No. G-8

ON THE STABILITY OF FOUNDATIONS OF EMBANKMENTS

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Notation.
$n_1; n_2; s$ = principal normal stresses and the principal shearing stress.
$n_x; n_z; s_{xz}$ = normal and shearing stresses in Z and X planes.
$n_f; s_f$ = normal and shearing stresses in planes inclined at $45 + \frac{1}{2}\varphi$ to the plane of major principal stress.
p = external pressure.
p_w = pressure due to the weight of soil or overburden.
φ = angle of internal friction.
$p_w \tan\varphi$ = in Fig. 2 to 5 is the portion of the shearing resistance which must be supplied by weight of the soil or the overburden in order to avoid the plastic state at the given depth.
c = shearing strength.
w = weight per unit volume.
$2b$ = width of a uniformly loaded strip.
$2L$ = width of a strip with triangular loading.
h = thickness of clay deposit.
T = total tension in a vertical plane through an embankment (see Fig. 6, 7 and 8)
Cosh, Sinh, Tanh = Hyperbolic functions.
Arc tan A = angle whose tangent is A.

To analyze the strength of the underground we shall follow the same method as is used in Structural Engineering, that is we first compute the stresses produced in the soil by external loads and then compare them with the strength of the material. If the stresses exceed the strength of the material the latter will be brought to plastic state. For the condition that this happen the stresses must be such that

$$\tfrac{1}{2}(n_1 + n_2) + c_0 \operatorname{Cot}\varphi \geqq s \operatorname{Cosec}\varphi$$

or

$$\tfrac{1}{2}(n_1 + n_2) + p_w \geqq s \operatorname{Cosec}\varphi \quad \text{(Equation)}$$

(Appendix 1 A) In using this formula as a condition of plasticity we have to remember that the internal friction can be fully developed only after a complete consolidation. In relatively impervious materials this may take a very long time. Thus for pervious materials, like coarse sand, we can use for φ its full value, but must consider it as zero in relatively impervious materials like clays. In clays the rapidly applied external stresses do not themselves contribute to the strength of the material and we can therefore rely only on the shearing strength the material possesses in its natural state. At a rapid application of load the above condition of plasticity for clays therefore reduces to $c_0 = s$, that is, the imposed shearing stress must be smaller than the shearing strength of the clay in its natural state. The methods for computing the stresses and for determining the shearing resistance of soils were discussed in earlier papers(4) (The numbers in brackets refer to Bibliography at end of report).

Knowing the stresses and the φ we can determine when and where the plastic state should appear. Consider, for example, the case of a uniformly distributed loading. The most unfavourable point will be at the edge of the loaded area. If we cut out a very small element at the edge, the stresses can be considered as corresponding to the case shown in Fig. 1. Substituting into the above equation the values for stresses as given in the figure we get for the condition of plasticity

$$p\alpha/\pi + p_w = p \operatorname{Sin}\alpha \operatorname{Cosec}\varphi/\pi$$

hence

$$p_w = c_0 \operatorname{Cot}\varphi = p(\operatorname{Sin}\alpha \operatorname{Cosec}\varphi - \alpha)/\pi$$

$$p = p_w \, \pi/(\operatorname{Sin}\alpha \operatorname{Cosec}\varphi - \alpha) \quad \text{Formula (B)}$$

If we have a cohesionless sand and $\varphi = 35^\circ$ the least favourable plane i.e. the plane where the required p_w is max. is at $\alpha = \frac{1}{2}\pi - \varphi = 55$ and $p = 6.72p_w$ or $p_w = 0.149p$. Thus in order to avoid plasticity there should be an overburden equal to about 15% of the foundation pressure, the required pressure $p_w = 0$ when $\alpha = 98$ which is the border plane of the region where plastic state cannot be produced by any pressure. If $\varphi = 25$ the least favourable plane is at $\alpha = 90-25 = 65$ and the required overburden $p_w = 0.32$ p.

If we have an impervious clay then we can rely only on the shearing resistance that the material has in its natural state and beyond that we consider φ as zero. The least favourable plane then will be at $\alpha = 90 - 0 = 90$. In order to avoid plasticity the material must have a natural shearing strength c_0 such that $p = \pi c_0$. Thus if $p = 5\text{kg/cm}^2$ the shearing resistance of the clay must be at least $c = 0.318p = 1.09$ kg/cm^2. If we had a sand with $\varphi = 35$, the pressure p_w must be at least $0.149p = 0.75$ kg/cm. If this pressure is to be supplied by weight of overlaying soil the depth must be about 4.3 m (14 feet) if it is all above the groundwater table and 6.9 m (23 feet) if submerged.

