No. K-1

ELECTRIC INVESTIGATION OF UNDERGROUND WATER FLOW NETS

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As has been explained formerly elsewhere (Vreedenburgh and Stevens. "Electrisch onderzoek van potentiaalstroomingen in vloeistoffen in het byzonder toegepast op vlakke grondwaterstroomingen". De Ingenieur (Holland) 1933. No. 32. ("Electric investigation of potential-flows in fluids especially applied to two-dimensional ground-water-flows"), the determination of the flow net of a water-flow percolating through a homogeneous earth-body with isotropic permeability, may be undertaken by applying the mathematical analogy existing between the equations of motion for this case and those for an electric current through a conducting sheet.

In the apparatus, constructed by us in the Laboratory for Technical Mechanics of the University of Bandoeng (Fig. 1), we use a liquid layer (diluted salt-solution) with boundaries similar to the ground-water-flow under consideration.

Thus far only two-dimensional ground-water-flows have been considered exclusively.

At the boundaries of the field of flow, where the piezometric-rise h of the ground-water, measured with respect to any assumed datum-level, has the same value, copper-electrodes are placed, as the remaining boundaries of the electrolyte correspond with the boundary-streamlines (Fig. 2).

When the ground-water flows through two earth-bodies with different coefficients of percolation k_1 and k_2, as for instance is presented by an earthen dam of dense material built upon a subsoil with larger porosity, the depths of the electrolyte should be proportional to these coefficients. Such a case has been investigated by us for which $k_2 = 7\ k_1$ (Fig. 2 and Fig. 4a). In order to satisfy the conditions of two-dimensional electric flow in the electrolyte where the depth changes suddenly, it is

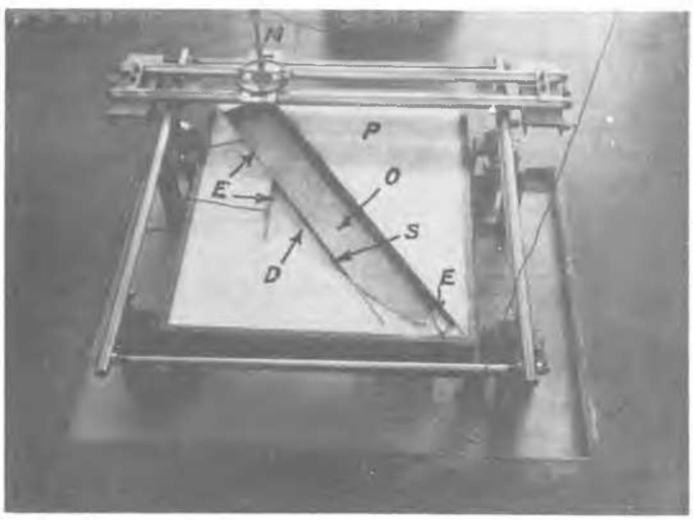
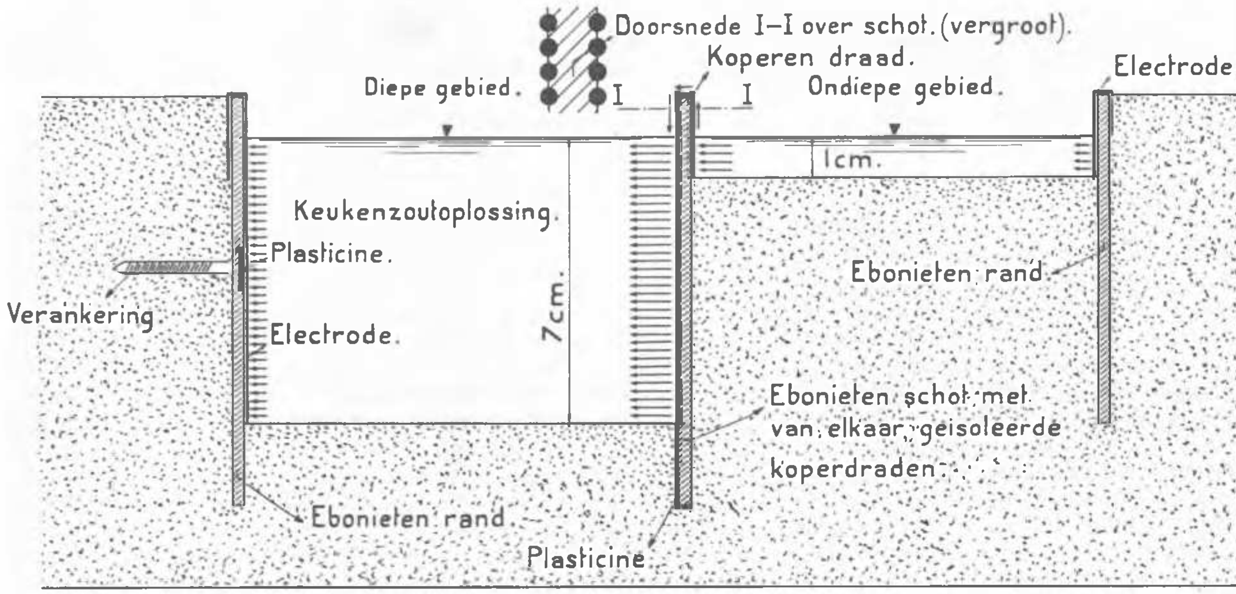


