This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.
ton. For a phreatic surface which slopes down from high water level to 2.30 m, this resistance only amounts to 25 ton. This shows that if the sliding resistance of the peat tends to adapt itself in a short time to the new loading conditions, as is probable according to investigations made in the laboratory, the equilibrium of the slope will not be assured. The same conclusion is derived by comparing computations of a sliding that has happened in the neighborhood.

There are also being made at the Laboratory of the Technical University of Delft model experiments to determine the water tensions for a not permanent current. These experiments are still in a preliminary stage, so that no more information is now available.

No. G-6

CRITICAL HEIGHT AND FACTOR OF SAFETY OF SLOPES AGAINST SLIDING
Dr. Karl v. Terzaghi, Professor at the Technische Hochschule in Vienna

In this discussion the following symbols will be used:

- $t_s$ = shearing resistance.
- $n$ = effective normal pressure on the surface of failure.
- $n_w$ = neutral pressure = pressure in the water.
- $c$ and $\tan \phi$ = the constant and the coefficient of shearing resistance in Coulomb's equation $t_s = c + n \tan \phi$.
- $\phi_f$ = angle of internal friction.
- $s$ = unit weight of soil and water combined.
- $s_0$ = unit weight of the water.
- $i$ = hydraulic gradient.
- $h$ = height to which the water rises in a standpipe in any point of the fill.
- $\alpha$ = angle of the slope to the horizontal.
- $H$ = height of a slope or an embankment.
- $z_0$ = depth to which the minimum lateral Rankine pressure is negative.
- $H_0$ = critical height, i.e. the height at which the slope fails by sliding.
- $S$ = factor of safety against sliding.

A) State of stress in an embankment. Fig. 1 shows the deformation of an earth fill under the influence of its own weight. It discloses expansion for the upper and compression for the lower part of the slope. Hence the upper part fails by expansion and the lower by compression in the direction of the slope. The failure of earth fills almost invariably occurs at a time when the effect of the weight of the fill is increased by the pressure exerted by percolating water. The percolation may be due either to seepage from a body of water stored up by the fill or to a heavy rainfall on the surface of the fill. In any case the state of stress in the percolating water is determined by two sets of curves, one of which represents the lines of flow and the other one those of equal standpipe level. These sets of curves are called the flow net. Fig. 2 shows an earth-fill consisting of fine-grained cohesionless material on an impermeable base during a rainstorm which is ample enough to maintain a continuous flow of water through the voids of the fill. For any point of the fill the neutral stress $n_w$ is equal to $h s_0$. The force which acts per unit of volume on the solid constituents of the earth consists of two components, shown in Fig. 3: a vertical component $s - s_0$ and a component $s_i$ acting in the direction of the flow. (2) The resultant of these components acts at a variable angle to the vertical as shown in Fig. 4. Such a variable orientation of the forces must be expected in any embankment subject to the pressure exerted by percolating water. Since the distribution of the stresses produced by such a system of forces cannot yet be computed, all our methods of computation are necessarily based on radically simplifying assumptions.

B) The equation of the surface of rupture. All the attempts which have been made to determine the equation of the surface of rupture are based on Réval's generalization of Rankine's theory. One of the assumptions involved in this theory is that the slope AB, Fig. 4, extends in both directions to infinity and that the force on the solid constituents per unit of volume of the fill acts in every part of the fill with the same intensity vertically downward.

The Rankine-Réval theory deals with two extreme states of stress in the earth corresponding to the minimum and the maximum earth pressure.

The minimum earth pressure is obtained if the earth is given an opportunity to expand in the direction of the infinite slope, AB, in Fig. 5a. In order to produce such an expansion we replace the earth located at the left of BB in Fig. 5a by a rough wall which adheres to the fill. If we allow this wall
to yield in the downward direction of the slope \( A B \) the earth pressure on this wall decreases until it assumes the minimum values indicated by arrows between the lines \( B'B_2 \) and \( b'b_2 \) in Fig. 5a. To a depth

\[ z_0 = \frac{2\alpha}{s} \tan \left( \frac{L_0 + \frac{\pi}{2}}{2} \right) \]  

(1)

these values are negative, that is, down to a depth \( z_0 \) the soil will be in a state of tension. The surfaces of least resistance against failure by shear intersect the surface \( A B \) at angles of \( L_0 + \frac{\pi}{2} \). With increasing depth below the surface the slope of one of the two sets of surfaces decreases, until at a depth

\[ z_1 = \frac{s}{\cos \theta} \cos \phi \]  

(2)

it becomes equal to \( \tan \alpha \), that is, equal to the slope of the surface. The depth \( z_1 \) is shown in Fig. 5a by the line \( C D \). One of the potential surfaces of failure by shear is shown by the line \( B_1E_1 \) in Fig. 5a. To a depth \( z_0 \) the soil fails by tension as indicated by the vertical crack \( E_1F_1 \). Between \( B_1 \) and \( B_2 \) the failure would occur by shear. This state of minimum earth pressure associated with an elongation of the soil in the direction of the slope corresponds to the upper part of a fill, in the vicinity of \( B_1 \), Fig. 1.

