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Fig. 2 shows the Debye-Scherrer röntgen diagrams for the two minerals. Beside kaolinite and montmorillonite, one or more unknown clay minerals have been frequently encountered in clays. Their properties lie between those of kaolinite and montmorillonite.

A thixotropic gel is one which may be liquified by a mechanical disturbance alone, without temperature change, and becomes solid again after the disturbance has ceased. The fact which has been observed and investigated by A. Casagrande that many soils have a smaller bearing capacity in a disturbed than in an undisturbed state is probably to be explained at least in part by the thixotropic properties of the clays.

Since the univalent, (Na and K) and to a smaller extent, the bivalent, bases (Ca and Mg) which are loosely bound on the surfaces of the clay particles, tend to unite with water, all dry clays swell more or less in water and tend to disintegrate into their constituent particles or into lumps or flakes. In Fig. 3 are shown a few characteristic pictures of clay specimens disintegrating in tap water, and Table II gives some data concerning these clays.

Fig. 3, in connection with Table II, shows very clearly that disintegration becomes more marked with increasing amounts of exchangeable bases. There is a very marked difference between the Ca-bentonite No. 3 and the Na-clay No. 4 in that the water surrounding the former is quite clear, owing to the flocculating effect of the Ca-ion, while that surrounding the Na-clay is cloudy.

If it were possible to destroy this swelling capacity of clay immersed in water, plastic flow of clay could be prevented. The electrochemical nature of the attachment of the exchangeable bases suggests that this may be accomplished by means of an electric current. Another abstract by the present writers presents the results of some investigations which have been carried out in this direction.

No. D-7 THE SHEARING RESISTANCE OF SATURATED SOILS AND THE ANGLE BETWEEN THE PLANES OF SHEAR  
Dr. Karl v. Terzaghi, Professor at the Technische Hochschule in Vienna

The stresses in any point of a section through a mass of earth can be computed from the total principal stresses  $n_I'$ ,  $n_{II}'$  and  $n_{III}'$  which act in this point. If the voids of the earth are filled with water under a stress  $n_w$ , the total principal stresses consist of two parts. One part,  $n_w$ , acts in the water and in the solid in every direction with equal intensity. It is called the neutral stress. The balance,  $n_I = n_I' - n_w$ ,  $n_{II} = n_{II}' - n_w$  and  $n_{III} = n_{III}' - n_w$ , represents an excess over the neutral stress  $n_w$  and it has its seat exclusively in the solid phase of the earth.

This fraction of the total principal stresses will be called the effective principal stresses. For equal values of the total principal stresses, the effective stresses depend on the value of  $n_w$ . In order to determine the effect of a change of  $n_w$  at a constant value of the effective stresses, numerous tests were made on sand, clay and concrete, in which  $n_w$  was varied between zero and several hundred atmospheres. All these tests led to the following conclusions, valid for the materials mentioned:

A change of the neutral stress  $n_w$  produces practically no volume change and has practically no influence on the stress conditions for failure. Each of the porous materials mentioned was found to react on a change of  $n_w$  as if it were incompressible and as if its internal friction were equal to zero. All the measurable effects of a change of the stress, such as compression, distortion and a change of the shearing resistance are exclusively due to changes in the effective stresses,  $n_I$ ,  $n_{II}$  and  $n_{III}$ . Hence every investigation of the stability of a saturated body of earth requires the knowledge of both the total and the neutral stresses.

If a saturated soil fails by shear, the normal stress on the surface of failure also consists of a neutral and an effective part. The relation between the normal stress  $n$  and the corresponding shearing resistance  $t_s$  is usually determined by means of shearing tests. In order to obtain from such tests not more than one value of  $t_s$  for each value of  $n$  the following conditions must be satisfied. All the specimens subjected to the test must have the same initial water content and in all the tests of the same series the normal pressure  $n$  must either be increased from zero to its ultimate value or all the tests must have been preceded by consolidating the samples under the same pressure  $n_0 > n$ . If these conditions are satisfied and if the neutral stress  $n_w$  has been kept equal to zero, the test results can be expressed by an empirical equation

$$t_s = \sigma_s + f_s (n) \quad (1)$$

wherein  $\sigma_s$  is a constant and  $f_s (n)$  is some function of  $n$ . By plotting the values of  $n$  as abscissae and the values of  $t_s$  as ordinates we obtain a curve such as  $M_s N_s$  in Fig. 1a. It is called the basic line of rupture. In most cases this line is almost straight.

Since the shearing resistance depends exclusively on the effective normal stress, a neutral stress  $n_w$  has no influence on  $t_s$ . Since the effective stress is equal to the difference between the total normal stress  $n'$  and the neutral stress  $n_w$ , we can replace equation (1) by

$$t_s = \sigma_s + f_s (n' - n_w) \quad (2)$$

The following discussions concerning the angle between the planes of rupture refer exclusively to the basic line of rupture represented by equation (1). According to Mohr's rupture hypothesis this line is identical with Mohr's envelope. The reasoning which led to Mohr's hypothesis is illustrated by Fig. 1b.

