ing to the spiral (10), and not to the spiral (12). In Fig. 7 (heavy line, left part) a slip line, (3a), passing through the edge of the loaded portion is represented. Original spirals are (3) and (4). In this case $\theta_0 = \pi/2$ and Equations (12) become (13). The point of intersection with the center line of the loaded portion is located at a depth of 2.30 b. (Equation 14.)

Shear Directions Within a Sandy Mass. In the case of the action of a concentrated force, F, at the boundary of a sand mass, the value of the plane stress, s, at a point (ρ, θ) may be expressed by Formula (15). Comparing it with Michell's radial distribution Formula (16), it may be concluded that equal stress, s, at the point (ρ, θ) of an isotropic elastic mass would be produced, if the loaded portion, M_oN_o, were loaded with a variable load $3/2p.\cos^3\theta$. Thus the problem is reduced to that solved in a paper printed elsewere in this volume (D. P. Krynine. Determination of Stresses Within a Two Dimensional Elastic and Isotropic Earth Mass.). A graphical solution is given in Fig. 8: the direction of the major principal stress, s, is found, and the slip lines forming an angle $90° - \alpha$ with it (Equation 1) are traced.

Conclusions. 1. There is a "zone of compressions" and a "zone of shears" within a semi-infinite earth mass loaded symmetrically at the boundary. The "limiting curve" separating them in a meridional section is practically a hyperbola.

2. The slip lines form equal angles with the direction of the principal stress; and the value of this angle is a function of the "concentration factor." In the case of clays, slip lines under a loaded portion of a two dimensional mass can be obtained from those under a concentrated load using the method of conformal representation.

3. The problem of finding shear directions within a uniformly loaded two dimensional sand mass can be reduced to that of a non-uniformly loaded elastic and isotropic mass.

No. E-4

DETERMINATION OF STRESSES WITHIN A TWO DIMENSIONAL ELASTIC AND ISOTROPIC EARTH MASS

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A graphical method for determining both normal and tangential stresses within a two dimensional elastic and isotropic earth mass, is given in this paper. The method can be applied in the case of plane stress distribution within any elastic isotropic body.

Notations.

a - arm or distance from the centroid of the area, F, to Point, O.
α - "angle of visibility" of the foundation
A - half, the area of a circle of a radius, R
b - half the width of the loaded foundation
C - center of pressures or the point of application of the resultant of the forces acting at the auxiliary arc, M'N'
CO - direction of the principal stress
f - fraction to show the distance from the center of a loaded foundation in terms of half its width, b
F_o - loading area formed by plotting vertical ordinates, p, at each point of the foundation, M_oN_o
F and F_1 - loading areas formed by plotting vertical ordinates p, and $\frac{p}{\cos\theta}$, respectively, at each point of the horizontal projection, M'N', of the auxiliary arc, MN
M - moment of the area, F, about point, O
dM - elementary moment
M_oN_o - loaded foundation placed at the horizontal boundary of the earth mass
m_on_o - small element of the loaded foundation, M_oN_o
MN - auxiliary arc or the projection of the foundation, M_oN_o, at the circumference of an arbitrary radius, R (center at O)
mn - small element of the auxiliary arc
M'N' - horizontal projection of the auxiliary arc, MN
m'n' - small element of the projection, M'N'
O - point where stresses are to be determined
p - unit load (variable) acting at the foundation, M_oN_o
p_o - average unit load at the foundation M_oN_o or an arbitrary standard unit load
p_z - vertical pressure at a point
dp_z - elementary vertical pressure at a point
ρ - radius vector
R - arbitrary radius of the auxiliary arc, MN
s - stress or sum of principal stresses
ds - elementary stress
τ_{xy} - shear stress acting at the horizontal plane
θ - vertical angular distance
I and II - ordinates