Formula B applies generally to any uniformly distributed loading. In the case of a strip load α is the angle subtended by the width of the loaded strip (5). Using the least favourable angle $\alpha = 90 - \varphi$ the formula becomes

$$p = \frac{p_w \pi}{\sin(90 - \varphi)\mathrm{Cosec}\ \varphi - 90^\circ + \varphi}$$

If the distribution of the external load is not uniform, the expressions for stresses become more involved. The solutions will then have to be made individually for each case as they are in general not independent of the scale. Fig. 2 shows the location of the most unfavourable points in the case of a terrace loading. The full lines show the points where the required p_w is max at the given depth. The broken lines are border surfaces of the region where the plastic state cannot be produced by any pressure. Thus when $\varphi = 35$ the most dangerous point at the depth $z = 0.75$ b is at 0.25 b outside the toe of the slope. At this point $p_w = 0.092 \times p \mathrm{ Cot }35 = 0.13$ p. The pressure due to overburden must be greater than this value in order to prevent a plastic state at this point. Fig. 3 shows the conditions under a strip with triangular loading. When $\varphi = 35$ the least favourable point in the horizontal plane at depth $x = 0.75$ L is at 0.25L inside the toe. At this point $p_w = 0.0667 \mathrm{ p\ Cot }35 = 0.098$ p. When $\varphi = 25$ the least favourable point in the same horizontal plane is at the center line of the loaded strip. To use such a diagram in a numerical case we would have to compare the $p_w \tan\varphi$ required with the shearing strength available. We can do this by comparing the $p_w \tan\varphi$ curve with the diagram representing the shearing strength caused by the overburden. There is no plastic state anywhere if the available strength is higher than the $p_w \tan\varphi$ required.

As an example consider the conditions under a triangular loading when $\varphi = 35$, $p = 5$ kg/cm^2 and the weight of soil $w = 1.83$ g/cm^3 (114 lbs/cu ft) The pressure p_w which is necessary in order to avoid the plastic state is shown in Fig. 4. Lines OA represent the pressures supplied by the weight of soil. When $L = 1.5$ m (line OA_1) the deficiency in pressure is greatest at about $z = 0.6L$. If we draw a line tangent to the p_w curve and parallel to OA_1, we can determine the pressure or the depth of surcharge necessary in order to avoid the plastic state. In the given case the required pressure is 0.28 kg/cm^2 or the corresponding surcharge $d_1 = 1.03L = 1.54$ m. If $L = 2.5$ m the least favourable point is at about $z = 0.25L$, $p_2 = 0.175$ kg/cm^2 and $d_2 = 0.96$ m. If $L_4 = 12$ m, $d_4 = 0.12$ m; $L_5 = 24$ m, $d_5 = 0.012$ m, etc. The least favourable point approaches the toe of the slope as the width increases. Theoretically the plastic state at the toe can never be avoided without a surcharge, but the magnitude of the latter becomes very small as the width increases and the depth of the least favourable point decreases. Its value at the toe is

$$p_w = \frac{p}{\pi} \cdot \frac{z}{2L} \sqrt{(2\ln \frac{2z}{L})^2 + (\pi - \frac{z}{L})^2}\ \mathrm{Cosec}\,\varphi - 2(1 - \ln\frac{2z}{L})$$

Fig. 5 shows a similar example for the case of terrace loading. If $L = 6$ m the deficiency in pressure is greatest at about $z = 0.22L$, $p_w = 0.13$ kg/cm^2 and the required surcharge $d_3 = 0.12L = 0.72$m. In any general problem where the expressions for stresses are involved we could determine the stresses for a few critical points and apply the condition A (page 1) to numerical values. For example, if we have the stresses $n_z = 5.61$ kg/cm^2, $n_x = 2.15$ and $s_{xz} = 1.76$ we get $n_1 = 5.61$, $n_2 = 1.26$ and $s = 2.18$ kg/cm^2 [Note: $s = \frac{1}{2}\sqrt{(n_x - n_z)^2 + 4s^2_{xz}}$; $n_1 = \frac{1}{2}n_z + \frac{1}{2}n_x + s$; $n_2 = \frac{1}{2}n_z + \frac{1}{2}n_x - s$.] If the material were an impervious clay it should possess a shearing resistance of at least $c = s = 2.18$ kg/cm^2. If the material is a cohesionless sand, the given point must be under a pressure

$p_w = s\ \mathrm{Cosec}\,\varphi - \frac{1}{2}(n_1 + n_2)$. This gives $p_w = 0.36$ for $\varphi = 35^\circ$, $p_w = 1.72$ for $\varphi = 25^\circ$ and $p_w = 4.98$ kg/cm^2 for $\varphi = 15^\circ$

As has been pointed out earlier the appearance of the plastic state cannot lead to a slide unless there is a possibility for a progressive failure. The question therefore arises what stress or what state to use as a criterion of stability of the foundation. The answer will obviously depend on the nature of the particular problem. In making earth fills the engineer may well allow loads beyond the plastic limit but in many other structures the settlements caused by the movement of grains at the mere approach of the plastic state may already be prohibitive. The best way would always be to base a new design on data obtained from analysis of some existing structure resting on an identical or a similar deposit. Analyses of existing foundations of which settlement records have been kept offer therefore most valuable material. Of particular value are the analyses of cases where trouble was experienced.