In order to produce the state of stress corresponding to the maximum earth pressure, we replace the soil located on the right side of \( B'B_2 \), Fig. 5a by a rough wall adhering to the soil and press this wall in the downward direction of the slope towards the soil. The potential surfaces of failure intersect the surface \( A B \) of the slope at angles of \( L_0 + \frac{\pi}{2} \). At a depth \( z_1 \), equation (2), the slope of one of the two sets of surfaces becomes equal to the slope, \( \tan \alpha \), of the surface as shown by the curve \( AB_1 \) in Fig. 5a. Hence at the depth \( z_1 \) the potential surfaces of rupture \( B_1B_2 \) and \( AB_1 \) have a common tangent. However, no slip can possibly occur along any of the potential surfaces of failure, unless the lateral pressure exerted onto \( B'B_2 \) becomes equal to the values indicated by the arrows between the lines \( B'B_2 \) and \( B'B_2 \). The state of maximum earth pressure associated with a compression in the downward direction of the slope corresponds to the state in the lower part of the slope of a fill, near \( A \), Fig. 1.

The shape of the curves \( B_1E_1 \) and \( AB_1 \) in Fig. 5a and the height \( H_1 \) and \( H_2 \) of the sections of the slope located above these curves are determined by a differential equation.

Frontard succeeded in solving these equations. He found that the curves \( AB_1 \) and \( B_1E_1 \), Fig. 5a, are deformed hypocycloids and he computed the values \( H_1 \) and \( H_2 \) (3). Yet he arbitrarily assumed that the critical height \( H_0 \) of the fill is equal to \( H_1 + H_2 \) as shown in Fig. 5b.

According to the Rankine-Resal Theory, the pressure exerted by the wedge \( BB_1E_1F \) in a downward direction at the instant of sliding along \( B_1E_1 \) cannot possibly exceed the values indicated by the length of the arrows along \( BB_1 \). The smallest pressure required to start any movement along \( AB_1 \) is indicated by the arrows shown along \( B'B_2 \). This is very much greater than the pressure exerted by \( BB_1E_1F \). The weight of any prism of earth \( B'B_2 \) located between \( B'B_1 \) and \( BB_1 \) is in equilibrium with the forces along \( B_1E_1 \). Hence the pressure conditions required to induce a slip along the two sections \( AB_1 \) and \( B_1E_1 \) in Fig. 5b cannot possibly be simultaneously satisfied.

In order to avoid the consequences of this incompatibility, Frontard seems to assume that an incipient rupture should suffice to propagate the failure through the balance of the body of earth, as is the case in a solid material. However, it
should be emphasized that this hypothesis is incompatible with the Rankine-Résal state of stress. The Rankine-Résal stress distribution applies exclusively to ideal plastic materials capable of unlimited deformation at unaltered values of $\sigma$ and $\phi$. If this condition is not satisfied, the stress distribution behind the slope will be something intermediate between the elastic and the plastic stress distribution whereupon Frontard's reasoning loses its validity.

In addition, Frontard's solution involves the following departures from reality. It assumes that the state of stress is everywhere identical with the Rankine stress. For the vicinity of the points $A$ and $E$, this assumption cannot be correct. Furthermore, it disregards the pressure exerted by the percolating water in directions other than the vertical and finally it is based on the Rankine assumption that the angle between the surfaces of rupture is equal to $90^\circ - \phi$. This assumption disregards the narrow limits to the validity of Coulomb's equation for clays (see Ref. 1) and may involve an error as high as $10^\circ$. Such an error has an important effect on the values of $H_3$ and $H_0$.

The above comments also apply to A. Caquot's theory of the critical height (4). The surfaces of rupture produced by the failure of simple slopes always exhibit the general character of Frontard's deformed hypocycloids, because the upper part of the failing earth always fails by expansion and the lower one by compression. Yet this similarity does not eliminate the fundamental defects of the theory, which are so serious as to render the results inapplicable to practical problems.