FIG. 1

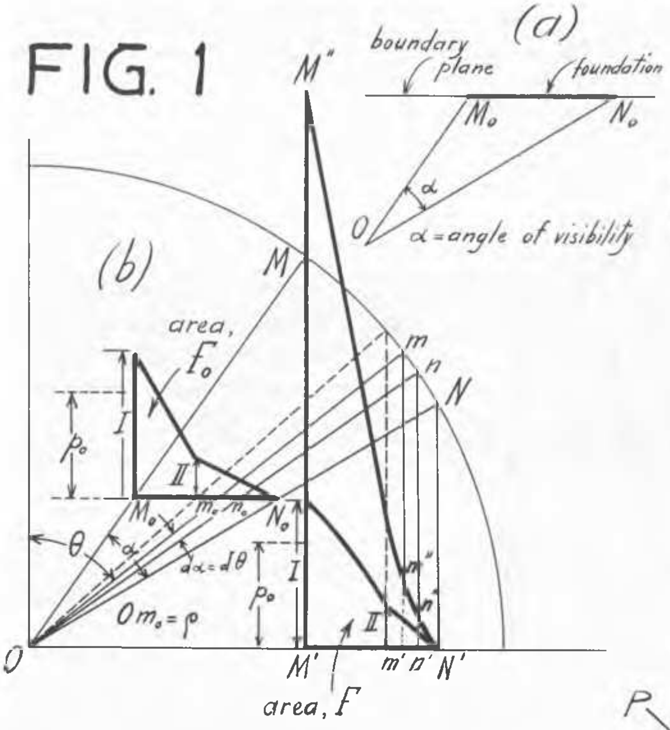


Fig. 1. Graphic determination of the vertical pressure at a point.

FIG. 3

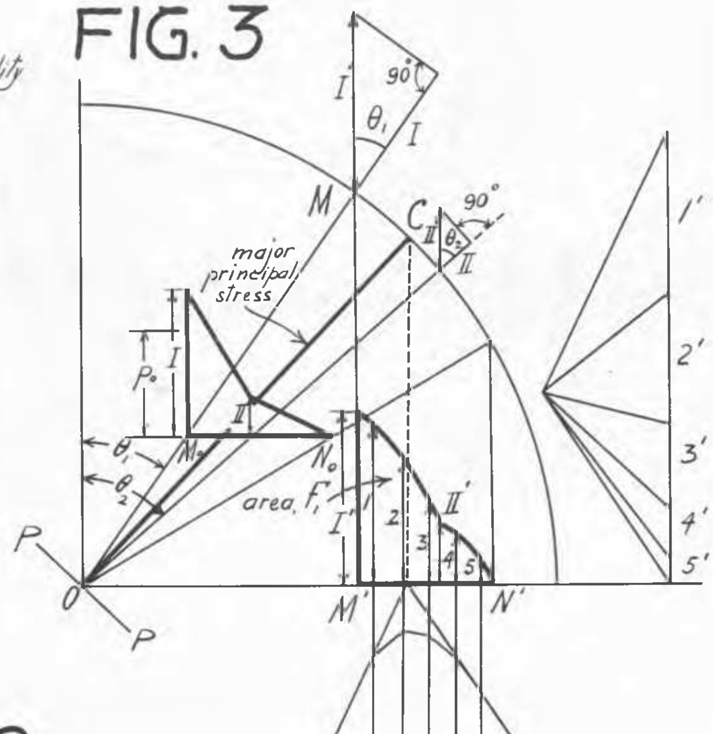


Fig. 3. Graphic determination of the value and the direction of major principal stress.

FIG. 2

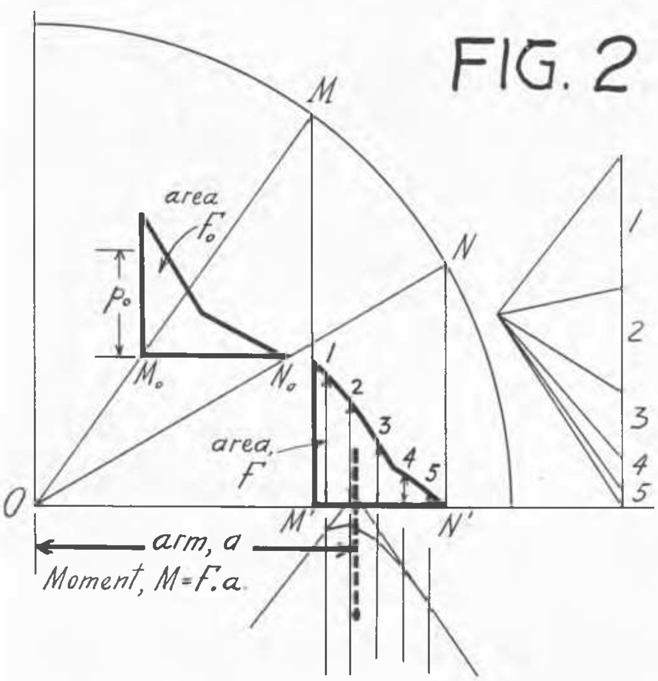


Fig. 2. Graphic determination of the shearing stress along a horizontal plane.

