Each of the ten samples contained grains within the range of two successive screens of the standard Tyler series. Each of the samples after screening was carefully washed to remove all fine material and was oven dried and stored in desiccators until tested.

T A B L E A

CRUSHED QUARTZ, LOOSE STATE

Grain Size in mm	Sieve Nos.	Test Nos.	Tan φ	φ
4.699-1.651	Through 4 on 10	196-214	0.7552	37°04"
1.651- .833	" 10 " 20	215-234	0.6424	32°43"
.833- .589	" 20 " 28	60-99	0.6045	31°09"
.589- .417	" 28 " 35	100-121	0.6168	31°40"
.417- .295	" 35 " 48	121-140	0.6145	31°45"
.295- .208	" 48 " 65	150-170	0.6335	32°21"
.208- .147	" 65 " 100	171-195	0.6492	33°00"
.147- .104	" 100 " 150	220-240	0.6715	33°53"
.104- .074	" 150 " 200	251-268	0.7059	35°13"
.074-	" 200	270-291	0.7484	36°49"

Tyler Standard Screen Scale Sieves (See Fig. 10) Area Sheared = 120 cm^2

Approximately twenty-five tests were made on each of these ten grain sizes under four different normal loads. The average ratios of the horizontal load to the vertical load were plotted in the manner shown in Fig. 5 and the resulting value of the straight line relationship was plotted in Fig. 10 against the average grain diameter. The results obtained, as shown in Fig. 10 are rather unusual. Sufficient information is not available at the present time to draw any conclusions from these results. It is very possible that the degree of angularity or sharpness of the grains varied with the grain size. In order to arrive at definite conclusions results of similar tests are now in progress on the crushed quartz in the dense state and on round grained Ottawa sand in the loose and dense states.

Acknowledgement. The author is indebted to Dr. A. Casagrande and P. C. Rutledge for suggesting the subject of this investigation and for their assistance in the design of the apparatus.

REFERENCES

(1) Research on the Shearing Resistance of Soils by A. Casagrande and S. Albert, Massachusetts Institute of Technology, 1932.

(2) Recent Developments in Soil Testing Apparatus, P. C. Rutledge, Journal of the Boston Society of Civil Engineers. October, 1935.

No. D-14 PROGRESS REPORT ON RESEARCH ON THE CONSOLIDATION OF FINE-GRAINED SOILS

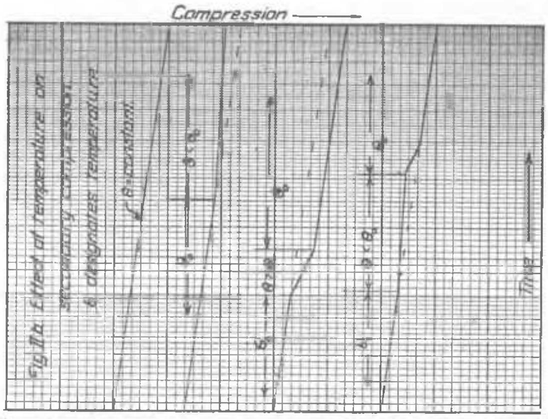
Hamilton Gray, Research Assistant in Soil Mechanics
Graduate School of Engineering, Harvard University

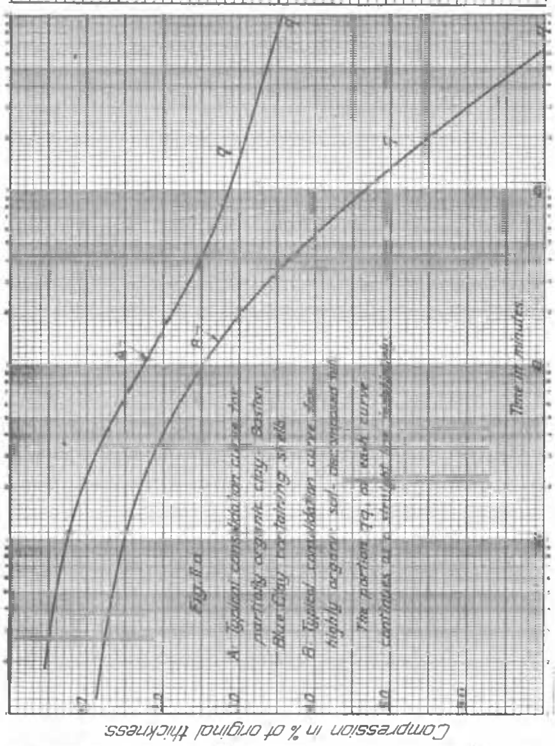
During the last year and a half an investigation of the differences between the theoretical and the empirical one-dimensional consolidation process of fine-grained soils has been in progress at the Soil Mechanics Laboratory of the Harvard Graduate School of Engineering. While the evaluation of the results is not yet complete, an outline of the work may prove of interest, and stimulate further investigations in this branch of Soil Mechanics.