C) Methods based on assumption of a circular sliding surface. This method was introduced into engineering practice by K. E. Patterson. It is based on the assumption that the slide occurs along a circular surface with its center at some point $O$, Fig. 6. The factor of safety against sliding is computed for several different circles selected at random and the center of the circle of least resistance is determined by trial. In order to compute the resistance against sliding along an individual circle the slice of earth, $A B C$, Fig. 6, located above the circle is divided into elements with a width of $\Delta l$, and a weight $\Delta G = z \sigma_{r} \Delta l$. The vertical sides of each element are assumed to be perfectly smooth. The frictional resistance along the base of the element is produced by the radial component $\Delta N = \Delta G \cos \alpha$ of the weight. If part of the slope is located beneath the water table $W W$, the total weight $\Delta G$ of the element is reduced by the hydrostatic uplift $z_{0} \sigma_{r}$. Failure occurs by rotation around $O$. The moment
which tends to produce failure is $Gd$. If $L$ is the total length of the arc $AC$ the moment resisting the rotation is

$$R \left[ k \alpha + \tan \phi \sum_{\alpha} \Delta G \cos \alpha \right]$$

Hence the factor of safety against sliding along $AC$ is

$$S = \frac{R \left[ k \alpha + \tan \phi \sum_{\alpha} \Delta G \cos \alpha \right]}{Gd}$$

D) Assumed sliding surfaces of the non-circular type. In reality the radius of curvature at the lower end of the surface of rupture is usually greater than it is at the upper end, as shown in Fig. 5. In order to adapt the assumed shape of the surface of rupture to this rather universal fact, L. Rendulic proposed that the circular arc in Fig. 6 be replaced by a section of a logarithmic spiral (5). This assumption involves no appreciable addition labor in the numerical computations.

If the fill is composed of sections with different Coulomb equations or if the base of the fill contains layers of clay with unusually low values of $\alpha$ and $\phi$, the surface of rupture must be assumed to consist of different sections with different curvature, for instance of straight lines, arcs of a circle or sections of logarithmic spirals, in such a fashion as to keep the surface of rupture entirely within the zone of least resistance.

E) Stability of fills acted upon by percolating water. The method of Petterson is applicable only to soils whose voids are filled with stationary liquid. Moreover, it disregards the effect of the tensile stresses in the vicinity of the upper rim of the slope, which may extend to a depth $z_0$, equation (1). During rainstorms these cracks are likely to be filled with water. In order to take account of this fact, we assume that the upper rim of the sliding surface is located at $C_1$ in Fig. 7, at a depth $z_0$ below the surface. The water accumulated in the crack $C_1D$ increases the driving moment around 0 by

$$M_w = \frac{1}{2} z_0^2 s_0 d_1$$

The method used by the author for computing the shearing resistance along a curved surface in an embankment subject to seepage is illustrated by Fig. 8. This figure represents a section through an embankment consisting of fine-grained, cohesionless material resting on a perfectly impermeable base $AB$ during a rainstorm. Since the cohesion of the material is supposed to be negligible, the depth $z_0$, equation (1), is equal to zero. The flow net for this fill is shown in Fig. 2. $ABC$ in Fig. 8 represents the surface of rupture. It is assumed to be an arc of a circle but the following procedure can also be used for any type of curve. By means of the flow net Fig. 2 we determine the height $h$ to which the water would rise in a standpipe in different points of the curve. All these points are located on the "standpipe curve" $K_a$. Beneath each element the water carries one part, $h s_0 d_1$, of the total weight of the element. Hence the effective normal pressure which acts onto the base of the element is
The balance of the procedure is identical with that described under the heading C. The method can be used for investigating the stability of slopes or embankments acted upon by any type of seepage. The flow net can be obtained either by means of the graphical procedure devised by Forchheimer or else by means of one of the known hydrological or electrical laboratory methods.

F) Stability of rolled clay fills in a state of incomplete consolidation. Immediately after a clay fill is constructed the water content of the clay is practically uniform throughout the fill. As time goes on the water content gradually adapts itself to the state of stress in the fill. If the fill is very high the water content of the central part of the fill, Fig. 9a, is likely to be appreciably greater than the water content after complete consolidation. If the embankment is used as a storage dam or if it becomes saturated during a rainstorm the flow of water associated with consolidation combines with the flow of the water which penetrates the fill from outside. An analysis of the resulting hydrostatic pressure conditions would require extremely elaborate investigations. The time and labor involved would not be justified except in connection with very important projects. In every other case it suffices to make a crude estimate of the stability conditions. For such investigations the following procedure is recommended. We assume a probable surface of rupture, placing its upper rim C₁ at a depth z₀, equation (1), below the crest of the fill. We next plot the standpipe curve K as explained above in connection with Fig. 6. Then we determine the effective normal forces, ΔN, which act on the surface of rupture, as shown in Fig. 8. From these values we compute the effective normal stresses on the plane of rupture and plot them in a diagram such as that shown in Fig. 9b. In this figure, the distance A C₁ represents the distance A C₁ in Fig. 9a, measured along the arc. Owing to the state of incomplete consolidation of the fill, the shearing resistance produced by these normal stresses is smaller than in a completely consolidated fill such as that represented in Fig. 8, though the values of c and φ may be the same. In order to obtain quantitative information regarding the effect of the excess moisture on the shearing resistance, three series of tests are made with the fill material in the state in which it is to be placed in the fill:

a) A series of very slow shearing tests, involving complete consolidation of the soil under both the normal and the shearing stresses.

