promoting discussion and developing constructive suggestions not only of possible application to the case just mentioned but for other similar structures, results from all of which, when published, would be of general value.

No. J-3 DISTRIBUTION OF THE LATERAL PRESSURE OF SAND ON THE TIMBERING OF CUTS

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Definitions and notations. The hydrostatic pressure ratio is the ratio between the normal stress on a vertical and on a horizontal element through any point within a mass of earth.

Hydrostatic pressure distribution is any distribution of the normal pressure over a plane section through a mass of earth, involving an increase of the normal pressure in simple proportion to the depth below the surface.

- s = unit weight of the fill
- H = depth of the fill
- φ = angle of internal friction
- δ = angle of wall friction
- ε = angle between plane of rupture and the back of the lateral support
- $\varepsilon_1 = 90 - (\varphi + \varepsilon)$
- E = total value of Coulomb's earth pressure on a vertical support
- E_h = horizontal component of E
- Q = reaction along surface of rupture
- q = vertical pressure per unit of area of a horizontal section
- n_h = horizontal pressure per unit of area of the back of the vertical support
- K_o = hydrostatic pressure ratio throughout the fill before the lateral support is allowed to yield
- $K_c = E_h/h^2 s$, the minimum hydrostatic pressure ratio compatible with the conditions of the problem
- K = hydrostatic pressure ratio at any point of the lateral support, with a value intermediate between K_o and K_c
- $c_i = K/K_c - 1$, a value called the confinement-index, which expresses the effect of the partial lateral confinement of the upper part of the wedge on the hydrostatic pressure ratio within this part.

Relation between lateral yield of support and the lateral pressure on the support. This relation has been analyzed in detail elsewhere. (Terzaghi, K., A fundamental fallacy in earth pressure computations. Journ. Boston Soc. C.E., April 1936.) Fig. 1 shows the results. This diagram is based on the assumptions that the backfill was placed before the lateral support was allowed to leave its original position a b and that the back of the lateral support remains plane throughout the subsequent lateral movement. On account of this assumption, the average yield is always equal to the horizontal movement of point A, Fig. 1, at one half of the height of the wall, regardless of the type of movement of the wall. The abscissae of a B represent the minimum lateral expansion of the fill required to reduce the ratio between the horizontal and the vertical pressure, n_h and q, respectively, to its smallest value, K_c, throughout Coulomb's wedge, c a b. In the following discussion it is assumed that the lateral yield of the wall occurs other than by tilting around its lower rim a, Fig. 1. As soon as the average yield becomes equal to $c_m H$, equal to the abscissa of point A, the total lateral pressure of the sand assumes the value E determined by Coulomb's earth pressure theory, because at that state the frictional resistance along the surface of rupture a c, Fig. 1, is already fully mobilised. At the same time, the lateral expansion of the upper part of the wedge does not yet suffice to reduce the hydrostatic pressure ratio K within this part to the minimum value K_c.