The tangent to the circle R at point A forms an angle  $90 - 2\epsilon$  with the horizontal. If we succeed in demonstrating that this tangent is identical with the tangent AT in Fig. 1a we may conclude that Mohr's hypothesis is correct.

If  $t_x$  is the ordinate and  $n_x$  the abscissa of any point on any circle of stress, we have the following relations:

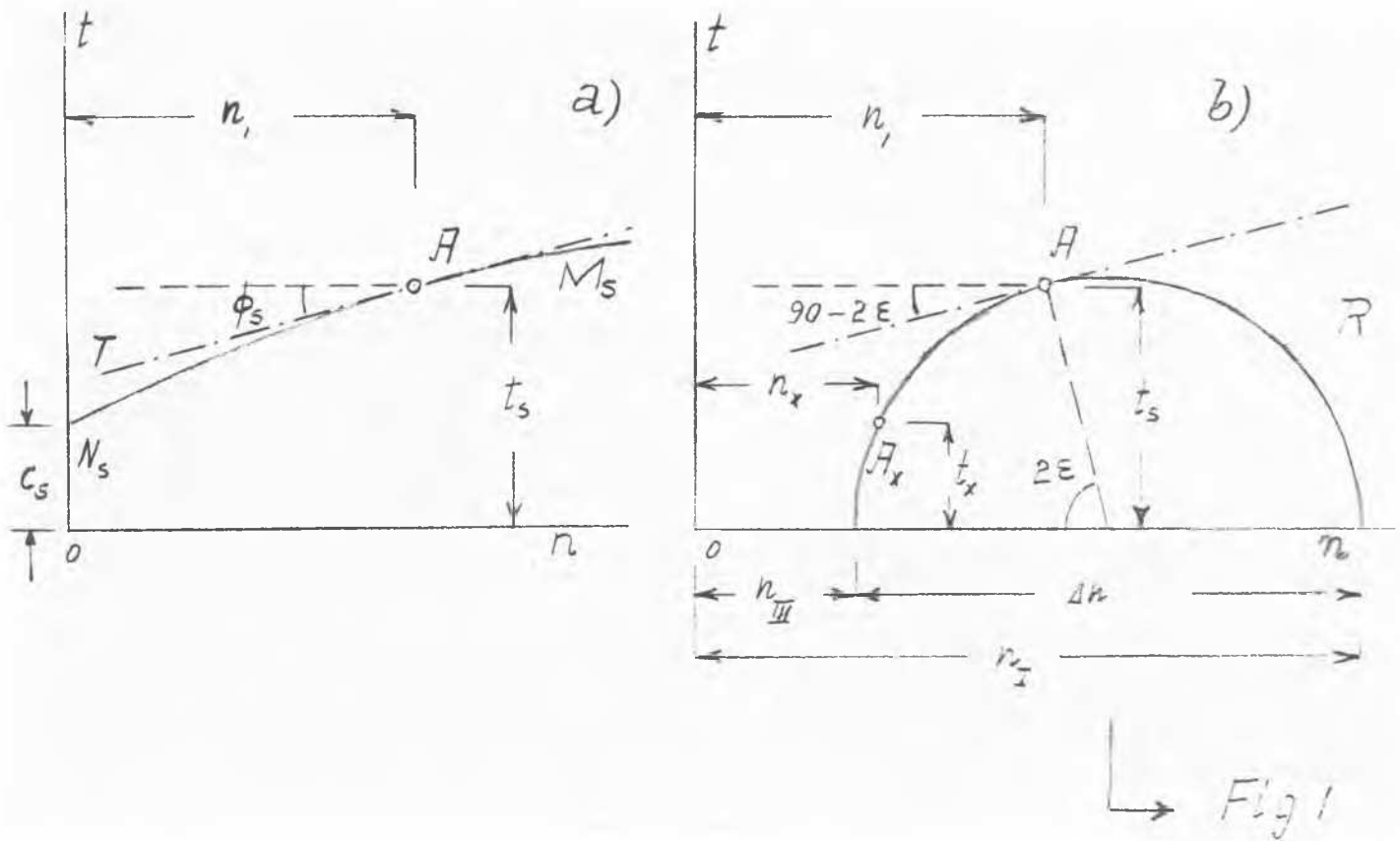
$$t_x = \frac{\Delta n}{2} \sin 2\epsilon_x \tag{3}$$

$$n_x = n_{III} + \frac{\Delta n}{2} - \frac{\Delta n}{2} \cos 2\epsilon_x \tag{4}$$

If the coordinates of a point represent the stress on the plane of least resistance they must in addition satisfy the condition  $t_s - t_x = \text{minimum}$ . Evaluating  $t_s$  by means of (1) we obtain

$$t_s - t_x = c_s + f_s(n) - t_x \tag{5}$$

The difference  $t_s - t_x$  is a minimum if



$$\frac{d(t_s - t_x)}{d\epsilon} = \frac{df_s(n)}{dn} \cdot \frac{dn}{d\epsilon} - \frac{dt}{d\epsilon} = 0 \tag{6}$$

