No. E-6 A RATIONAL METHOD FOR THE DETERMINATION OF THE VERTICAL NORMAL STRESSES UNDER FOUNDATIONS

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Art der Belastung type of loading	lotrechte Normalspannung vertical normal stress	Abbildungen figures
Punktlast Q single force	$\sigma\frac{c}{Q}=\frac{3}{2\pi}\left[\frac{c}{R}\right]^4$	a)
Linienlast von der Länge a p = Kraft pro Längeneinheit lineload over the length a p = load per unit of length	$\sigma\frac{c}{p}=\frac{1}{2\pi}\left[\frac{c}{R_1}\right]^4\frac{a[3R_1^2+2a^2]}{[R_1^2+a^2]^{3/2}}$ $R_1^2=b^2+c^2$	b)
un endlich lange Linienlast p = Kraft pro Längeneinheit infinite long line load p per unit of length	$\sigma\frac{c}{p}=\frac{2}{\pi}\left[\frac{c}{R_1}\right]^4$ $R_1^2=b^2+c^2$	c)
rechteckige Lastfläche q = Kraft pro Flächeneinheit rectangular square with ~~the~~ the load q per unit of area	$\frac{\sigma}{q}=\frac{1}{2\pi}\left[\mathrm{arctg}\frac{ab}{cR}+\frac{abc}{R}\cdot\frac{R_1^2+R_2^2}{R_1^2R_2^2}\right]$ $R^2=a^2+b^2+c^2,\ R_1^2=a^2+c^2,\ R_2^2=b^2+c^2$	d)
unendlich lange Streifenlast q = Kraft pro Flächeneinheit infinite long strip with the load q per unit of length	$\frac{\sigma}{q}=\frac{1}{\pi}\left[\mathrm{arctg}\frac{b}{c}+\frac{bc}{b^2+c^2}\right]$	e)

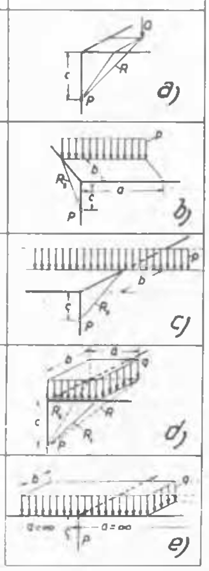

Fig. 1.

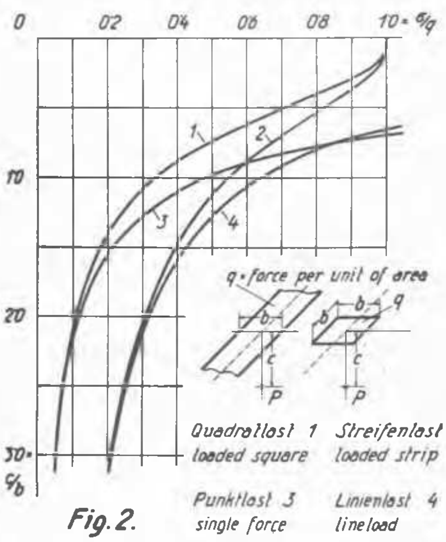

Fig. 2.

The basis for a reliable forecast of the settlement of a structure is the determination of the stresses in the underground, caused by the load. To give an idea of the extent of this preparatory work, it may be mentioned that for every point of the foundation, where the settlement is to be predicted, the function representing the variation of the vertical normal stress σ with the depth must be known. With this function determined the settlement of the point in question is given by,

$$s=\int\frac{a}{1+\varepsilon}\,\sigma\,dz$$

where z is the depth under the base of the foundation and $\frac{a}{1+\varepsilon}$ an empirical coefficient, determined in the laboratory. As $\frac{a}{1+\varepsilon}$ varies both with the material and the depth, the integration is difficult to carry out, however, a graphical integration generally suffices. It is further to be noted that, in the case of cohesive soils with low permeability, the total computed settlement will only be reached in the course of several years.

In order to arrive at even a rough approximation of the stress distribution in the underground (1) (Numbers refer to Bibliography), it is necessary, first of all, for an infinite depth theoretically to replace the generally very inhomogenous soil by a homogenous and fully elastic material (the elastically isotropic, semi-infinite body). Under this assumption Boussinesq has determined the vertical normal stress in point P (see Fig. 1a) as caused by the concentrated load Q, acting on the surface of the semi-infinite body. If we further assume that the reaction of the underground on the foundation structure is evenly distributed, we obtain through integration (2), and for the various cases of loading shown in Fig. 1., the vertical normal stress at the depth c under the bottom of the foundation. By determination of the stress distribution under a building foundation the case of loading and appurtenant formula to be applied depends, not only on the type of foundation structure, but also on the distance between the particular part of this structure and the point in the underground at the moment under investigation. For example, we are to determine the stresses in the underground close to the foundation structure, then we must use the formula for stripload or rectangular load, respectively, for wall or column foundations. On the other hand, in greater depth it is possible to replace these cases of loading by the simpler cases of line load or concentrated load, as the curves show (see Fig. 2) that, at depths greater than twice the smallest diameter of the loaded area under consideration, the difference in stress caused by the exact and the simplified case of loading, respectively, is negligible. At depths smaller than the smallest diameter of the loaded area it is, however, absolutely necessary to use the exact formulas. Under these conditions an exact stress determination requires a very large amount of calculation, and several writers have already published tables and charts, which reduce the amount of work and without which it, in some cases, would be impossible to carry the work out at all. At the soil mechanics laboratory at the Technische Hochschule in Vienna several such charts for rectangular-, line-, and concentrated loads are in use and will be described in the following.