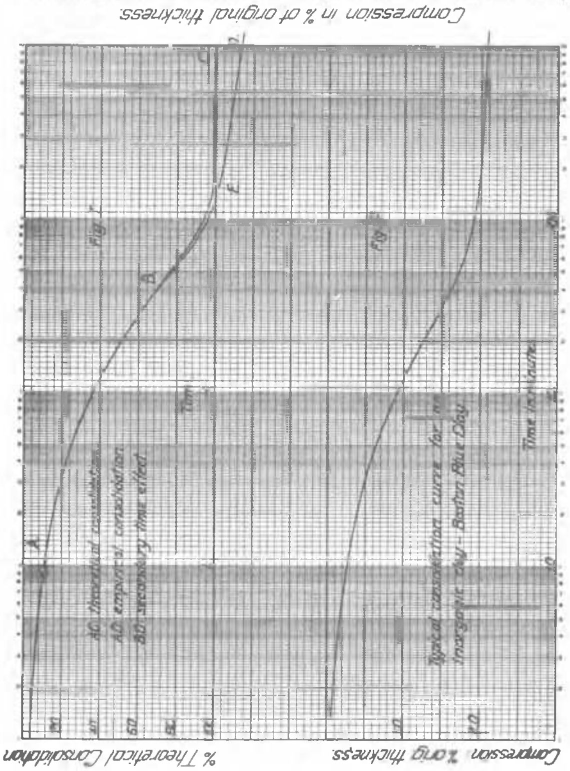
Theoretical Considerations. Up to the present time, the theoretical time-consolidation relationship which has found extended use is that resulting from the solution, with the proper boundary conditions, of the following differential equation:

$$a\frac{\partial p}{\partial t} = \frac{\partial}{\partial z}\left(\frac{k}{1+e}\cdot\frac{\partial p}{\partial z}\right) \qquad (1)$$

under the condition that a and $\frac{k}{1+e}$ are constants, p is the pressure in the solid matter, k the permeability, e the void ratio, and a the coefficient of compressibility. This equation, the same except for the coefficients, as that established by Fourier for the flow of heat, was first developed in connection with soils by Terzaghi (Erdbaumechanik). Since in reality the coefficients are not constant, but functions of p, this solution is necessarily an approximation, though a very good one provided the total change in e under the load increment is not large. This solution is well-known and the results are given here merely for the sake of clarity.







$$\text{(i)}\quad p = p_2 - \frac{4}{\pi}(p_2 - p_1)\sum_{n=0}^{n=\infty}\frac{1}{2n+1}\left[\sin\frac{(2n+1)\pi}{2H_o}\cdot z\right]\epsilon^{-\frac{(2n+1)^2\pi^2 k}{4a(1+e)H_o^2}t}$$

$$\text{(ii)}\quad Q = 1 - \frac{8}{\pi^2}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)^2}\cdot\epsilon^{-\frac{(2n+1)^2\pi^2 k}{4a(1+e)H_o^2}t}$$

Q is the degree of consolidation of a layer of finite thickness (= $2H_o(1 + e)$), drained top and bottom, p_2 the ultimate stress in the solid matter, p_1 the initial stress in the solid, and p the stress in the solid at any time during the process of consolidation. The graph of (ii) is ABC in Fig. I, for the case $\frac{k}{a(1 + e)H_o^2} = 0.01$.