FIG. 4

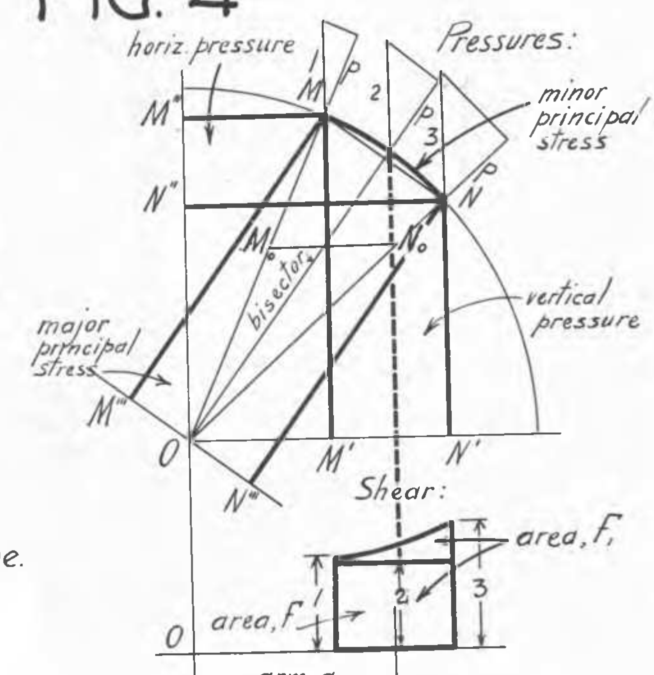


Fig. 4. Stresses in the case of uniform loading of a foundation.

FIG. 5

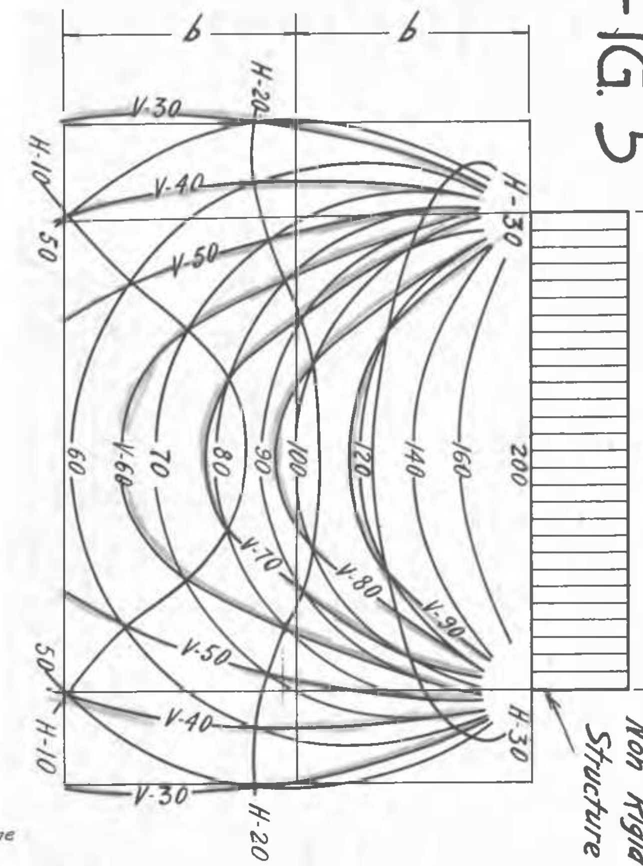


Fig. 5. Sum of principal stresses; vertical and horizontal pressure in per cent of the unit load.

$$d_s = \frac{2\left[\frac{p.\rho.d\alpha}{\cos\theta}\right]}{\pi \rho} . \cos\theta = \frac{2p}{\pi} . d\alpha \qquad (1)$$

$$d_s = \frac{2[p.R.d\alpha]}{\pi . R} = \frac{2p}{\pi} . d\alpha \qquad (2); \qquad s = \frac{2p}{\pi} . \alpha \qquad (2a)$$

$$dp_z = d_s . \cos^2\theta = \frac{2p}{\pi} . \cos^2\theta . d\theta = \frac{2p}{\pi R^2} . \left[R . \cos\theta . d\theta\right] . \left[R . \cos\theta\right] =$$

$$= \frac{p_o}{A} . \left[R . \cos\theta . d\theta\right] . \left[\frac{p}{p_o} . R . \cos\theta\right] \qquad (3)$$