In the case of loading shown in Fig. 1d, the point P, at the moment under investigation, lies exactly under the corner D of the loaded rectangular square A B C D (see Fig. 3). In explaining the method of stress determination to be proposed here, we confine our considerations at first to the vertical normal stress at a definite depth $z = c$ under point D and obtain the coefficient of influence σ/q as a function of the two variables a and b. This means that the coefficient of influence is completely determined by the position of the corner point B opposite to point D. The function $\sigma/q = f(a/c, b/c)$ is shown in Fig. 3 by the curves of equal influence. As these curves only depend on the ratios a/c and b/c a single influence chart can be used for the determination of the stresses at any depth by just changing the scale to which the foundation plan A B C D is drawn. In the

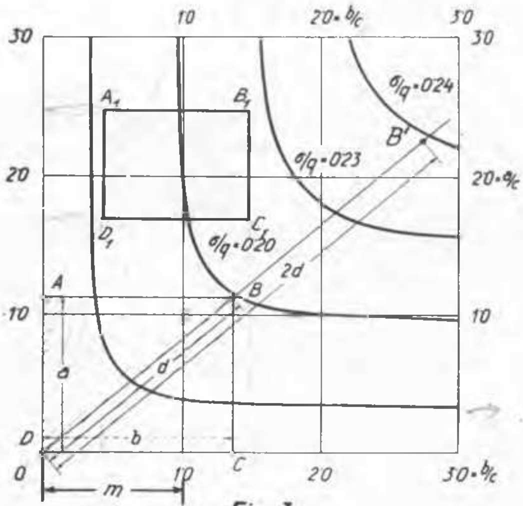

Fig. 3.

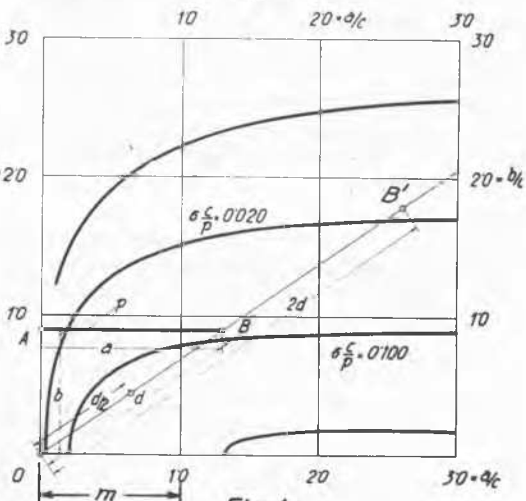

Fig. 4.

chart shown in Fig. 3 we call the distance (0 - 1.0) = m the unit length of the chart, if the foundation plan is drawn to the scale 1:x the chart will give us the coefficient of influence for the depth c = mx. In order to determine the stresses at other depths, the foundation plan may be drawn to various scales, each giving the coefficient of influence for the depth corresponding to the scale. However, such a redrawing of the foundation plan to various scales is unnecessary, as the same result can be reached in a much easier way. If we have determined the coefficient of influence in the depth c/n we draw a line from the origin to point B and extend it to a point B' in n times the distance d from the origin (OB' = n.OB = n.d) and we can then read the coefficient of influence for the depth c/n directly at B'. For example, the coefficient of influence for the depth c/2 can be read at the distance 2d from O, for the depth 2c at the distance d/2. As subdivision of the distance d is more troublesome than multiplication by a whole number it is advantageous to draw the foundation plan to a scale that corresponds to the greatest depth c at which the stresses are to be determined. The asymptotes of the curves in Fig. 3 represent the influence values for infinitely long evenly loaded strips. As the law of superposition here can be applied, we can use the influence chart even in the more general case, where the point P does not lie under a corner of the rectangular loaded square. In this case we replace the original rectangle A_1 B_1 C_1 D_1 by four rectangles, each with a corner point in O, and determine the coefficient of influence for A_1 B_1 C_1 D_1 as the sum of the influence values read directly at A_1, B_1, C_1 and D_1 giving each value its proper sign according to the position of the rectangle. From this it will also be clear, how the charts are to be applied to investigate the stress distribution under a building, the foundation of which can be subdivided into a number of rectangles. For the practical determination of the foundation stresses a plan at the level of the underside of the footings is drawn on transparent paper and placed on the influence chart in such a manner that the coordinate axis of the chart are parallel to the sides of the footings, while the origin of the chart coincides with the point of foundation at the moment under investigation. The total vertical stress is then the sum of the various loads multiplied by the respective coefficients of influence.

The process of determination of the stress in a depth z = c under a line load p per unit of length is essentially the same as described above. The foundation plan is placed in such a way on the chart that the point under investigation coincides with the origin O and the axis "a" of the chart is parallel to the line load in question. The influence value of a line load with the length a and in a distance b from the origin O, as shown in Fig. 4 (see also Fig. 1b) is read at the endpoint B. In order to investigate a line load in a position, where one endpoint does not coincide with O, a reading is made for each end and added, the proper sign being considered. After the above explanation, also the use of the chart for concentrated loads, the influence lines of which are circles, will be easily understood.

To make a sufficiently accurate determination of the various possible coefficients of influences, the original charts, designed under the supervision of the writer according to the formulas given above, are 70/70 cm large (3). The preparation of these charts required a very large amount of time and effort, which however was more than regained in elimination of tedious mechanical computations especially when investigating building foundations with a large number of various types of loadings

BIBLIOGRAPHY

1. O. K. Fröhlich: Druckverteilung im Baugrund, Wien Springer 1934.
2. Steinbrenner: Tafeln zur Setzungsberechnung, Die Strasse 1. Oktober 1934.
3. The complete charts have not been published, but can be obtained directly from the writer.