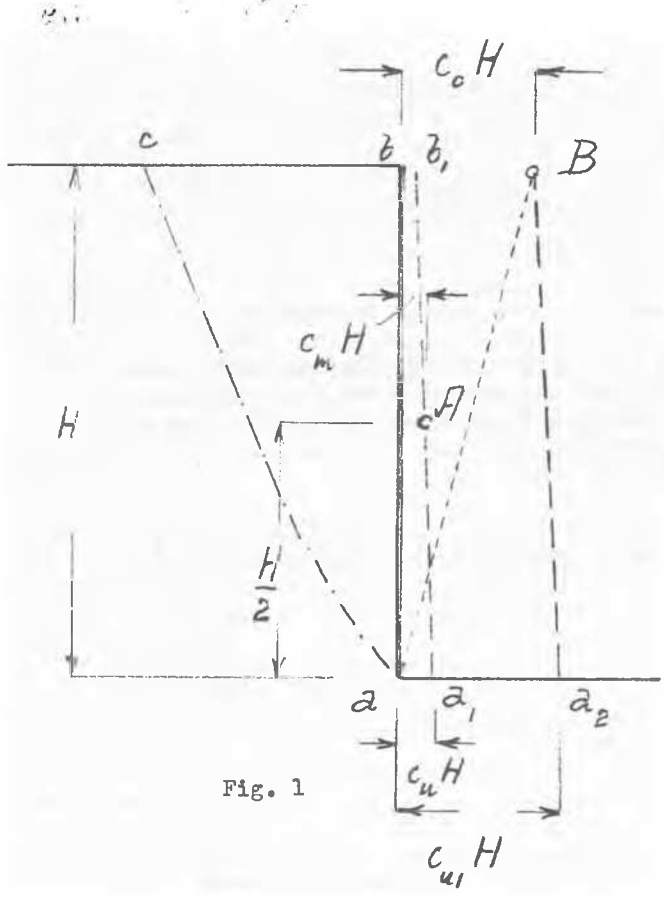

Fig. 1

According to Coulomb's earth pressure theory the horizontal component of the lateral pressure of a mass of sand with a horizontal surface on the vertical back of a yielding lateral support is

$$E_h = \frac{\tan \varepsilon}{\tan \delta + \cot \varepsilon_1} \cdot \frac{1}{2} H^2 s$$

