No. H-1

# THEORY OF LATERAL BEARING CAPACITY OF PILES

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Introduction. Some years ago (1928) the writer was consulted about the lateral sliding of a small retaining-wall founded on concrete vertical piles, and was surprised at the lack of useful information concerning this matter. He therefore worked out a method of analysis, published in French, page 505 of No. 8 - 1928 of "La Technique des Travaux" (Address: 196 rue Grétry, Liège - Belgium), and found that his method was also applicable to poles or masts embedded in the ground, to sheet-piling, etc.

As it is likely that the paper referred to has passed unnoticed abroad, the theory is summed up in this contribution, and the equations are given.

In November 1928, and later in January 1930 opportunity was found to carry out a few experiments with the purpose of testing the proposed theory, but measurements were not very accurate and little could be gathered about the types of soils, as no laboratory for investigation of this kind was available. Fortunately, the writer happened to read Mr. Feagin's paper on "Lateral Pile-Loading Tests" and the discussion of it by A. E. Cummings in November 1935 "Proceedings".

Here was ample information about the subject, and the possibility of comparing the method of calculation proposed by Mr. Cummings with the writers own equations.

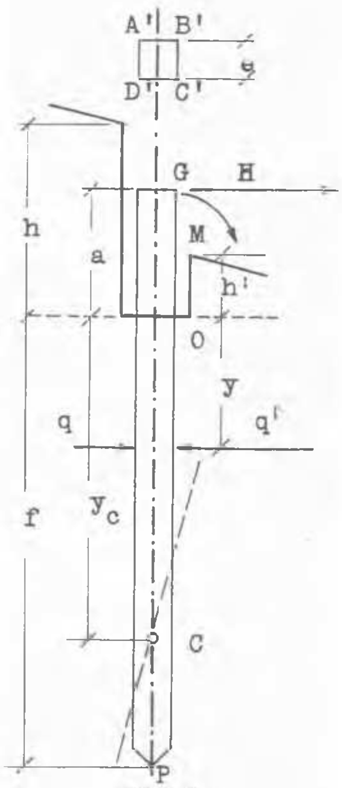

fig.1

## Theory Published in 1928.

Lettering of Fig. 1:
- G = Head of pile
- P = Bottom of pile
- O = Ground line
- C = Pivot
- A'B' C'D' = Section of pile
- A'D' = B'C' = e
- h, h' = Height of earth-load not touching the pile
- H = Horizontal resulting force at G
- M = Resulting couple of forces
- q, q' = Unit earth-pressures (depth y)

Projecting horizontally:

$$H + e\int_o^P q.dy - e\int_o^P q'.dy = o \qquad (1)$$

Taking moments about G:

$$M - e\int_o^P q(a+y)dy + e\int_o^P q'(a+y)dy = o \qquad (2)$$

If the soil is granular and without cohesion the equations of q and q' will be, as soon as the earth-particles are gliding on one-another,

between O and C : $q = A.\Delta(h+y)$
$q' = B.\Delta(h'+y)$

between C and P : $q = B.\Delta(h+y)$
$q' = A.\Delta(h'+y)$

in which $\Delta$ = weight per unit volume of soil
A = dimensionless coefficient of active earth-pressure
B = dimensionless coefficient of passive earth-resistance.

Integrating of (1) and (2) gives,

$$\frac{H}{e.\Delta(B-A)} = y_c^2 + y_c(h+h') - \frac{Bh - Ah'}{B-A}\cdot f - \frac{f^2}{2} \qquad (3)$$

$$\frac{M}{e.\Delta(B-A)} = \frac{f^3}{3} + \frac{f^2}{2}\left(a - \frac{Bh-Ah'}{B-A}\right) + fa\cdot\frac{Bh-Ah'}{B-A} - \left[\frac{2}{3}y_c^3 + y_c^2\left(a + \frac{h+h'}{2}\right) + y_c\cdot a(h+h')\right] \qquad (4)$$