It may be well to call the attention of American engineers to investigations made by Dr. Froelich (2). Dr. Froelich has made an independent derivation of the formula (B) and using $\varphi = 35$ and $w =$ (w = when submerged) has applied it in analyses of a series of foundations of bridge piers and other structures resting on sand. The agreement of the theory with the observed behaviour of foundations is striking. The foundations were satisfactory in all cases where the stresses were kept below the magnitudes at which the plastic state would begin and trouble was experienced in all cases where this limit was exceeded.

The distribution of stresses will be greatly modified if the ground is not homogeneous. The extreme cases discussed earlier were the case of a frictionless plane at a moderate depth below the foun-

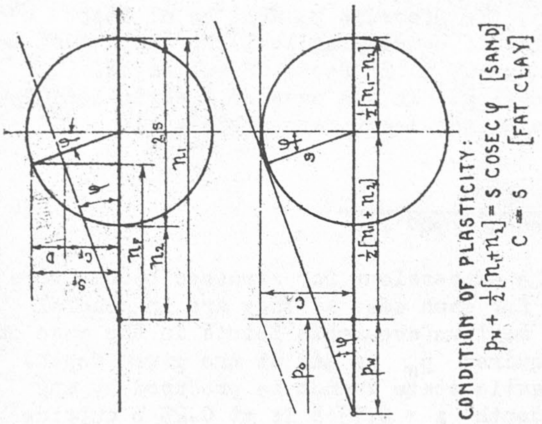
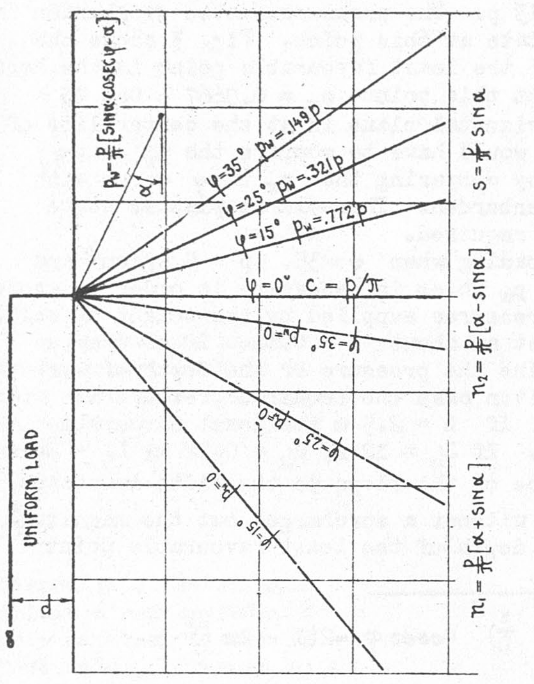