b) A series of shearing tests to be performed by very rapid application of the shearing force after preceding complete consolidation of the soil under the vertical loads. The results of these two series of tests are shown in Fig. 9b by the straight lines A₁ M₂ and A₂ M₂ respectively.

c) A series of tri-axial compression tests on cylindrical specimens whose surface is completely covered with a water-tight membrane in order to keep the water content constant throughout the tests. The corresponding line of rupture A₁ M₂ in Fig. 9b may be considered practically identical with the envelope of Mohr's circles obtained from these tests (See Ref. 1). The slope tan φ of this line depends essentially on the air content of the soil. For a clay without any air this line is almost horizontal. The ordinates of the broken line A₁ B C M₂ represent the smallest values which the shearing resistance t can assume within the fill. The points of intersection B and C between the three lines...
of rupture have the abscissae \( n_1 \) and \( n_2 \) respectively. By transferring these values into Fig. 9b we sub-divide the normal stresses \( n \) on the surface of rupture \( AC_1 \) into three groups with an intensity between \( C \) and \( n, n_1 \) and \( n_2 \). For the stresses of each group the shearing resistance is determined by the corresponding sections of the broken line \( A_a B C M_b \) in Fig. 9c and the areas subject to the stresses of each group are shown on the horizontal axis of Fig. 9b. The equations for computing the shearing resistance for these sections are

\[
\begin{align*}
T_1 &= c_1 + n \tan \phi_1 \\
T_2 &= c_2 + n \tan \phi_2 \\
T_3 &= c_3 + n \tan \phi_3
\end{align*}
\]

The remainder of the investigation is identical with the procedure described under the heading C). To make the tests a to c requires an intimate knowledge of soil-testing procedure. The only source of error on the unsafe side involved in the theoretical part of the method lies in the neglect of the excess hydrostatic pressure which may develop in the space between the unconsolidated core and the toe \( A \) of the fill on account of the drainage from this core towards the toe. However, under normal conditions this error is more than compensated by the opposite error associated with the assumption that the shearing resistance in the peripheral parts of the fill has the lowest conceivable values.

REFERENCES

(1) Terzaghi, K., The shearing resistance of saturated soils and the angle between the planes of shear. Paper No. D-7, pp. 54 - 56.
(2) Terzaghi, K., Erdbaumechanik. Vienna 1925. p. 129.

No. G-7

STABILITY OF SLOPES OF NATURAL CLAY

Dr. Karl v. Terzaghi, Professor at the Technische Hochschule of Vienna

Explanation of terms and symbols.

\( w \) = water content of undisturbed sample taken at a depth \( z \) below the surface.
\( w' \) = water content of the same clay in a remoulded state after it was reduced from the liquid limit by consolidation under a pressure equal to the effective weight of the overburden at depth \( z \).
A clay is said to be normally compacted if \( w \) is equal to or greater than \( w' \).
Overcompacted clays are those for which \( w \) is appreciably smaller than \( w' \).
\( w_p \) and \( w_l \) = plastic and liquid limits, respectively.
\( I = \frac{w - w_p}{w_l - w_p} \), a ratio called the liquidity index. Its value indicates the consistency of the clay after remoulding it without changing the water content.
\( s \) = unit of weight of the clay, solid and water combined.
\( s_0 \) = unit weight of the water.
\( t_s \) = shear resistance
\( n \) = effective normal pressure \( (1) \)
\( c \) and \( \tan \phi \) = the constants in Coulomb's equation \( t_s = c + n \tan \phi \)
\( c_m \) = average shearing resistance per unit of area of a surface of rupture computed from the data pertaining to a real slide.
\( A \) = compressive strength of the clay obtained from a simple compression test.
\( z_0 \) = depth to which open vertical cracks in the clay can possibly extend.
\( z_1 \) = greatest depth of a surface of rupture below the surface of the ground.
All the numerical data are given in tons and meters.

Shearing resistance and secondary structural features. For the purposes of investigating the shearing resistance of undisturbed clay, this material may be classed into the following principal categories:
A) Soft, intact clays, free from joints and fissures. The results of the shearing tests are consistent with those obtained from triaxial compression tests.