It is sometimes assumed that the pile pivots about the tip. This happens only when $y_c = f$, substituting in (3) and (4), putting

$$h + h' - \frac{Bh - Ah'}{B-A} = r$$

and eliminating r, gives,

$$\frac{M}{e.\Delta(B-A)} = -\frac{f^3}{12} - \frac{H}{e.\Delta(B-A)}\cdot\left(\frac{f}{2}+a\right)$$

H and a being positive, M must be negative. As H is maximum when $y_c = f$ , a negative moment is favourable to lateral resistance. Fixing of the pile-heads against rotation develops a similar couple of forces.

Calculation of simultaneous values of M and H may be difficult, but this is another problem.

Description of Tests. Piles No. 1 and No. 2--On the suggestion of Professor G. Magnel (University of Ghent), who was good enough to provide the apparatus for measurement, with the kind permission of Mr. Hauspye, Chief-Town-Engineer, and assistance of Messrs. A. Fougnies & P. Cornelis, Contractors, tests were made in 1928 on two square concrete piles of 0.35 m x 0.35 m section, located in the South-Dock of the Ghent-harbour-extensions.

Those piles had been driven only to ascertain the effects of jetting in given conditions, and they had to be pulled out.

A windlass, fixed on the land, was connected by means of a steel cable with the pile, at the point where deflection was observed. Between two links of the attaching chain an apparatus, containing a Brinell-ball, was placed. By measuring the diameter of the Brinell-mark, the loads were determined with an accuracy of 5%. The equipment used was not very satisfactory. After the third test on pile No. 1 the cable broke and the Brinell-apparatus was lost.

The soil consists of sand, under water and fully saturated, but no samples were taken.

Pile No. 3--In 1930 Professor G. Magnel again arranged a lateral test on a concrete pile of 0.30 m x 0.31 m section, located in the building-site of the factory "Sarga" near Ghent. This time the loads were determined from an accurately calibrated gauge, belonging to the university-laboratory for concrete.

The soil consists of sand, ground-water elevation at 3 m below the surface. Professor Magnel examined samples in his laboratory and found an apparent specific weight of 1.525 $T/m^3$ for dry sand taken above the ground water elevation, and 1.492 $T/m^3$ for dry sand taken below ground-water elevation. The absolute specific weight of both sands was 2.65 $T/m^3$.

The apparent angle of friction $\varphi$ was not determined.

Pile No. 164--In 1935 Mr. Feagin tested a single concrete pile at Alton (U.S.A.). The results are taken from "Proceedings" November 1935 and expressed in metric Tons and cm.

R E S U L T S

| Date and Location | Pile No | Load H in Tons | Deflection reading d in cm | Permanent defl. d' in cm | Rotation of top section $\alpha$ | Permanent rotation $\alpha'$ |
|---|---|---|---|---|---|---|
| 14 Nov. | 1 | 1.45 | 4 | 0 | - | - |
| 1928 | | 2.15 | 8 | 1.2 | - | - |
| South | | 2.85 | 12 | 2.2 | - | - |
| Dock. | | | | | | |
| 21 Nov. | 2 | 1.30 | 6.1 | 1.1 | 0.0099 | 0.0012 |
| 1928 | | 2.05 | 12.2 | 2.3 | 0.0201 | 0.0026 |
| South | | 5.20 | 30.7 | 6.2 | - | - |
| Dock. | | | | | | |
| 16 Jan. | 3 | 1.00 | 0.35 | - | 0.002 | - |
| 1930 | | 1.40 | 0.58 | - | 0.0031 | - |
| "Sarga" | | 2.00 | 0.88 | - | 0.0045 | - |
| | | 2.40 | 1.2 | - | 0.0063 | - |
| | | 3.00 | 1.5 | - | 0.009 | - |
| | | 3.40 | 4.5 | - | 0.027 | - |
| | | 3.60 | 8.7 | 7.3 | - | - |
| Sept. | 164 | 3.3 | 0.17 | - | - | - |
| 1935 | | 6.6 | 0.35 | - | - | - |
| Alton | | 9.9 | 0.65 | - | - | - |
| U.S.A. | | 13.2 | 1.22 | - | - | - |
| | | 14.9 | 1.58 | - | - | - |
| | | 18.15 | 2.54 | 0.91 | - | - |
| | | 19.80 | 3.24 | 1.52 | - | - |