While the k, a, and e values are changing during the consolidation process, the ratios $\frac{a}{k}$ and $\frac{a(1 + e)}{k}$ are practically constant for most fine-grained soils.

Using a transformation devised by Van Dusen (Bureau of Standards), the author has solved the general differential equation (1) for the case wherein the coefficients are functions of p, subject to the restriction that $\frac{a(1 + e)}{k}$ = a constant, and for a layer of finite thickness. (M. A. Biot has previously solved the same problem for a layer of infinite thickness. Annales de la Soc. Scientifique de Bruxelles, Serie B, 1935.) The result for a finite layer is:

$$u = u_2 - \frac{4}{\pi}(u_2 - u_1)\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)}\left[\sin\frac{(2n+1)\pi}{2H_o}z\right]\epsilon^{-\frac{(2n+1)^2\pi^2 k}{4a(1+e)H_o^2}t} \qquad (2)$$

where $u_j = \int_{p_o}^{p_j}\frac{a}{\beta}dp$, and $\beta = \frac{a(1+e)}{k}$ a constant.

Since empirical pressure-void ratio curves have the form: $e = e_o - c \log p$, and since $a = -\frac{de}{dp} = \frac{c}{p}$, one may write $u_j = c_1\int_{p_o}^{p_j}\frac{dp}{p}$, whence (2) becomes

$$\text{(iii)}\quad \log\frac{p}{p_2} = \left(\log\frac{p_1}{p_2}\right)\frac{4}{\pi}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)}\left[\sin\frac{(2n+1)\pi}{2H_o}z\right]\epsilon^{-\frac{(2n+1)^2\pi^2 k}{4a(1+e)H_o^2}t}$$

The corresponding expression for Q cannot be easily obtained in this case, but the Q-t curve can be plotted without excessive labor. This new curve differs from the solution (ii), but not greatly.

Empirical Results. The chief difference between any theoretical Q-t curve and empirical consolidation curves is the so-called "secondary time effect". The meaning of this term is made clear in Fig. I. This effect has been noted and described by numerous observers, not only in the laboratory but in field settlement records. It is found to a greater or less degree in all soils, but is most pronounced in soils containing organic matter. For a given soil it is less marked on the recompression branch than on the virgin branch of the pressure-void ratio diagram. Remoulding also tends to reduce this effect in those soils which undergo a definite structural change when disturbed. It appears that the magnitude of the secondary compression is a function of the type and quantity of the organic content of the soil, modified by these other considerations. The curves in Fig. II and IIa show the types of time-consolidation curves observed for different materials.