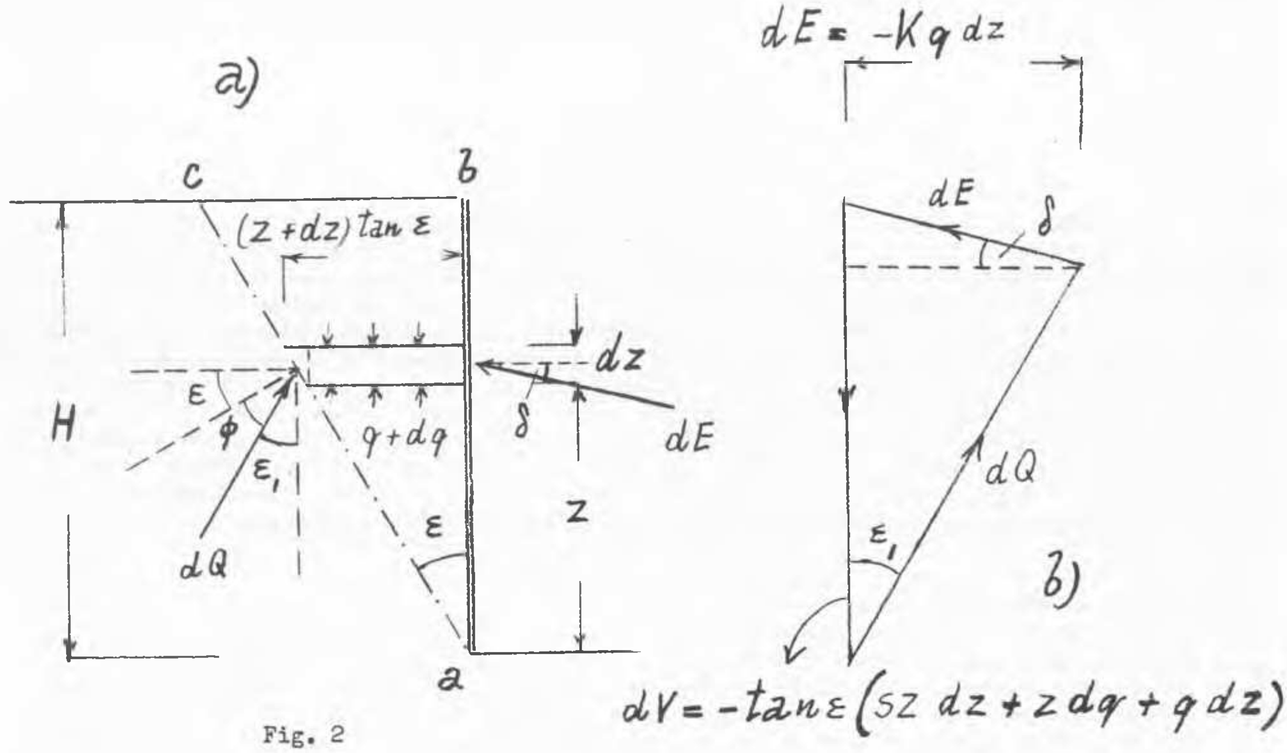

Fig. 2

or

$$K_o = \frac{\tan \varepsilon}{\tan \delta + \cot \varepsilon_1} \tag{1}$$

If the support yields in any fashion except by tilting around its lower rim, the value c_i assumes a maximum after the center of the wall has passed through the point A in Fig. 1. Any further outward movement of the wall causes K to decrease, which in turn involves a decrease of the value c_i. Finally, after the crest has arrived at B, the value of c_i becomes equal to zero.

Distribution of the lateral pressure of sand over the back of a lateral support. In order to compute this distribution we use an approximate method similar to Koenen's method for computing the lateral pressure of the filling material on the side walls of prismatic grain bins. In our case the bin has the shape of a wedge, with one vertical wall, a b, and one inclined wall, a c, Fig. 2 a. The angles of friction between the contents of our bin and the walls are δ for a b and φ for a c. In addition the hydrostatic pressure ratio decreases from K at the top of our wedge-shaped bin to K_o at the bottom. For this decrease we assume the simplest law compatible with these conditions. This is the straight-line law

$$K = K_c \left(1 + c_i \frac{z}{H}\right) \tag{2}$$

In accordance with the method used by Koenen we divide the content of our bin into horizontal elements with a thickness dz. The conditions for the equilibrium of one such element are shown in the polygon of forces, Fig. 2 b. As will be seen from this figure, the equilibrium of the vertical components requires that

$$z(s\,dz + dq) + q\,dz = q\,\frac{\tan \delta + \cot \varepsilon_1}{\tan \varepsilon}\,K\,dz$$

If we introduce the values K_o, equation (1) and K, equation (2), into this equation, we get

$$z(s\,dz + dq) + q\,dz = q\left(1 + c_i \frac{z}{H}\right) dz$$

The constant of integration is determined by the condition q = 0 for z = H. The result of the integration is

$$q = s\,\frac{H}{c_i}\left(1 - e^{-c_i \zeta}\right) \tag{3}$$

wherein $\zeta = \frac{H - z}{H}$. For $c_i = 0$ this equation becomes

$$q = s\,H\zeta = s(H - z)$$

indicating an increase of the vertical pressure in simple proportion to depth. The horizontal pressure n_h is equal to

$$n_h = q\,K = s\,\frac{H}{c_i}\,(1 - e^{-c_i \xi})\,(\frac{c_i + 1}{c_i} - \xi) \qquad (4)$$

Fig. 3 a shows the relation between the elevation z above the base of the fill and the ratio $n_h/K_o\,H\,s$ for values of $c_i =$ 0, 1, 2 and 4, according to equation (4). The value $K_o\,H\,s$ represents the horizontal Coulomb's pressure at the base of the support for hydrostatic pressure distribution. With increasing values of c_i the lateral pressure at the foot of the lateral support decreases and at the same time the pressure on the upper part of the lateral support increases. Yet the total lateral pressure is always equal to

$$E_h = K_c \cdot \frac{1}{2} H^2 s$$

regardless of the distribution of the lateral pressure.

The elevation z_o of the center of the pressure above the foot of the lateral support is determined by the equation

$$z_o = \frac{\int_0^H n_h\, z\, dz}{\int_0^H n_h\, dz} = H(\frac{2}{3} + \frac{2e^{-h} - c_i^2 + 2c_i - 2}{c_i^3}) \qquad (5)$$

For $c_i = 0$ we obtain from this equation $z_o = \frac{1}{3}$ H and for $c_i = \infty$ the value $z_o = \frac{2}{3}$ H.