Calculation of Limit-Loads (Single piles). The hydrostatic pressures are in equilibrium and must not be considered, but the presence of water alters the value of $\varphi$ and the specific weight of the immerged soil is reduced. Supposing the same density of sand for the three tests (No. 1--No.2--No. 3), the apparent specific weight of sand above water will be $\Delta = 1{,}525$ $T/m^3$ and under water

$$\Delta' = 1.492\ (1 - 1/2.65) = 0.929\ T/m^3$$

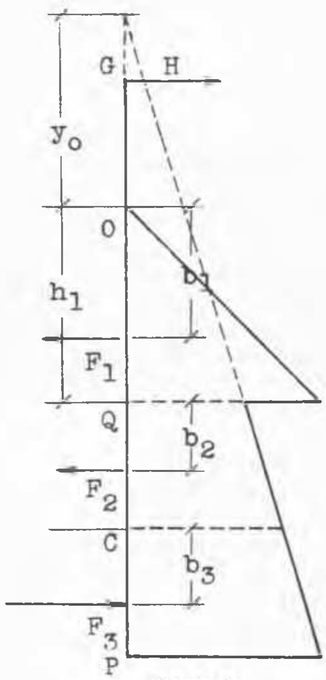


fig.2

The earth-pressures on two opposite sides of the piles are supposed to be proportional with the pressures on a wall of indefinite length, as computed by Rankine. The skin friction on the two other sides is supposed to affect only the coefficients of proportionality. This justifies the use of the values

$$A = \frac{1 - \sin\varphi}{1 + \sin\varphi} \quad B = \frac{1 + \sin\varphi}{1 - \sin\varphi} \quad B - A = \frac{4 \sin\varphi}{\cos^2\varphi}$$

for calculation of the limit-load H, the unknown coefficients altering simply the margin of security.

For piles No. 1 and No. 2 equations (3) and (4) are applicable. As $M = o$ $h = h' = o$, they give

$$H = e.\,\Delta\,(y_c^2 - \frac{f^2}{2})\ (B - A) \qquad (5)$$

$$y_o^3 + \frac{3}{2}\,ay_o^2 - \frac{1}{2}\,f^2\ (f + \frac{3}{2}a) = o \qquad (6)$$

For pile No. 1 it is assumed that $\sin\varphi = 0.6$
$B - A = 3.75$
For pile No. 2 it is assumed that $\sin\varphi = 1/3$
$B - A = 1.5$

because in the report dated 29-3-1928 about the driving of this pile it is stated that:

"About the elevation ( -5.00 ) the tip passed through a layer of unresisting soil, and the penetration had to be increased by 0.80 m.

For pile No. 3 it is assumed that B - A = 3.75 above groundwater and that B - A = 1.5 under water, as Prof. Magnel told us that the subsoil consisted of quicksand.

$\Delta$, A and B changing at the depth $h_1$ of the groundwater elevation (Fig. 2) the equations (5) and (6) are useless, and other equations must be written, which involves no difficulties.

For concrete pile No. 164 described in Mr. Feagin's paper ("Proceedings", November 1935), equations (5) and (6) are applicable, the water elevation reaching nearly the ground surface and the head of the pile being not fixed.

The pile has a conical shape and e has been taken equal to the medium-diameter. According to Mr. Cummings the apparent specific weight is 112.5 lb per cu ft dry or 1.8 $T/m^3$ and there are 30 to 37% of voids. The absolute specific weight will be

$$\Delta_a = 1.5 \times 1.8 = 2.7\ T/m^3$$

which is the usual value, and the apparent specific weight under water will be

$$\Delta' = 1.8\ (1 - 1/2.7) = 1.33\ T/m^3$$

The sand being very dense it will be assumed that

$$B - A = 3.75$$

under water.