Fig. 1.
Model of a Dam with Smaller Permeability than the Subjacent Soil
D = dam; O = subjacent soil; S = cascade; E = electrode; P = paraffin; M = exploring point.

Fig. 7
Measuring Table
M = measuring canal; F = tone filter; Tr = transformer; V = variometer; T = telephone; Fr = frequencymeter; Vo = voltmeter; E = electrode; MP = measuring point.

found necessary to use a "cascade" consisting of a large number of copper wires placed side by side with a small space between each other. From this procedure we expected the electricity to flow immediately out of the thin electrolyte-sheet in the dam-region into the total height of the thick electrolyte sheet in the subsoil region without appreciable disturbance of the two-dimensional electric flow (Fig. 3). This procedure satisfied in every respect as was shown by measurements carried out in the immediate neighbourhood of the partition between the two regions. At a very short distance on both sides of the partition the electric potential measured at points situated in one vertical line showed always the same value. The results of the measurements are illustrated by Fig. 4b. With regard to this flow net we remark that at the line of separation of the dam and the subsoil, the streamlines are broken in such a way, that the ratio of the tangents of the angles with the normal to the line of separation is $k_1 : k_2$, that is for the case under consideration $1 : 7$.

In Fig. 5 the results of the measurements relating to the flow net of water percolating through the embankment of a canal in high embankments founded on a pervious subsoil, are shown. Moreover the capillary-flow has been taken into account. If wanted we may introduce also the funiculary-flow into the investigations just as well. The flow through this region may be considered as a flow through a kind of soil with a coefficient of percolation, which increases in some way from the normal value at the capillary-boundary-layer to the value zero at the funiculary-boundary-layer. The depth of the electro-



Doorsnede over de „cascade" tusschen het diepe en ondiepe gedeelte van het electrolyt.