Temperature variations have a fundamental effect on

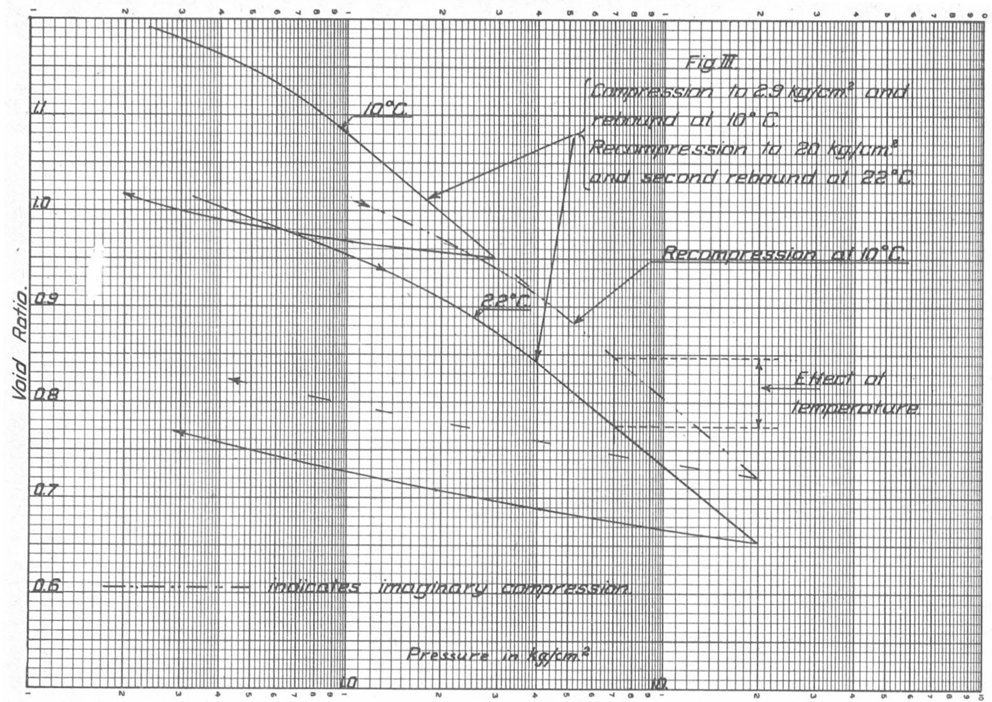


Fig. III.
Compression to 2.9 kg/cm² and rebound at 10° C.
Recompression to 20 kg/cm² and second rebound at 22° C.
10°C.
22°C.
Recompression at 10°C.
Effect of temperature
indicates imaginary compression.
Void Ratio.
Pressure in kg/cm²
1.1
1.0
0.9
0.8
0.7
0.6
1.0
10

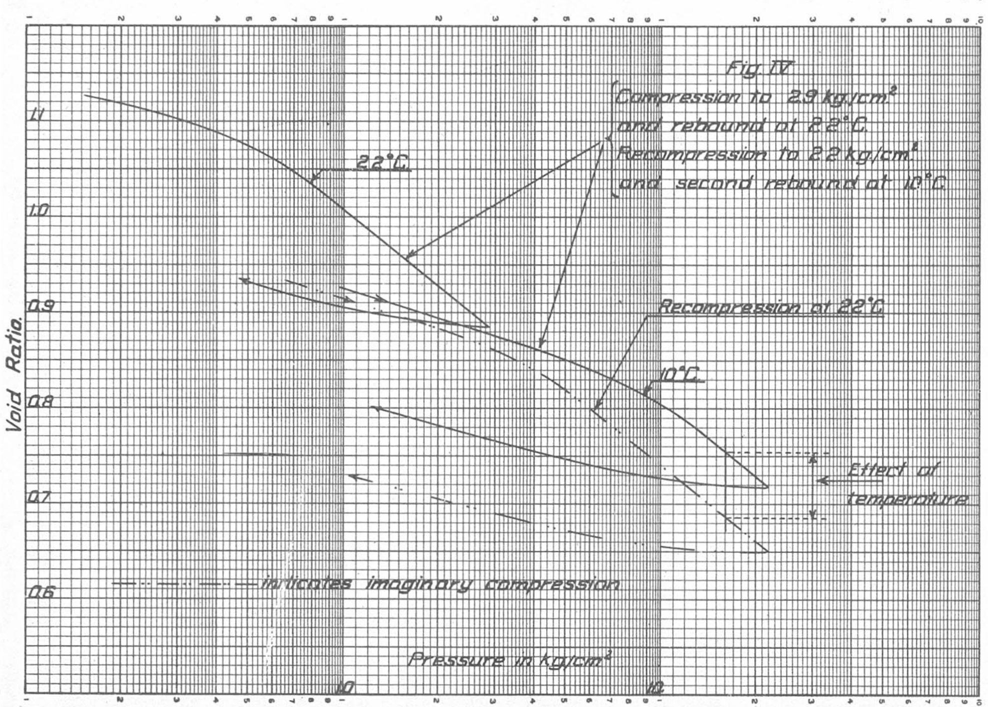


Fig. IV.
Compression to 2.9 kg/cm² and rebound at 22° C.
Recompression to 2.2 kg/cm² and second rebound at 10° C.
22°C.
Recompression at 22°C.
10°C.
Effect of temperature
indicates imaginary compression.
Void Ratio.
Pressure in kg/cm²
1.1
1.0
0.9
0.8
0.7
0.6
1.0
10