Fig. 3 b represents the relation between the value c_i (abscissae) and the height z_o of the center of pressure above the base of the fill. For small values of c_i the height z_o increases rapidly with increasing values of c_i. For $c_i = 3.6$ we obtain $z_o = 0.5$ H.

The value of c_i for a given back fill cannot exceed the value of the ratio K_o/K_a unless the upper part of the support is forced against the fill. In practice this is never the case. For a given sand, the value of K_o/K_a decreases rapidly with decreasing relative density of the sand. For equal values of K_o/K_a, the maximum value which c_i can assume depends on the type of wall movement. It increases with increasing values of the ratio between the yield $\Delta \ell_u$ of the lower rim and the yield $\Delta \ell_o$

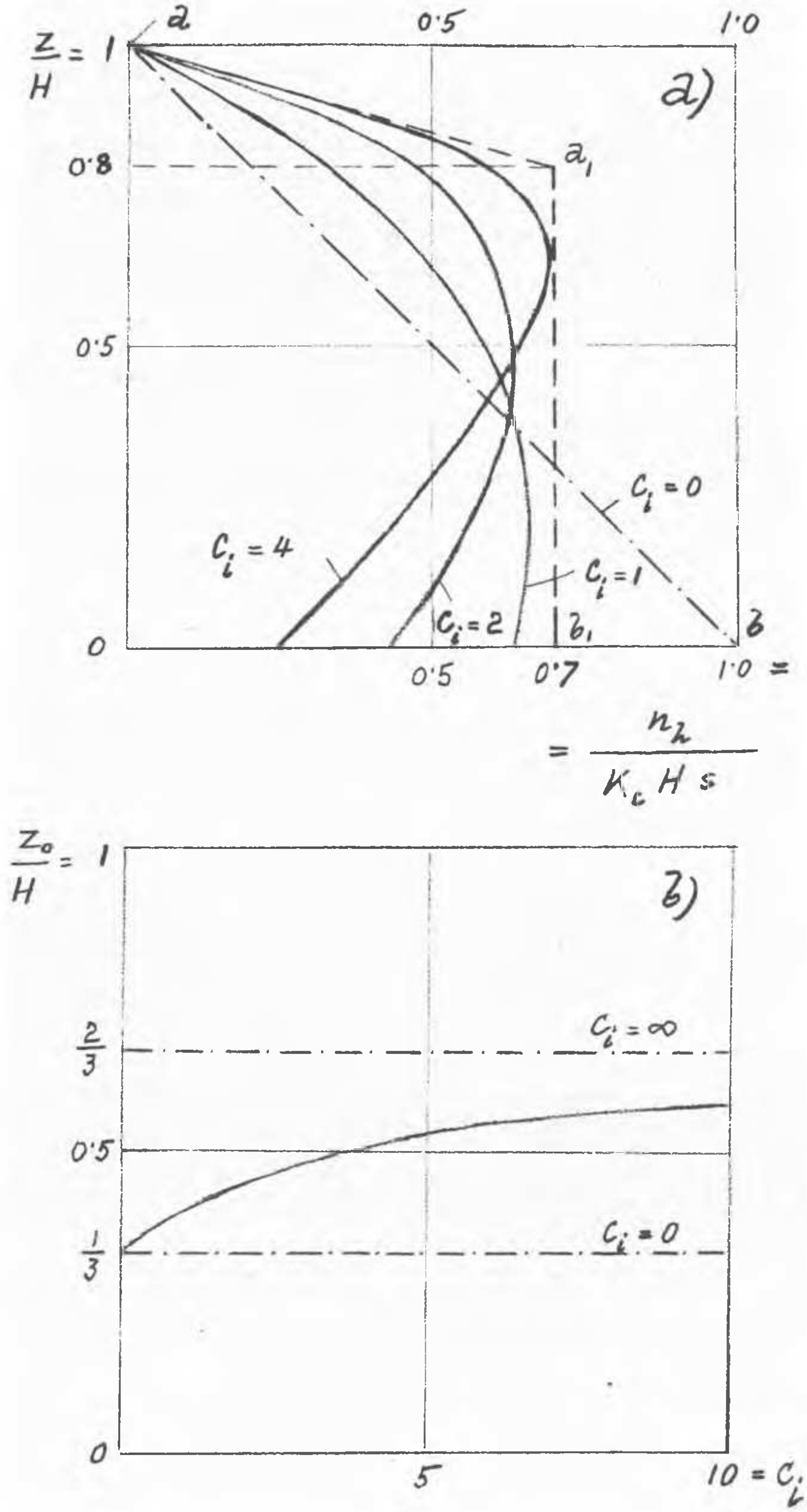

Fig. 3

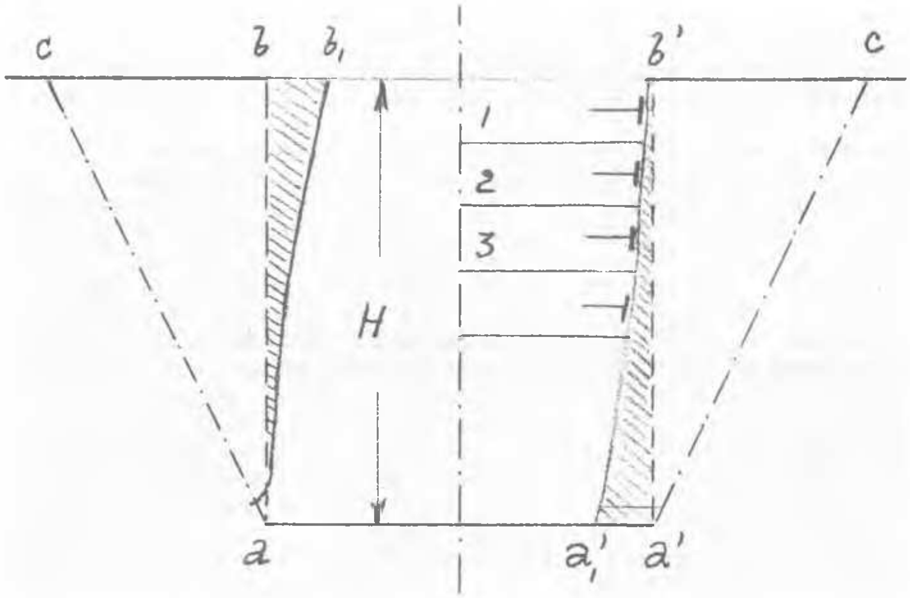

Fig. 4

of the upper rim of the support. From the large scale earth pressure tests at M.I.T. quoted in (Terzaghi, K., A fundamental fallacy in earth pressure computations. Journ. Boston Soc. C.E., April 1936) we derived the following values:

Loose sand, $\Delta l / \Delta l_o = 0.23$, $c_{i\,max}$ ca. 0
Dense sand, " $= 0.23$, $c_{i\,max}$ ca. 1
Dense sand, " $= 1.00$, $c_{i\,max}$ ca. 2

In each case c_i assumed its maximum value during the process of lateral yield, immediately after the total lateral pressure became a minimum. After the wall had passed through the critical position corresponding to $c_{i\,max}$, the value c_i again decreased in each case and finally became equal to zero, while the total lateral pressure increased slightly.

Lateral pressure on the timbering of cuts in sand. Fig. 4 shows the lateral expansion of Coulomb's wedge produced by the process of excavating and timbering a cut in a stratum of sand. The distance between the lines a b and a b_1 on the left side shows the lateral expansion of the sand which would be required to keep the value c_i equal to zero throughout the process. However, in practice, the excavation operations never permit a lateral expansion as indicated by a b b_1 on the left side of the figure, because in sands the operation of strutting follows the process of excavation without any delay. During the removal of the uppermost slices, in Fig. 4, the total lateral expansion of the sand is insignificant because the distance to which it extends into the sand is very small. As excavation proceeds towards greater depth the alteration of the original state of stress produced by the excavation operations becomes more and more important, with the result that a more energetic local expansion of the sand occurs. Hence one should expect the total lateral expansion to increase with the depth, by the width of the shaded area a' a_1' b', as shown on the right side of Fig. 4. However, in no case is one entitled to assume that the lateral expansion of the sand in the vicinity of the bottom of the cut could be less important than that near the top.