FIG. 1a



LOCATION OF LEAST FAVOURABLE PLANES

FIG. 1

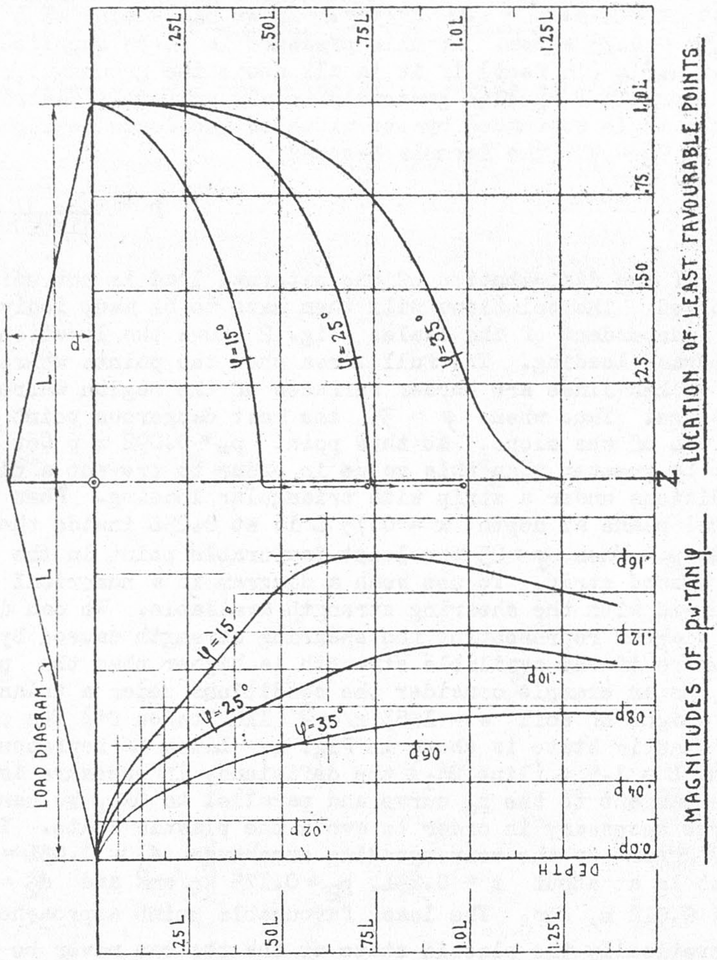
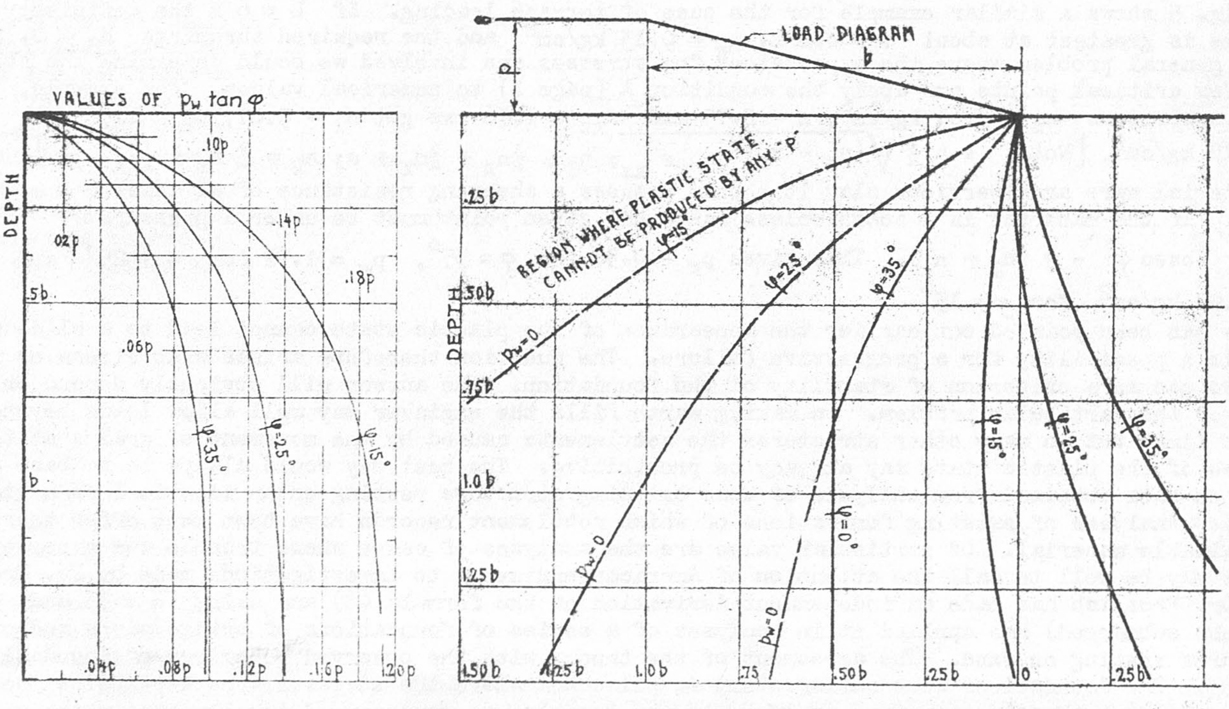


FIG. 3



LOCATION OF LEAST FAVOURABLE POINTS

FIG. 2

dation and the case of a rigid surface where the friction is not restricted. As far as the theoretical condition of plasticity is concerned the conditions become less favourable than in the case of homogeneous ground. According to Carothers' solution for the case of a rigid rough surface at a moderate depth below a uniformly loaded strip (1) the vertical and the horizontal normal stresses at the rigid boundary are equal $n'_z = n'_x$. Therefore $\frac{1}{2}(n_1 + n_2) = n'_z$, and the condition of plasticity becomes $p_w = s' \ \mathrm{Cosec}\,\varphi - n'_z$. If the depth is small the least favourable points are below the edges of the loaded area, and the stresses here are higher than in the homogeneous case. On the other hand, the material underneath the central portion of the strip is in more favourable condition than in the homogeneous case. As the formulae for stresses are rather involved, they are omitted in this report. A table of computed values was given in (4) p. 226. It would be of very much interest if such analyses could be compared with analyses of existing foundations in corresponding conditions. The appearance of the plastic state would either change the distribution of pressure from uniform to one increasing toward the center or else subject the foundation slab to a considerable bending moment. The case is much less favourable if instead of being rough the hard surface is frictionless and offers no shearing resistance. The condition of plasticity in the case of a uniform strip load now becomes $p_w = p \ \mathrm{Cosec}\,\varphi \ \mathrm{Tan\,h}\,(\pi b/2h)$ (Appendix 1 C). This requires that p_w at the frictionless boundary be larger than p; e.g. if $\varphi = 35$ and $b = h = 6$ m we get $p_w = 1.6$ p. If the material is an impervious clay the above condition becomes $c = p \cdot \mathrm{Tan\,h}\,(\pi b/2h)$. In the case of a triangular loading the condition is

$$p_w = p \ \frac{2h}{L} \ \mathrm{Cosec}\,\varphi \ \ln \mathrm{Cosh}\,(\pi L/2h) \quad \text{(Appendix 1 D)}$$

e.g. if $\varphi = 35$ and $L = h = 6$ m. $p_w = 1.07$ p. In case of clay the condition becomes $c = p \ \frac{2h}{L} - \ln \mathrm{Cosh}\,(\pi L/2h)$ e.g. if $h = L = 6$ m, $c = p \ \ln \mathrm{Cosh}\,\frac{1}{2}\pi = 0.613$ p.