The values  $dn/d\epsilon$  and  $dt/d\epsilon$  can be computed from (3) and (4). Thus we obtain

$$\frac{\Delta n}{2} \frac{df_s(n)}{dn} \sin 2\epsilon - \frac{\Delta n}{2} \cos 2\epsilon = 0$$

or 
$$\tan(90 - 2\epsilon) = \frac{df_s(n)}{dn} \tag{7}$$

For  $n = n_1$  the ratio  $\frac{df_s(n)}{dn}$  is equal to the slope of the tangent AT in Fig. 1a. Therefore the tangents in Fig. 1a and Fig. 1b are identical. This result implies the identity of the line  $M_s N_s$  on Fig. 1b, the envelope of the circles of rupture in Fig. 1b. Equation (7) also implies that the planes of rupture intersect at an angle  $90 - \varphi_s$ . However in reality the angle between the planes of rupture is very seldom equal to  $90 - \varphi_s$  and in many cases the difference is very important. For concrete and marble,

detailed information regarding this difference has been published by Ros (Ros, M. and Eichinger, A. Versuche zur Klärung der Frage der Bruchgefahr. II. Nichtmetallische Stoffe. Eidgen. Materialprüfungsanstalt an der E.T.H. Zurich, 1923.) Still more conspicuous is the difference between theory and fact for clays. No such contradiction would be possible unless there were a fallacy in the preceding reasoning. This fallacy resides in the step from equation (5) to (6) which is based on the assumption that the value  $c_s$  in equation (5) is a constant. However, for concrete we have no proof that the cohesion is independent of the state of stress and for clays we know from direct observation that the cohesion changes with the water content  $w$ . Since an increase of the normal pressure on a layer of clay reduces the water content, the corresponding cohesion increases.

In order to visualize the consequences of this fact let us consider the results of a set of very slow shearing tests on a clay whose original water content is equal to the liquid limit. When plotting the results of such a series of tests in a diagram we always obtain a straight line A B, Fig. 2, which passes through the origin of the system at an angle  $\varphi_s$  to the horizontal. This line can be described by an empirical equation,

$$t_s = n \cdot \tan \varphi_s \quad (8)$$

similar to the equation for a perfectly cohesionless material. If we subsequently reduce the normal pressure  $n$  to zero the value  $t_s$  is considerably greater and the water content considerably smaller than at the beginning of the test, though the normal pressure was in both cases equal to zero. Hence equation (8) is only valid if the normal pressure  $n$  is associated with a definite water content  $w$ . This important condition can be expressed by the symbols

$$w = f_1(n) \quad \text{or} \quad n = f_2(w) \quad (9)$$

It also applies to equation (1), which was previously introduced into equation (5). Equation (5) refers to the state of stress represented by the circle R in Fig. 1b. Different points of this circle represent different values of the normal stress  $n$  corresponding to the same value of  $w$ , because the abscissae of these points represent the normal stress on different sections through the

same specimen. Hence the condition (9) cannot possibly be satisfied for more than two sections through the specimen. Nevertheless the transition from (5) to (6) was performed as if equation (1) were valid for any section. This inadmissible procedure explains the contradiction between reality and the theoretical result.

If the normal pressure on a layer of clay is reduced from  $n_1$  to lower values, the corresponding relation between the normal pressure  $n$  and the shearing resistance  $t$  will be as shown by the line B C in Fig. 2. Some investigators assume that the cohesion for the state B is equal to  $c = A C_1$ . In reality the cohesion corresponding to the state B must be appreciably greater because the transition from the state B towards C is associated with an increase of the water content which in turn reduces the cohesion. In order to express this fact in analytical terms M. J. Hvorslev suggested a general equation,

$$t_s = f(w) + n \tan \varphi_r \quad (10)$$

wherein  $\varphi_r$  is an empirical value smaller than the angle  $\varphi$  in Fig. 2. This angle is called the angle of internal friction. Equation (10) is valid for all cases, irrespective of the previous history of the specimen. If we replace equation (5) by

$$\Delta t = t_s - t_x = f(w) + n \tan \varphi_r - t_x$$

and repeat the computation which led from (5) to (7) we obtain

$$2\theta = 90 - \varphi_r \quad (11)$$

This result eliminates the contradiction between theory and experiment. It explains the reason why the angle between the planes of shear has nothing in common with the angle  $\varphi_s$  in Coulomb's equation and for cohesive soils it excludes an identity between the line of rupture obtained from shearing tests and the envelope of Mohr's circles. Yet in spite of the importance of the difference between  $\varphi_s$  and  $\varphi_r$ , the difference between the ordinates of these two lines seldom exceeds five per cent. Hence for practical purposes this difference can be neglected.

In order to obtain reliable information on the cohesion-function  $f(w)$  in equation (10) and on the angle of internal friction  $\varphi_r$  it is necessary to make several series of shearing tests involving different values of  $w$  at equal values of  $n$  and to supplement these tests by an accurate determination of the water content. Successful tests of this type were recently made by J. Hvorslev.