These facts determine the values of c_i and as a consequence the distribution of the pressure of the sand over the lateral support. According to the results of the M.I.T. tests cited above, a uniform expansion of a dense sand over the full height of the lateral support involves a value $c_i = 2$. The greatest value of c_i for a dense, perfectly cohesionless sand is approximately equal to 4. The lateral pressure distribution corresponding to these two values is shown in Fig. 3 a by the curves marked $c_i = 2$ and $c_i = 4$. Yet, in practice, the value of c_i can never be ascertained with an accuracy exceeding that of a crude guess, because it depends on several unknown factors such as the amount of axial compression of the struts, the compression of the wedges, and on the care with which the timbering is constructed. Hence, until we obtain numerical data by direct measurement under different field conditions we are obliged to design the timbering in such a way as to provide for any possible distribution of the lateral pressure. In Fig. 3 a the abscissae of the straight line a b represents the lateral pressure according to the hydrostatic pressure theory. In the upper part of the cut in a dense sand the lateral pressure is likely to be more than twice the theoretical pressure while in the lower part it may be considerably smaller. Hence it seems advisable to assume for the computation on the timbering of normal subway cuts a pressure distribution such as that shown by the broken line a a_1 b_1 in Fig. 3 a. However, in cuts in dense sand with exceptionally long struts and in cuts in loose sand it may be possible that the lateral pressure acts as shown in Fig. 3 a by the straight line a b.

Lateral pressure on the timbering of cuts in cohesive soils. In nature many soils have a certain amount of cohesion. For such soils experience suggests that the greatest lateral pressure is likely to act within the upper half of the cut and that the lowest part of the sides of the cut can be left, at least for some time, without any lateral support. A theoretical investigation similar to the one illustrated by Fig. 2 led to similar results. It also demonstrated that the earth can be left without any lateral support other than a protective lining between the bottom of the cut and a maximum elevation of

$$H_o = \frac{2c}{s} \tan\left(45 + \frac{\varphi}{2}\right) \qquad (6)$$

provided that the struts located above H_o are strong enough to take up the total lateral pressure exerted by Coulomb's wedge. In this formula s is the unit weight of the earth and c is the constant in Coulomb's equation $t = c + n \tan\varphi$ for the shearing resistance t of the soil under a normal pressure n. For an ideal plastic material the height of the unsupported section could even be made equal to 2 H_o, equation (6). According to Rankine's earth pressure theory the lateral pressure on the timbering should be equal to zero from the upper rim of the cut down to the depth

$$H_o = \frac{2c}{s} \tan (45 + \frac{\varphi}{2})$$

and below this depth it should increase in simple proportion to depth. Hence the preceding conclusions are utterly incompatible with Rankine's theory while, at the same time they are strikingly confirmed by experience.

The limitations to the validity of Coulomb's method for computing the total lateral pressure exerted by cohesive soils on the timbering of cuts are briefly mentioned (loc. cit.). This validity seems to be limited to cohesive soils with a high sand content.

No. J-4 EFFECT OF THE TYPE OF DRAINAGE OF RETAINING WALLS ON THE EARTH PRESSURE

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Notations.

s = unit weight of the fill, solid and water combined
s_o = unit weight of water
φ = angle of internal friction
δ = angle of wall friction
q = smallest amount of rainfall per unit of area and unit of time required to maintain a continuous flow in the backfill
k = coefficient of permeability of the backfill
h = height to which the water rises in a standpipe at any point of a fill
$n_w = h\, s_o$ stress in the water neutral stress in the fill
ε = angle between an arbitrary plane section through the heel of the wall and the back of the wall
ε_1 = angle between the plane of rupture and the back of the wall
H = height of the wall
E = lateral pressure required to maintain the equilibrium of an arbitrary wedge
E_1 = lateral pressure corresponding to the wedge of maximum thrust
w = total liquid pressure on an arbitrary plane section through the fill

The types of drainage and of saturation are indicated by indices in the following manner:
E, E_1, ε, ε_1, etc., refer to backfill with inclined filter (Fig. 3) during rainstorm;
E', E_1', ε', ε_1', etc., to a backfill with a vertical filter (Fig. 2) during rainstorm;
E'', E_1'', ε'', ε_1'', etc., to a backfill with any type of drainage after a rainstorm.