The following chapter treats the resistance of a clay deposit under a triangular loading more in detail.

Embankments resting on clay. A deposit of clay must often serve as a foundation for a dam. Being a relatively impervious material it helps to make the foundation watertight and safe against erosion, but for the same reason it consolidates very slowly and therefore has a lower shearing resistance than pervious materials. There thus arises the question of strength.

The stresses caused by the external loads will contribute to the strength of the clay only at the rate the consolidation is proceeding. In the case of relatively impervious material and thick deposits the process of consolidation is so slow that the increase in strength due to external stresses must be neglected. We can thus rely only on the strength the clay has in its natural state. The criterion of plasticity will then simply be $s = c$.

In the following we shall simplify the outline of the dam to a triangle of equal area and width. Assuming that the stresses are proportional to the ordinates of the height we shall thus get a strip loading with a triangular distribution (Fig. 6). The formulae for stresses in the elastic case were given in (4) pages 227-236. The max shearing stress is $s = 0.256$ p and occurs at depth 0.50 L below the base. There is no plastic state anywhere in the underground as long as the pressure p is below 3.91 c.

The picture of the distribution of stresses will be greatly modified if the ground is not homogeneous. Consider the case where at a moderate depth there is a rough rigid boundary where the friction is not restricted such as rock or a deposit of gravel. According to the theory of elasticity the principal shearing stress at the rigid boundary is

$$s = \frac{2d \ \cdot \ p}{L \ \cdot \ \pi} \cdot \left[2 \ \mathrm{Arc\,tan}\ e^{\frac{\pi}{2}\frac{x}{d}} - \mathrm{Arc\,tan}\ e^{\frac{\pi}{2}\frac{x+L}{d}} - \mathrm{Arc\,tan}\ e^{\frac{\pi}{2}\frac{x-L}{d}}\right]$$

(4) p. 222. If d is small relative to L the shearing stress is max at about $x = \frac{1}{2}L +$. The formula then becomes

$$s = \frac{2d \ \cdot \ p}{L \ \cdot \ \pi} \left[2 \ \mathrm{Arc\,tan}\ e^{\frac{\pi}{4}\frac{L}{d}} - \frac{\pi}{2} \qquad - \mathrm{Arc\,tan}\ e^{-\frac{\pi}{4}\frac{L}{d}}\right]$$

With the aid of these formulae we can compute the pressure p at which the plastic state appears. Thus when $L/d = 5$, $s = 0.19$ p; and we have a fully elastic state as long as the pressure is below $p = 5.27c$. The above formulae do not hold when the ratio L/d is very small as we shall then have a homogeneous ground. The limiting condition at small values of L/d therefore is $p = 3.91$ c. Curve A, Fig. 8, shows the relation between L/d and p/c. At high values of L/d i.e. when the clay deposit is very thin $p = cL/d$.

In many cases the appearance of the plastic zone cannot yet lead to a failure; the condition $s = c$ is therefore not a complete criterion of the stability of the foundation. It is often desirable to have also an estimate of the ultimate resistance. When we increase the loading beyond the point where the plastic state first developed, the plastic region will begin to spread. As such a semiplastic state where part of the material is in elastic and part in the plastic state is very difficult to analyze mathematically, we shall consider only the stage where the plastic region has spread so far that a slide must occur. All the material that offers resistance will then be in plastic state i.e. its stresses are no longer proportional to the strains.

In our analyzes we shall use Hencky's principle of equilibrium of a plastic material and apply the solution given by Prandtl in (3). To simplify the formulae we shall neglect the resistance offered by the material which is not underneath the main body of the dam. Further refinement is discouraged also