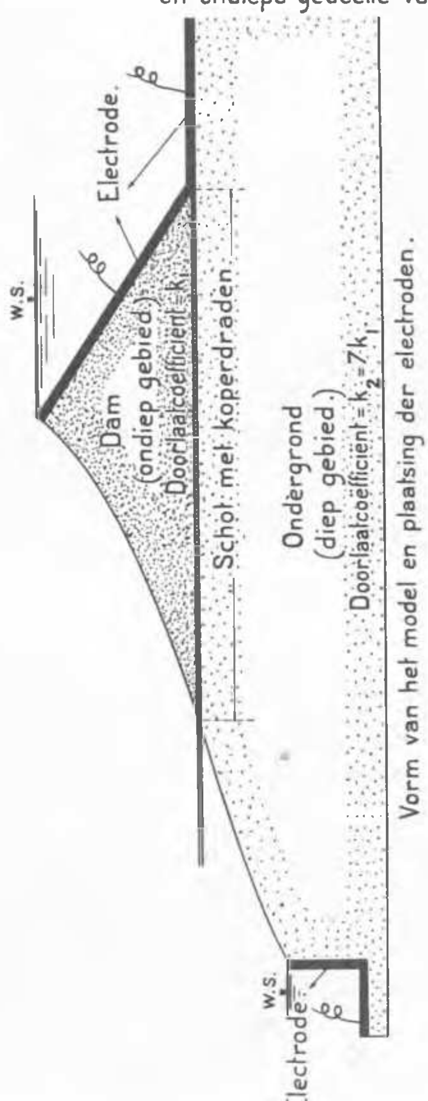
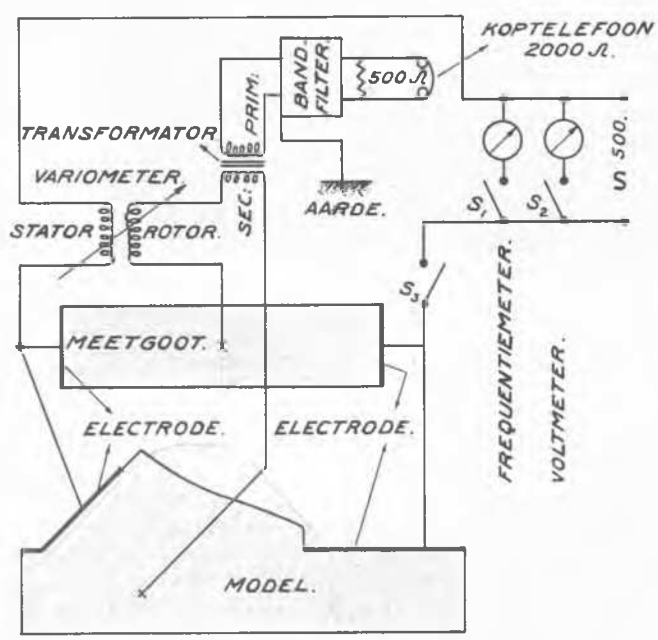


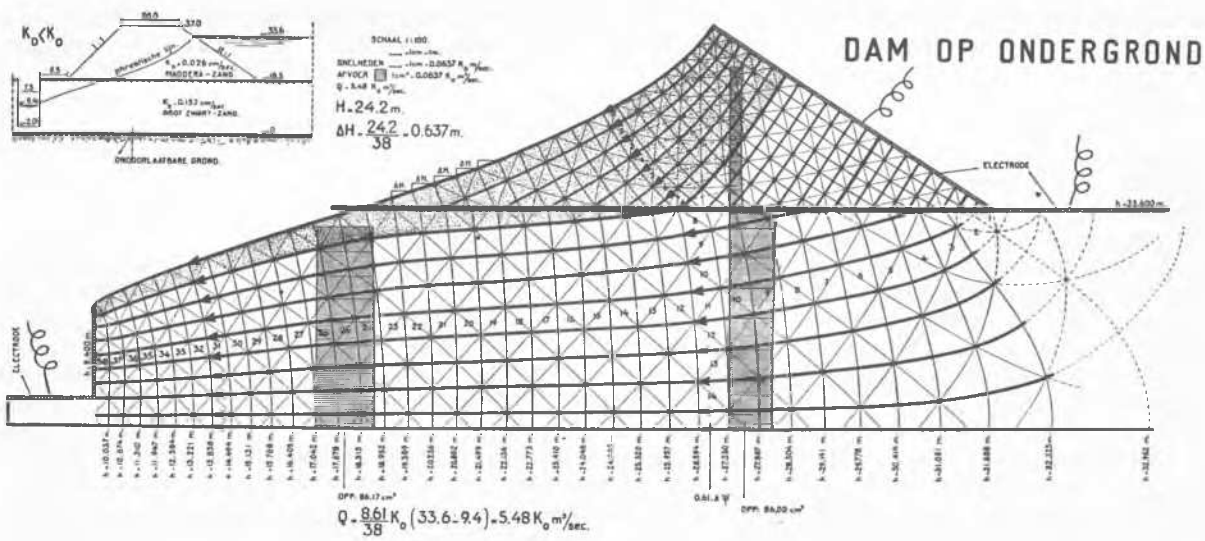
Fig. 2. Shape of the model and situation of the electrodes for the dam according to Fig. 4a.