Effect of rainstorms on the stability of retaining walls. The failure of retaining walls backfilled with fine-grained material occurs almost exclusively during heavy rainstorms. The increase of the unit weight of the fill due to the rain is always irrelevant. The physical characteristics of the backfill material also remain unaltered. Therefore the effect of the rain can only be due to a change in the

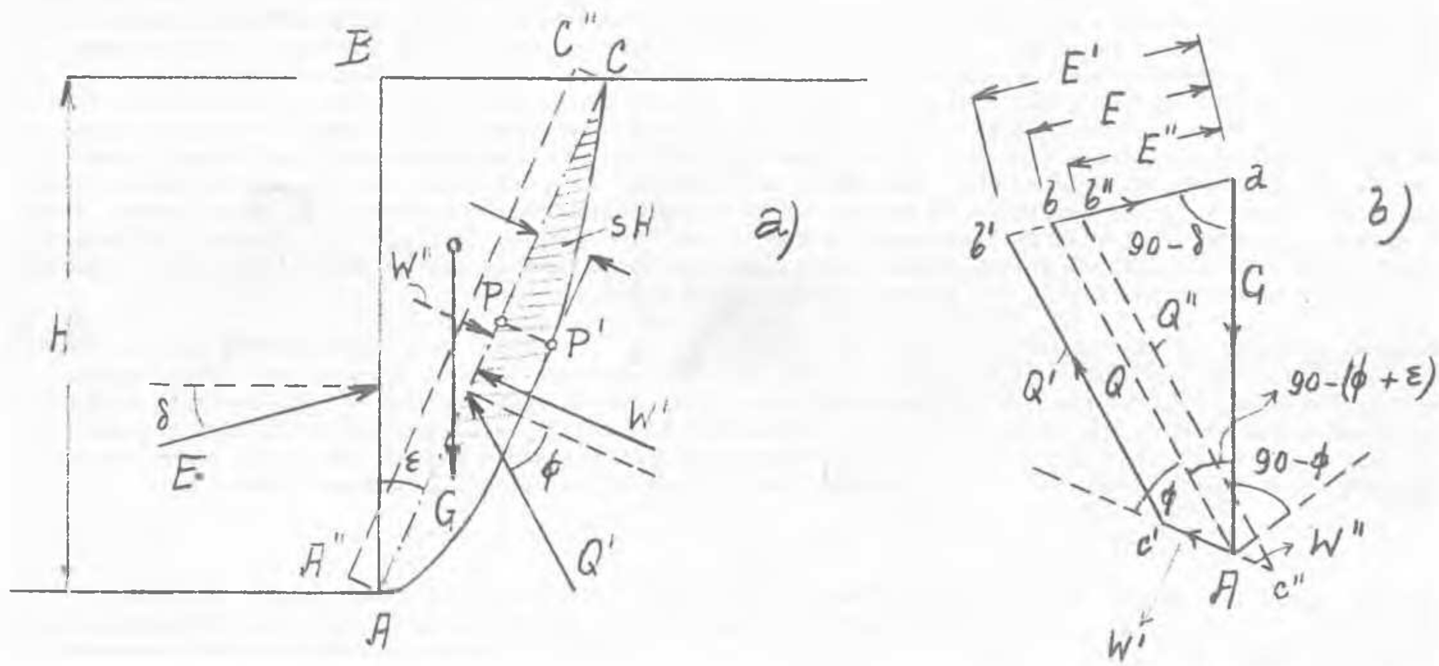

Fig. 1