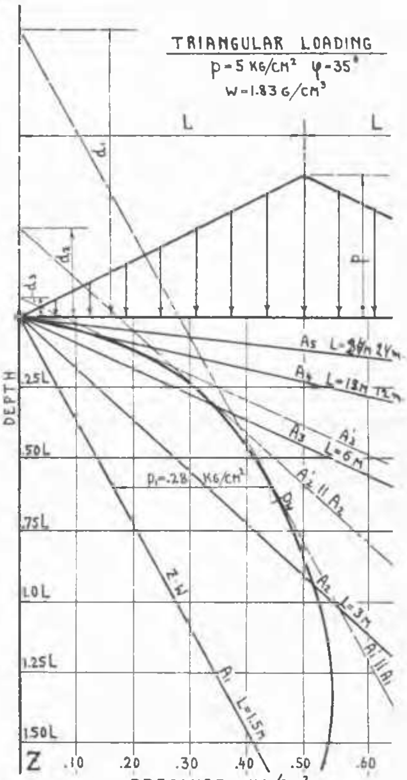


FIG. 4

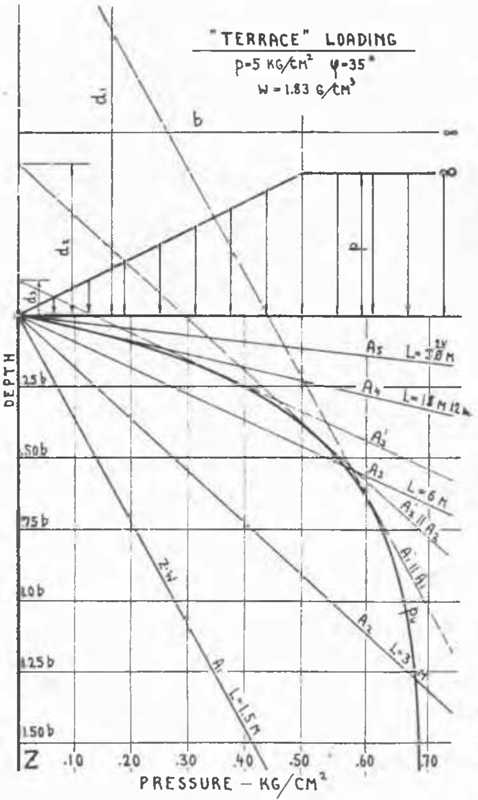


FIG. 5

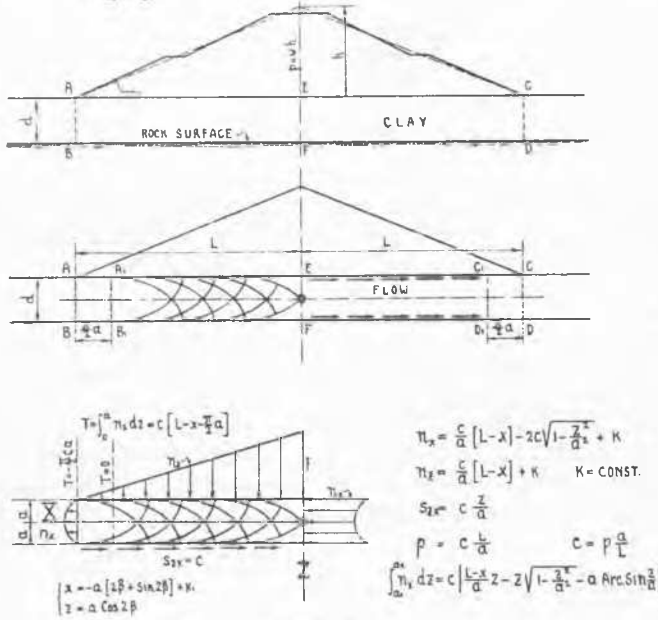


FIG. 6

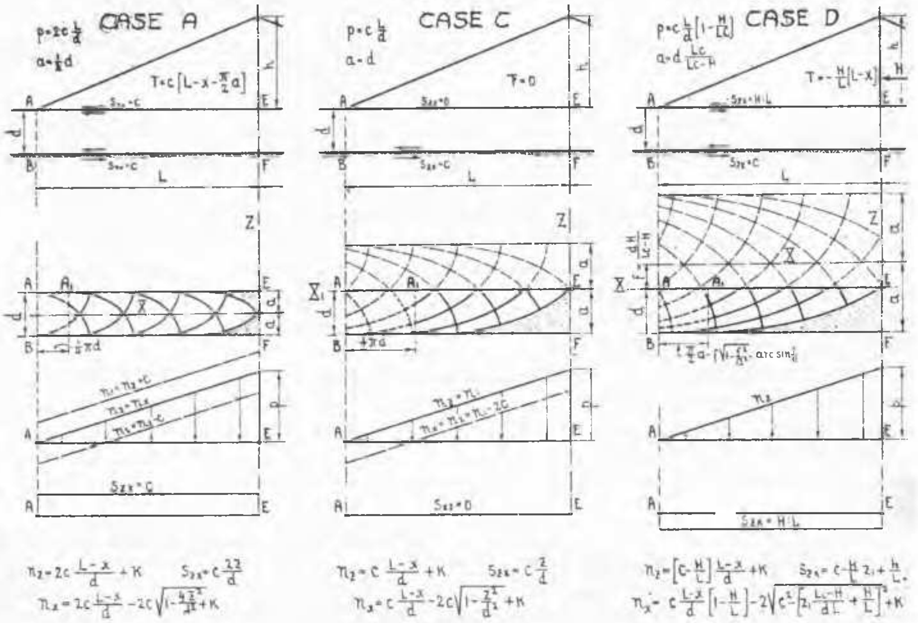


FIG. 7

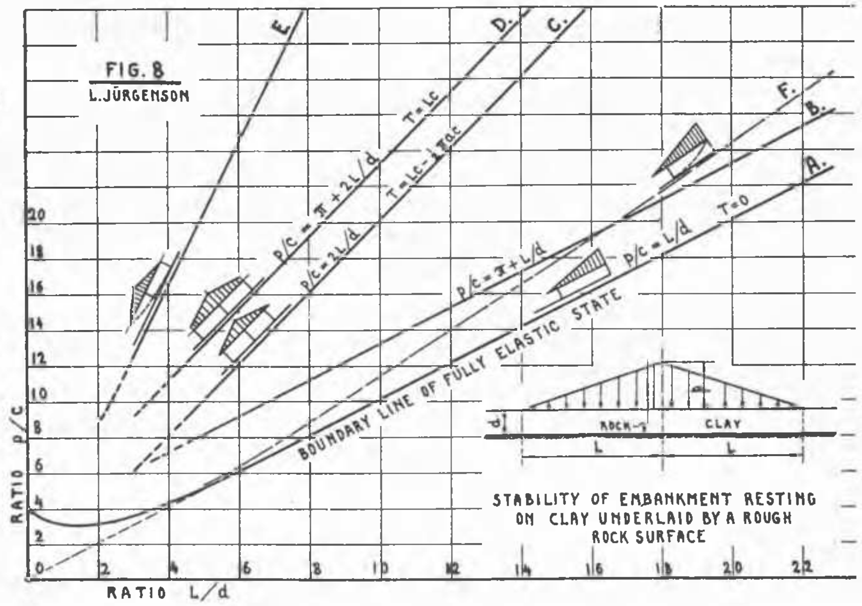


FIG. 8
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STABILITY OF EMBANKMENT RESTING ON CLAY UNDERLAID BY A ROUGH ROCK SURFACE by the fact that the actual flow conditions at the toes are rather complicated and the theoretical stresses as given by Prandtl's solution are not rigorously correct at the toes. Strictly speaking we are not evaluating the state of the incipient sinking of the embankment in the state where the ultimate resistance of the ground to penetration is fully developed, but are evaluating an arbitrary but defined state which should be close to it. If the shape of the dam is not equilateral we must consider the steeper side first as the less favourable. In the stress analysis we shall have to distinguish between several cases which depend upon the tensile strength of the embankment and upon the magnitude of the horizontal force acting. The latter could be produced by the horizontal component of the dammed up water or by pressure of the liquid core in the case of hydraulic fill embankments. For methods of analyses of the stability of the body of the dam itself the reader is referred to publications on the theory of Cylindrical Slides and the method of Dr. Gilboy. We shall here be concerned with the strength of the embankment itself only as much as it affects the stability of the foundation. As is shown in the Appendix 1 B the formulae for the different cases are:

Case A. The embankment is able to resist the full shearing stress $s_{xz} = c$ on its base and the resulting tension and bond stresses (Fig. 7, Case A). $p = 2cL/d$; $c = pd/2L$; shearing stresses at the base $s_{xz} = c$. Tension in the dam at centerline $= cL$ diminishing to zero at the toes.