Fig. 3. (at left) Cross section on the "cascade" between the deep and shallow region of the electrolyte.



SCHAKELSCHEMA.

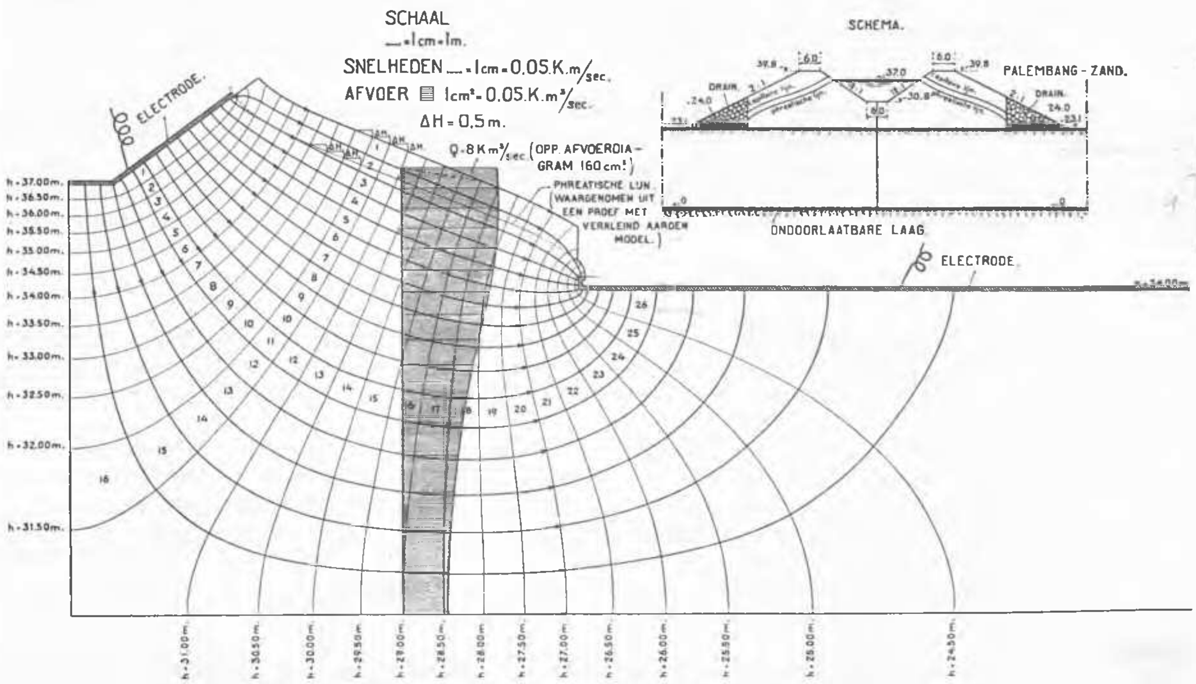
Fig. 6. Wiring Diagram.



Dam with smaller permeability than the sub-soil.

Fig. 4.

POTENTIAAL- EN STROOMLIJNENBEELD BIJ LEIDING IN OPHOOGING VOOR PHREATISCHE EN CAPILLAIRE STROOMING.



Flow-net of the phreatic and capillary flow for a canal in high embankments.
Coefficient of percolation of dam and sub-soil = k .

Fig. 5.

lyte should increase accordingly. In this connection it should be noticed that it is practically impossible to investigate the flow net including the capillary-zone and funiculary-zone by means of an earthen model on reduced scale.

In Fig. 6 the wiring-diagram of our apparatus is shown, whilst Fig. 7 shows the measuring-table. For the exploration of the field of flow we use alternating current with a periodicity of 500, producing a tone whose vanishing-point may be detected by means of a set of earphones connected in a Wheatstone's bridge. When using a tone-filter the inconvenient overtones may be eliminated and the tone disappears completely when the electrical arrangement is equipped with a variometer. Consequently the potential-measurements may be effected with a very high degree of accuracy.