$$dp_z = p_o . \frac{area\,(m'm''n''n')}{A} \qquad (4); \qquad p_z = p_o . \frac{area\,[M'M''N''N']}{A} \qquad (4a)$$

$$d\tau_{xy} = ds . \cos\theta . \sin\theta = \frac{2p}{\pi} . \cos\theta . \sin\theta . d\theta = \frac{2p}{\pi . R^2} \left[R . \cos\theta . d\theta\right] . \left[R . \sin\theta\right] =$$

$$= \frac{p_o}{A} . \left[\frac{p}{p_o} . R . \cos\theta . d\theta\right] . \left[R . \sin\theta\right] = \frac{p_o}{A} . dF . \left[R . \sin\theta\right] = \frac{p_o . dM}{A} \qquad (5)$$

$$\tau_{xy} = p_o . \frac{M}{A} \qquad (6)$$

$$p_z = \frac{2p}{\pi} . \frac{1}{\sqrt{1-f^2}} \qquad (7)$$

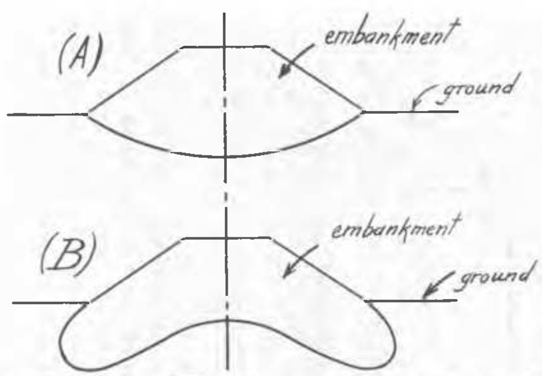
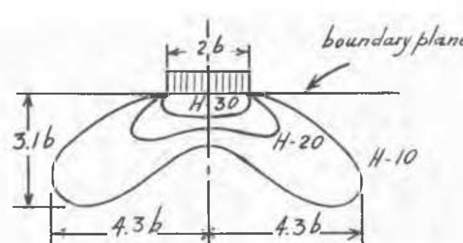


FIG. 7 Two cases of settlement of an embankment



Isolines of horizontal pressure.

FIG. 6

Sum of Principal Stress at a Point; Angle of Visibility. To determine the value of the stress at a point of a two dimensional elastic isotropic mass, Michell's radial distribution formula will be used (Timoshenko, "Theory of Elasticity", page 82 1934). The value of the elementary stress, ds, at Point O caused by a loaded infinitely small element, m_on_o, of the foundation, M_oN_o, placed at the horizontal boundary of the mass (Fig. 1) is expressed by Formula (1). The angle under which the foundation, M_oN_o, would be seen from Point O, if the mass were transparent, will be called "angle of visibility" (terminology of Professor N. M. Gersevanov); so that the angle $d\alpha$, is the "elementary angle of visibility." A circle of an arbitrary radius, R, is traced from Point O and the foundation, M_oN_o, is projected on it (auxiliary arc, MN). The projection of the element, m_on_o, is mn. Both elements, m_on_o, and mn, have the same elementary angle of visibility. If the element, mn, were loaded normally with the same unit load, p, as the element m_on_o, the elementary stress, ds, at Point O, would be expressed by Formula (2), since in this case $\theta = 0$ and R is to be substituted for ρ. Hence instead of considering stresses at Point, O, coming from the element, m_on_o, those coming from the element, mn, may be considered. Repeating this reasoning for all elements of the arc, MN, it may be said that for the purposes of determining stresses at Point, O, the arc, MN, may be substituted for the foundation, M_oN_o. If the unit load is constant through the foundation ($p = p_o$), an integration of Formula (2) leads to Formula (2a), which means that in the case of uniform loading of a foundation the sum of principal stress, s, at a point is proportional to the angle of visibility of the foundation from that point.

Vertical Pressure at a Point. The elementary vertical pressure, p_z, at Point, O, (Fig. 1) is expressed by Formula (3). From the two expressions in brackets, the former, $R.d\theta.\cos\theta$ equals the length of the element, m'n'; and the latter $p/p_oR.\cos\theta$ is the length of the ordinate, m'm, multiplied by the ratio of the actual unit load at Point, m_o, to an arbitrary standard load, p_o. The loading area, F_o, is transformed into the loading area, F, by projecting each point, m_o, at the base of the area, F_o, first on the arc (Point, m), and then to the line M'N' (Point m'), and plotting at Points, m', ordinates equal to those at Points, m_o. The ordinates of the area, F, furnish values of factors p/p_o to be used in constructing the figure, M'M''N'. The vertical ordinates of this figure equal $p/p_o.R\cos\theta_o$; hence the value of the vertical pressure, at Point, O, may be expressed by Formulas (4) and (4a).