Case B. The embankment can resist a shearing $s_{xz} = s'_{xz} < c$ on its base and the necessary bond and tension $p = c\ \frac{L}{d}\ (1 + \frac{S_{xy}}{c})$; $c = pd/L - s'$; $s = s'$ Tension in the dam at centerline $= s'_{xz}L$.

Case C. The embankment can resist no shear at its base i.e. its tensile strength is zero (Case C, Fig. 7) $p = cL/d$; $c = pd/L$; $s_{xz} = 0$; Tension in the dam $= 0$.

Case D. Hydraulic fill dam with core pressure H. The shear is assumed to be uniformly distributed over the base

$$s_H = H/L\ ;\ \ p = c\ \frac{L}{d}\ (1 - \frac{H}{Lc});\ \ c = p \cdot \frac{d}{L} + \frac{H}{L}$$

Case E. Dam with active water pressure H. L_1 = the distance of the impervious blanket from the center line of the dam measured positive toward the upstream.

$$p = c - \frac{L}{d}\ (1 \cdot \frac{H - cL_1}{L\ c});\ \ c = p \cdot \frac{d}{L + L_1} + \frac{H}{L + L_1}$$

Discussion of the formulae. If the dam can resist the necessary tension, the failure load is 100% higher than the load at which the plastic state first developed. If the dam can take no tension at all the ultimate load drops to the point where the plastic state first appears. Thus line A. (Fig. 8) coincides with the line representing the ultimate pressure in the case of a dam that resists no tension. (Case C.)

The carrying power of the ground is in all cases proportional to the ratio L/d. The resistance of the underground could thus be increased by widening the embankment or by some means of reducing the depth of the clay stratum such as a partial replacement of the clay by a more resistive material. In some circumstances it might also be possible to take advantage of the consolidation of the clay and the resulting increase in the shearing resistance c. The height of the dam would then have to be built up by stages in horizontal layers.