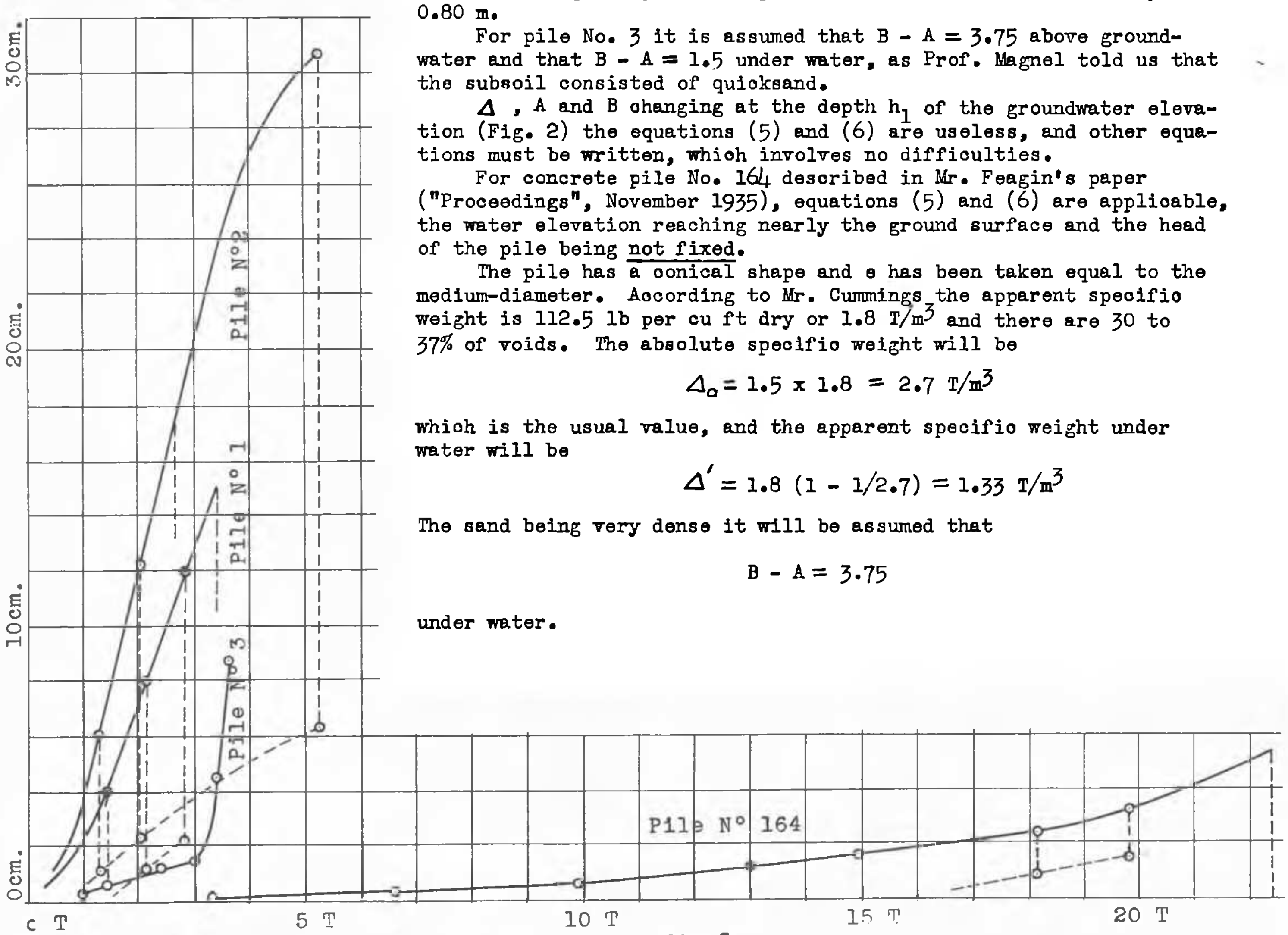


fig.3

CALCULATED LOADS

| Pile No | Value of a in m | Value of f in m | Value of $h_1$ in m | Value of B-A above water | Value of B-A under water | Value of $y_o$ in m | Load H in Tons |
|---|---|---|---|---|---|---|---|
| 1 | 5.000 | 6.50 | 0.00 | - | 3.75 | 4.89 | 3.40 |
| 2 | 5.700 | 7.30 | 0.00 | - | 1.50 | 5.67 | 2.68 |
| 3 | 0.500 | 6.80 | 3.00 | 3.75 | 1.50 | 4.88 | 6.11 |
| 164 | 0.356 | 9.15 | 0.00 | - | 3.75 | 7.50 | 22.35 |

Conclusions. Diagram Fig. 3 shows that if, for every pile, about one-third of the calculated load is taken, no permanent deflections are to be feared.

But, as the value of the angle of friction is essential, calculations are only intended on this occasion to see if acceptable results are obtainable with usual values of $\varphi$.

The question of piles with heads fixed in monoliths will be developed in a future contribution. The above theory is applicable without difficulty, and no "a priori" assumptions as to the shape of the elastic curve are necessary.

The efficient experiment described by Mr. Cummings in his paper (p. 1363 "Proceedings", November, 1935) confirms wholly the existence of a pivot, as it is stated that: "When the upper ends of the rods were moved about 1 in. to the right, the bottom of the rod moved about 1/4 in. to the left."

No. H-2 ON THE COMPUTATION OF PILES, BASED ON THE THEORY OF AXIAL IMPACT

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Based on considerations of the progress of elastic waves produced by the impact of two cylindrical bodies of different lengths, and of the velocities and the stress conditions produced by the impact, a formula is derived for the determination of the penetration of the pile under the last blow which exceeds in theoretical accuracy all such formulae heretofore derived.

