forces acting in the soil on account of changes in hydrostatic pressure were taken into account. The settlements at the surface of the ground were calculated as the change in length of laterally confined vertical columns reaching a depth of some 300 m.

In regard to relative size and direction of tilting the settlements observed later on agreed essentially with those estimated. The observed values were, however, only about one-third of the values estimated when using the coefficients for clayey Devon. This was in itself not surprising, since the deposit contained a very great percentage of sandy layers with very small compressibility. The factor of  $1/3$  to be applied to the results calculated for entirely clayey Devon was found very soon during the early stage of the work when only the bottom course of concrete had been laid. For all subsequent works correctly estimated values of the settlement could be taken into account. The calculations made in regard to settlement and tilting have thus entirely fulfilled their purpose.

Although the working hypothesis has proved very useful in practice it was later on found that it does not entirely correspond with what actually happens. When continuing the test, the results of which are given in table 3, it proved that the values obtained deviated more and more from the calculated ones, which can be gathered from a comparison between Fig. 1 and 2. Full agreement could of course not be expected as the constants were calculated for another test. Also the fact that the test shows permanent deformation of the sample should not give too serious concern as the permanent deformation observed might be due to disturbances of the sample caused by its cutting out and its transport to the laboratory and does not indicate that, contrary to our assumptions, the actual ground is capable of permanent deformation.

It is more serious that the shape of the experimental unloading curve is not similar to the one determined by the working hypothesis. This indicates weakness in the latter. Further suspicion is aroused by the fact that, according to the working hypothesis, the unloading and the loading curve at partial unloading and subsequent loading should coincide as shown on Fig. 2. Tests carried out later on with sand indicate that at such change in loading the expansion and compression follows a hysteresis loop as shown in Fig. 1.

According to the working hypothesis the relation between the vertical and lateral pressures should be affected by hysteresis but the relation between the deformation and the sum of the principal stresses should be free from hysteresis. Later tests described in a paper by Mr. W. Kjellman have shown that for sand this latter relation is also affected by hysteresis and in all probability this holds true also for clay. Tests with sand have also shown that the lateral stresses by unloading vary in another manner than assumed in the working hypothesis and this is probably also true for clay.

It is thus clear that the working hypothesis described in the preceding paragraph gives only in a somewhat crude way the compressions and expansions in soil and that it should be considered as a first trial only by means of which the laboratory experience can be applied to calculations of settlements of actual structures.

No. D-4

IMPROVED METHODS OF CONSOLIDATION TEST  
AND OF THE DETERMINATION OF CAPILLARY PRESSURE IN SOILS  
Prof. N. Gersevanoff, Institute for Scientific Research  
of Building Foundations, Moscow, USSR

Summary. (1) It is to exclude the possibility of swelling of the cohesive soil samples during the consolidation test. Otherwise care should be taken that the water which is brought in contact with the sample should be of the same chemical composition as the ground water at the place from which the sample has been taken.

(2) The variations in soil moisture during the consolidation test are to be determined by means of measuring the deformations of the sample.

(3) The ratio of the increment of the lateral pressure to the increment of the vertical one is a constant and depends only on the structure and other natural properties of the soil, while the ratio of the lateral pressure to the vertical pressure may assume any value.

(4) The value of the capillary pressure of cohesive soil samples can be measured by means of one and the same apparatus by different methods, both direct or indirect.

The principles of soil research suggested by Professor Terzaghi in his remarkable work "Erdbaumechanik" are wide-spread in the USSR.

The consolidation test is used in the USSR only with the view of predicting settlements which are to be expected as a result of the compression of soil strata underlying structures with given loadings.