The value of a normal stress acting in any other direction, may easily be found, using the procedure described. In such a case, a plane normal to the given stress is traced through Point, O, and the auxiliary arc, MN, is projected on it instead of being projected on a horizontal plane.

Shear Along the Horizontal Plane. The value of the elementary shear stress, $d\tau_{xy}$, is expressed by Formula (5). The expression in brackets, $p/p_o.R.\cos\theta.d\theta$ is nothing else than the area, F, shown in both Fig. 1 and 2; and the elementary shear stress, $d\tau_{xy}$, is proportional to the moment, dM, of an elementary vertical slice, dF, of that area about Point O. Integrating, Formula (6) is found to express the value of the shear stress τ_{xy}. To compute the value of the moment, M, the area, F, is measured, and its centroid (center of gravity) located. A similar procedure can be applied for determining the shear stress at any plane.

Principal Stresses. Area, F, in Fig. 1 and 2 expresses the value of the vertical projection of the forces acting at the auxiliary arc, MN. Plotting at each point of the horizontal projection, M'N', of the auxiliary arc, MN, values of $p/\cos\theta$ and not those of p, area, F_1, would be obtained. Area, F_1, expresses the sum of all forces, acting at the auxiliary arc, MN; and its centroid, upon being projected on the arc, MN, furnishes Point C, "center of pressures", or the point of application of the resultant of all the forces acting at the arc, MN. Pressure exerted on a plane PP, which is normal to the direction CO, is greater than in any other direction; besides, in this case arm $a = 0$. Hence, the direction CO is that of the major principal stress. The latter value may be found in the same way as that of any other normal stress (Fig. 1).

Uniformly Loaded Non-Rigid Structures. Fig. 4 represents the stressed condition at a point in the case of a uniformly loaded non-rigid foundation. The major principal stress and the minor principal stress are proportional: (a) to the area limited by the arc MN and the lines NN'''; N'''M''' and M'''M; and (b) to the area between the arc MN and the chord, MN, respectively. The maximum shearing stress is proportional to half the area of the rectangle MNN'''M'''. In Fig. 5 isolines of the sum of principal stresses; vertical pressure (V) and horizontal pressure (H) are given; numerical values are in per cent of the average unit load, p_o, acting at the structure. The horizontal pressures (H) are clearly seen in the sketch, Fig. 6. Two cases of settlement of an embankment are shown in Fig. 7. If the ground presents more resistance horizontally than vertically, case (A) takes place; but, if some consolidation along the center line of the embankment develops, the material runs laterally following practically an isoline of horizontal pressure (Fig. 6).

Action of a Rigid Structure. A rigid structure is overloaded at the edges according to Formula (7). The whole mass along the center of the foundation is relieved; and there are two stress concentration zones at the edges.

Conclusions. 1. The sum of the principal stresses at a point of a two dimensional elastic isotropic mass is proportional to the "angle of visibility" of the loaded portion.

2. Vertical pressures and shear at a point; and also the value and the direction of the principal stresses can be determined graphically.

3. Sets of isolines of vertical and horizontal pressures in the case of non-rigid structures are given in this paper. The different character of settlement of embankments on swampy land can be explained by the prevailing action either of vertical or horizontal pressure.

No. E-5

STUDIES OF SOIL PRESSURES AND SOIL DEFORMATIONS BY MEANS OF A CENTRIFUGE

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The study of soil problems by means of modeling with the aid of a centrifuge is used in various institutions of the USSR. (The laboratory for Soil Physics of the All-Union Foundation Research Institute, the laboratory for Physics of the Military-Engineering Academy, the laboratory for Soil Mechanics of the Research Institute for Water-Supply and Hydro-Geology, and the Research Laboratory of Moskva-Volgostroi).

The centrifuge gives the possibility to create a complete mechanical similarity and exactly reproduces the loadings called forth by the weight of the given system.

This principle has been put forward in the USSR by Professor N. N. Davidenkov and Professor G. J. Pokrovsky, independently of the American investigator, P. Bucky.