The ultimate resistance to penetration will depend very markedly upon the tensile strength of the dam itself. It is 100% higher when the embankment is able to resist the required tension compared to the case where it resists none. This is a rather important consideration as it is often possible to increase the tensional resistance by special reinforcement such as the use of fascines. The theoretical magnitude of the necessary tension and bond can easily be computed. Placing the fascines on a thin blanket of gravel would evidently be better than placing them directly on the clay. Using the theory of Consolidation it is possible to estimate how the shearing strength of the material increases with the time. The fascines may become superfluous a long time before they deteriorate. The total tension to be provided for in order that the dam can be considered as rigid is proportional to the distance from the toes and is $T = cL$ in the center. The same as in a reinforced concrete slab the amount of bond to be provided for is equal to the shear i.e. c units per lineal unit of the width of the dam. As is evident from formulae the safest way of building up an embankment is to build it in horizontal layers starting with the full width.

Homogeneous Ground. The formulae given above do not hold when the ground is homogeneous, i.e. when the rock is at a great depth relative to the width of the dam. However, the criterion given for the stresses in elastic state holds. The plastic state will not be produced anywhere as long as p is smaller than 3.91 c. The ultimate resistance will depend upon the strength of the dam itself. If it were absolutely rigid the failure could not occur before p 2 (2)c (Prandtl's solution).

As all methods of estimating the ultimate resistance must involve assumptions which must be kept in mind when such methods or formulae are used, it appears that in practical applications the comparison of the allowed pressure with the pressure at which the plastic state is bound to develop is generally a better criterion of the stability of foundations.

Note: Corrected Formula for the Squeezing Test. The formula given earlier was $c = \frac{P\ a}{B\ L}$ Journal Boston Society of Civil Engineers, July, 1934, Fig. 7, page 250. The constants of integration were determined from the condition that the stresses are zero at the outer edge of the plate ($n_z = n_x = 0$ when $x = 1$ and $z = a$. It appears to be more reasonable to determine the constants from the condition $\int_{-a}^{+a} n_x \, dz = 0$ when $x = L$. The formula for the shearing strength of the clay then becomes:

$$c = \frac{Pa}{BL(L + a)} = \frac{Pa}{BL\,(L + a)}$$

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No. G-9

STABILITY OF EARTH SLOPES

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Introduction. Slopes of artificial earthworks: cuts, fills and embankments remain stable, if they are not made steeper than a certain maximum angle: β. If a slope steeper than β is desired, the slope must be supported by a retaining wall.

The forces that keep a slope stable are the passiv internal forces: friction and cohesion. Acting against the direction of sliding, these forces prevent movement on the surface of rupture. Relation between the normal /n/ and tangential /t/ stresses that act on the surface of rupture /Fig.1/ may be expressed by the well known Coulomb formula:

$$t = n \tan \varphi + c \tag{1}$$

where φ is the angle of internal friction of the earth, and c kg/cm² is the shearing resistance, or cohesion.

This hypothesis /Eq.1/ as condition of rupture has been verified by experiments, and may be accepted as a basis for stability computations.

According to observation flatter slopes stay stable at greater heights than steep ones, in other words to a certain slope at an angle β belongs a definite maximum height: h, at which the slope can remain still stable /Fig.1/. The slightest increase of this height may cause the AB slope to begin to slide on the AC surface of rupture.

It also has been observed that such a surface of rupture is of cylindrical shape. In the past, however, numerous investigators - Francais, Culmann - have assumed this surface to be simply a plane.

Substitution of a plane for the curved surface is not quite reasonable. The difference between them is greater than to warrant this simplification. Only the simpler mathematics and analogy with the earth pressure problem may, temporarily, make computations based on assumption of a plane acceptable.

Indeed, based on investigations by Petterson, Sven Hultin and Fellenius of the Statens Järnvägars Geotekniska Komission /1/, generally, in stability computations, the surface of rupture is regarded as circular cylinder.

For computing stability of earth-slopes with the assumption of cylindrical sliding surface various graphical methods were devised by W. Fellenius, H. Krey /2/, and K.v. Terzaghi /3/.

These graphical methods are correct, but not practical. They involve a great number of trials and demand considerable amount of work even, if one is experienced in applying the methods.

An exact, theoretical solution has been attempted by Resal /4/, Frontard /5/, Caquot /6/ of the French school, but a mathematically correct solution that would be free of contradictions is still unknown.

In the following the author assumes the cylindrical sliding surface of the Swedish method, but instead of the prolix graphical solution he gives a simple, analitically derived formula for determining the maximum steepness of a slope at a certain height.

As the Culmann parabola of the plane of rupture-theory expresses the relation between height and steepness of slopes, so a limiting curve of this new theory presents the same relation; the values, however being quite different. The values supplied by the limiting curve are on the safe side, those given by the parabola are not /Fig.2/.

According to the cohesion parabola a slope, with a height h will be stable at a maximum angle β_1* /Fig.2/. Length of the slope, assuming a plane of rupture, can be expressed by

$$sl = \frac{4\,c}{\gamma} \, \frac{\cos \varphi}{1 - \cos(\beta_1 - \varphi)} \tag{2}$$