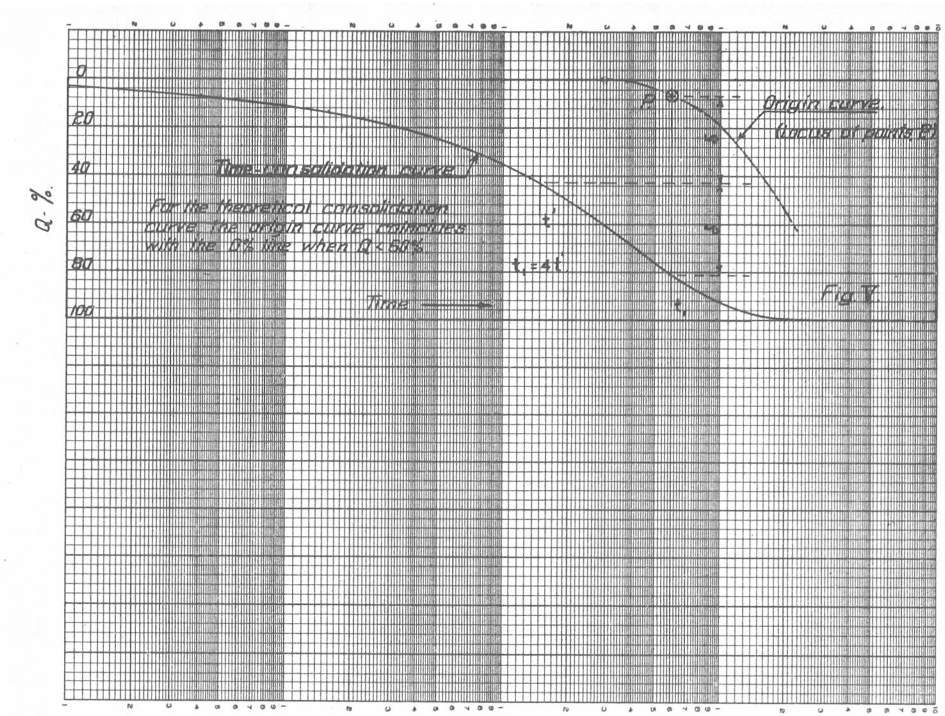


Fig. V.
Origin curve. (Locus of points P)
P
Time-consolidation curve
For the theoretical consolidation curve, the origin curve coincides with the 0% line when Q < 60%.
t₁ = 4t'
Time
Q - %.
0
20
40
60
80
100