Centrifuges with effective radii from 0.8 to 1.5m have been made for experiments and the following problems have been studied by means of these centrifuges: (Fig. 1)

1. Stability of slopes in earth banks and cuts.

2. Distribution of pressure under foundations.

The results are shown in Fig. 2. The curve II is a theoretical one; the curve III shows the results of model experiments and the curve IV of field experiments.

The experiments have been carried out on sand. The pressures were measured by means of aerostatical dynamometers, which consist of a small vessel filled with coloured viscous liquid, and closed by a rubber membrane. The height to which the liquid rises at the end of the test in the capillary tube immersed in the liquid, indicates the pressure exerted on the apparatus during the test. (Fig. 3)

3. A similar apparatus has been used for determination of pressure on culvert pipes buried in earth. By a special device not only the normal, but the tangential pressures as well could be measured. Fig. 4 shows the results. (The normal stresses are shown by the dotted line and the tangential stresses by the full line.)

4. Settlement of foundations. In Fig. 5 the results of model and field experiments are compared. The curve I shows the relation between time and settlement. The curve II-the relation between load and settlement.

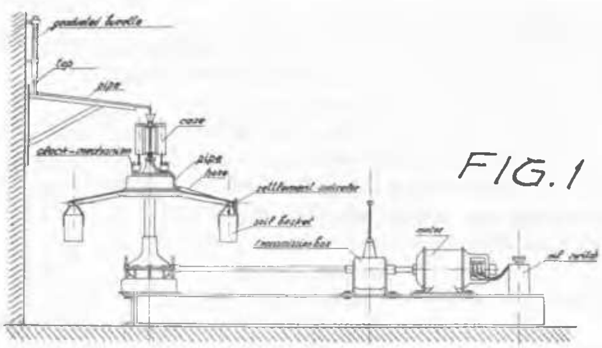

FIG. 1

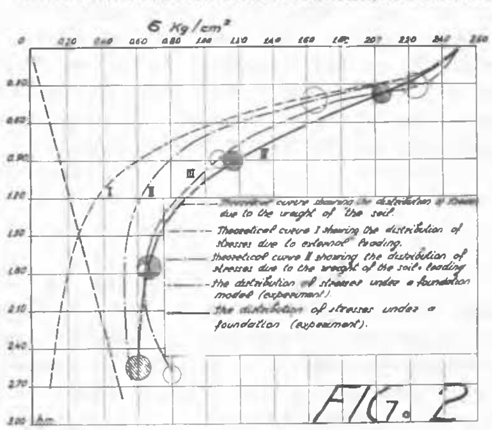

FIG. 2

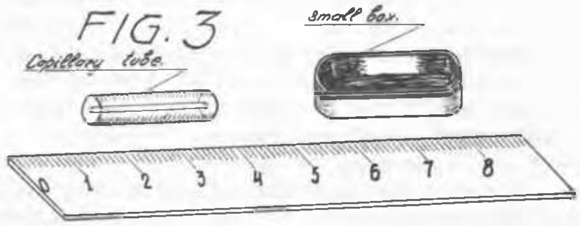

FIG. 3

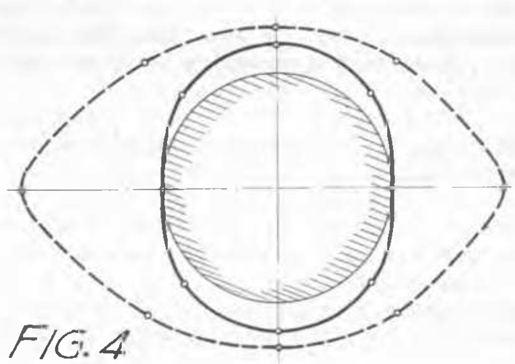

FIG. 4

Comparision of soil deformations through model and field experiments.

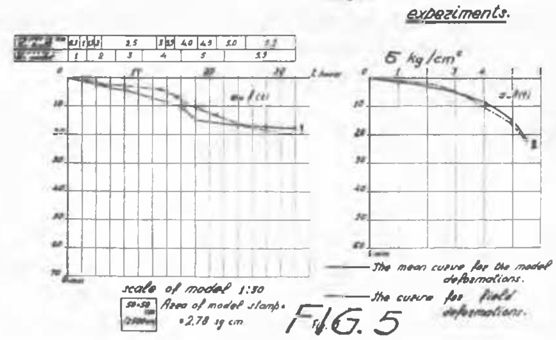

FIG. 5