The derivation of this new formula is based on the following considerations: Utilizing the conditions for the progress of the elastic wave the magnitude of the downward velocity $v_b$ of the tip of the pile is computed, due to the impact of the hammer on the pile, and expressed in the form

$$v_b = f_1 \; (h \; L)$$

in which $h$ = height of fall of the hammer, and $L$ = distance covered by the elastic wave. The upward velocity $v_r$ of the tip of the pile produced by the resistance of the soil against penetration is derived and expressed in the form

$$v_r = f_2 \; (R)$$

where $R$ = the entire penetration resistance of the pile.

Using the condition that the algebraic sum of these two velocities is equal to zero, that is

$$f_1 \; (h \; L) - f_2 \; (R) = 0$$

and assuming, further, that the elastic wave is covering a certain distance on its return from the tip to the butt of the pile, one can compute that height of drop for the hammer for which the velocity of the tip of the pile becomes zero in the moment when the elastic wave has covered the known distance L for the entire pile.

Using the expressions for $v_b$ and $v_r$ for the velocities of the tip of the pile, one can finally determine the penetration $S$ of the pile, that is the distance which the tip of the pile covers in the period from the beginning of the impact to the moment when no further penetration of the pile takes place.

The considerations outlined above lead to formulae for the computation of the height of fall of the hammer and the penetration of the tip of the pile $S$ which can be expressed mathematically as follows:

$$h = \frac{\mu_1^2 R^2}{2g \, \omega_p^2 E_p \varrho_p \mu_2^2 \mu_3^2 \, 4 \alpha^2 \beta^{m-2}}$$

$$S = L_b \cdot \frac{R}{\omega_p} \cdot \frac{\mu_1}{E_p} \cdot \left[ \frac{2(\beta^{m/2} - 1)}{\beta^{\frac{m}{2}} - \beta^{\frac{m}{2}-1}} - m \right]$$