In case of a two-dimensional flow through soils with homogeneous-anisotropic permeability, which means, that the permeability in two main-directions perpendicular to each other, is different, the electric analogy may be useful too. If the main-percolation-coefficients are denoted by k_1 and k_2, the plane of flow is determined, by compressing or stretching the given boundaries (as a rule two h-lines and two streamlines) in one of the main-directions, so that the ellipse of direction with the semi-axes $\sqrt{k_1}$ and $\sqrt{k_2}$ becomes a circle. (Vreedenburgh. 2nd paper. "De stationnaire waterbeweging door grond met homogeen anisotropische doorlaatbaarheid". "The steady flow of water through soils with homogeneous anisotropic permeability". See also: Schaffernak and Dachler. "Die Wasserwirtschaft". 1933. No. 30.) After the determination of the flow net by means of the electric analogy apparatus of the thus transformed plane of flow, the entire flow net is retransformed so that the boundaries coincide with their original shape. The h-lines and streamlines of the flow net which have been distorted just as well, represent now the flow net through anisotropic soil.

No. K-2

ON THE STEADY FLOW OF WATER PERCOLATING THROUGH SOILS WITH HOMOGENEOUS-ANISOTROPIC PERMEABILITY

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Summary. The extremities of the velocity-vectors for a piezometric-rise-gradient = 1, are situated on the velocity-ellipsoid. The ellipsoid of direction gives the direction of flow with regard to the surfaces of equipiezometric-rise and moreover the coefficient of percolation in any arbitrary direction. The flow net may be transformed by means of a linear transformation in a potential-flow through earth with homogeneous-isotropic permeability. The coefficient of percolation may be chosen so that through corresponding tubes of flow equal amounts of water run off.

If any point of a given earth-body be considered, then the permeability of the ground may be the same in every direction. The permeability is then said to be isotropic. If at every point of the earth-body the permeability is isotropic and moreover the percolation-coefficient the same, the soil is said to be homogeneous-isotropic. In the case of any steady flow and of any arbitrary rectangular coordinate system OXYZ, the components of the velocity of percolation at the point under consideration are:

$$v_x = \frac{\partial h}{\partial x} \; ; \quad v_y = \frac{\partial h}{\partial y} \; ; \quad v_z = \frac{\partial h}{\partial z} \; ; \qquad (1)$$

as h denotes the so called piezometric-rise of the groundwater at that point. By this is meant the level to which the water rises, measured in cm's above any assumed arbitrary but fixed datum-level, in a piezometric tube inserted in the dam and whose open lower-extremity coincides with the point under consideration.

If at any point of the earth-body the permeability is not the same in every direction, the permeability is said to be anisotropic. If this anisotropy is of the same nature at every point of the earth-mass, the soil is said to be homogeneous-anisotropic. In the following only homogeneous-anisotropy will be supposed and only steady flows will be considered.

As has been proved by Versluys any anisotropic soil may always be replaced by an equivalent porous mass with three systems of pore-tubes perpendicular to each other, which fictitious mass shows the same run-off in every direction, provided the circumstances with regard to the piezometric-rises in the field of flow, remain the same.

The directions of these three systems of pore-tubes are called the main-flow-directions of the earth-mass. If the coefficients of percolation in the three main-directions are respectively k_1, k_2 and k_3 (k_1 k_2 k_3) the velocity-components of the water at any point, parallel to the axes of a fixed rectangular coordinate system OXYZ, as OX, OY and OZ are respectively parallel to the directions of k_1, k_2 and k_3, are:

$$v_x = k_1 \frac{\partial h}{\partial x} \; ; \quad v_y = k_2 \frac{\partial h}{\partial y} \; ; \quad v_z = k_3 \frac{\partial h}{\partial z} \; ; \qquad (2)$$

For any homogeneous-anisotropic soil, the main-flow-directions and the main-coefficients of percolation are apparently the same at every point.

Let us now consider any point P in a homogeneous-anisotropic soil and any surface of equipiezometric-rise (h-surface), besides any streamline both passing through this point. We assume furthermore the axes of the rectangular coordinate-system OXYZ parallel to the main-flow-directions. Let the normal to the