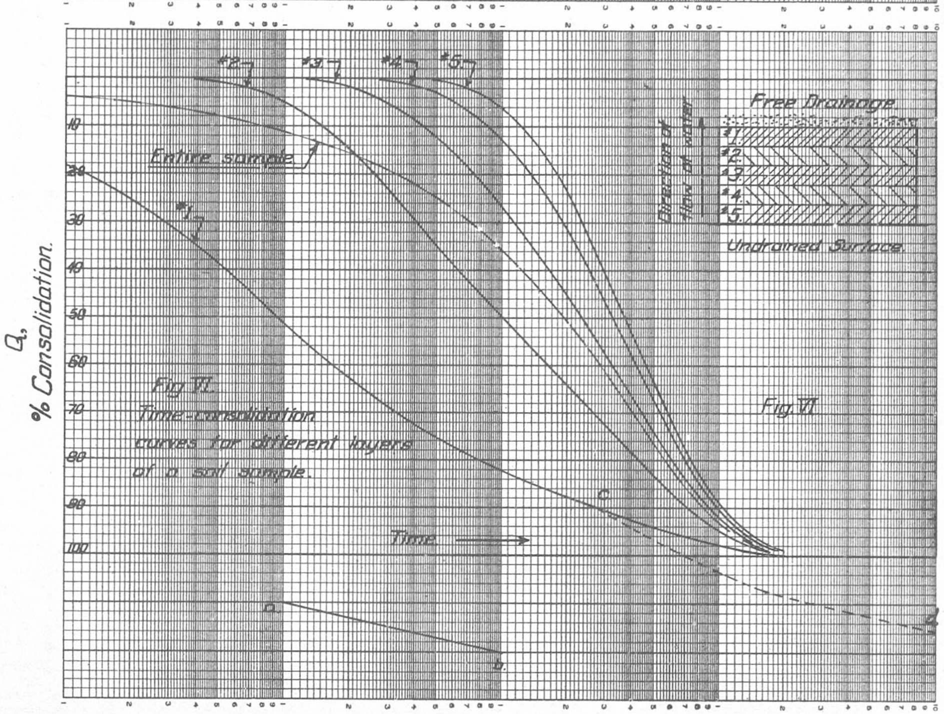


Fig. VI.
Time-consolidation curves for different layers of a soil sample.
Fig. VI
Free Drainage
Undrained Surface.
Direction of flow of water
#1
#2
#3
#4
#5
Entire sample
Time
a
b
c
d
Q, % Consolidation.
10
20
30
40
50
60
70
80
90
100

the secondary portion of the Q-t curves. The influence of this factor is explained in Fig. IIb. By running tests on the same material at different temperatures it is found that the portion ED, of the empirical curve is generally steeper for those tests which are run at high temperature. This difference is not always very great, however. Fig. III and IV indicate how this phenomenon influences the pressure-void ratio curves. It should be noted that those materials whose pressure-void ratio relations show the greatest sensitivity to temperature show also the greatest difference between the slopes of those portions ED of the time curves. Since unloading and reloading a soil may disturb its structure, an effect like that shown in Fig. III may occur without any temperature change. Hence it is possible to be deceived by such a soil because the temperature effect shown in Fig. III would be exaggerated and that of Fig. IV minimized. In some soils the pressure-void ratio diagrams are not at all affected by temperature differences, but in others, consideration of this factor leads definitely to more precise results in making settlement analyses.

In order to determine accurately the characteristics of a soil for use in practical settlement analyses, it is necessary to know what the empirical Q-t curve would be like if the secondary effect did not exist. Lacking a theory which considers this effect, we must be able to compare the experimental Q-t curve with a theoretical curve which has been adjusted in such a manner that it simulates the experimental one. The following method has been found satisfactory. Referring to Fig. V, the "origin" curve has been found to reflect the properties of the Q-t curve in such a manner that one is enabled to detect readily any deviation from the theoretical shape. Inspection of experimental Q-t curves shows that usually their shape coincides with the theoretical during the early stages of the consolidation process. We may therefore assume that the secondary effect is negligible until the consolidation has reached a definite point. But since the material nearest the drainage surfaces consolidates first, it is reasonable to suppose that the secondary effect will be felt first in the outer (i.e. most quickly drained) portions of the soil. Accordingly the Q-t curves for constituent layers of the soil mass have been computed. Fig. VI.

The process of altering the theoretical Q-t curve is in its simplest form as follows. A secondary effect of the type expressed by the line, ab, of Fig. VI is assumed to begin at any point of the soil as soon as the consolidation at that point has reached some definite value, say 90%. This corresponds to point c in Fig. VI. Another practical assumption would be to start at a point where a parallel to ab is tangent to the curve. Thereafter, the ordinates to the curve, ab, are added to the elementary Q-t curves as shown in the figure. This is done for <u>all</u> the elementary layers, and the average found graphically, i. e. by scaling the ordinates at any time and finding their average value. This average gives us the adjusted Q-t curve. By varying the shape of ab slightly and by assuming its effect to start at various values of Q a number of different adjusted curves are obtained. Each has a characteristic origin curve; and from them one may approximate with fair accuracy any empirical curve which has a point of inflection as in Fig. II and IIa, A. A curve such as B, Fig. IIa cannot be so surely imitated. Such soils are exceptional and ordinarily make such poor foundation material that one would never found a structure of any consequence upon them. For those soils which are ordinarily encountered it is possible to separate the consolidation process into two distinct parts from one of which, the theoretical, we may determine the desired soil characteristics by means of known methods.

<u>Acknowledgement.</u> This subject was suggested, as providing suitable material for research, by Prof. A. Casagrande. Many of the methods employed in this investigation were originated by him. Mr. D. D. Leslie computed the curves of Fig. VI and otherwise aided in the investigation.