No. H-4

CONDITIONS FOR THE STABILITY OF PILES

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The dynamic formulae determine the load carrying capacity of the piles almost exclusively on the basis of data connected with the pile itself (material, dimensions etc.), whereas the static formulae (by Grex, Dörr and others) take as basis for this determination solely data connected with the soil.

The resistance is assumed to consist of:

a) resistance of the lateral surface assume to be proportional to the pression of soil against the lateral surface of the pile, and

b) resistance of the point which is assumed to be proportional to the depth of the pile's point plunging into the soil.

The load carrying capacity of the pile is determined on the basis of the condition of equality of the external force and forces of resistance (Fig. 1), i.e. we assume

$$R = \alpha l + \frac{\beta l^2}{2} \qquad (1)$$

Here α and β depend on the volume weight and the angle of friction of the soil.

There are no data relating to the pile in formula 1 and this cannot but influence the results attained with this formula.

In order to find out the influence of data connected with the pile on the latter's carrying capacity we will analyse the following case:

A semi-infinite elastic mass is pierced by a semi-infinite rod possessing an elasticity differing from that of its surrounding medium. The rod is connected with the medium it is piercing by forces arising on the surface of contact of these bodies. The condition we have to analyze is such where there exists no relative displacement of the rod in regard to the surrounding medium.

Let us draw from point O several concentric hemispheres and assume that all these hemispheres, without change in size have dropped down to some small space inversely proportional to the squares of radius of the hemispheres, i.e.

$$S = \frac{A}{R^2} \qquad (2)$$

where A is the coefficient of proportionality.

Let us take two hemispheres of infinite proximity with radius R and R $+ dR$, tracing at angle β to normal line N (axis Z) a radius ab (Fig. 2), we get the change in length of element ab $= dR$ of this radius.

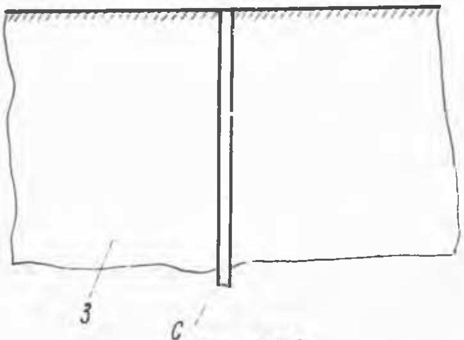


Fig N1

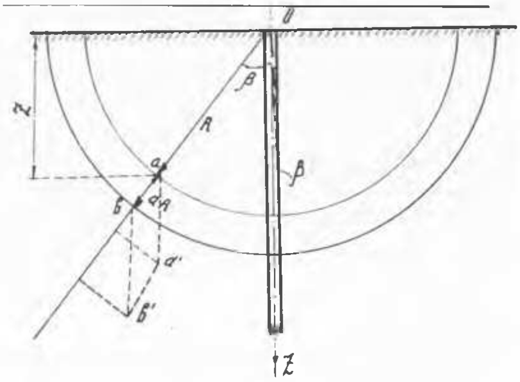


Fig N2

The ends a and b of the element will have vertical displacements:

$$aa' = \frac{A}{R^2}$$

$$bb' = \frac{A}{(R + dR)^2} = \frac{A}{R^2 + 2\,RdR + (dR)^2}$$

The change in length of ab will be equal to

$$a'b' - ab = (bb' - aa')\cos\beta = \left(\frac{A}{R^2 + 2RdR + (dR)^2} - \frac{A}{R^2}\right)\cos\beta$$

or

$$a'b' - ab = \frac{-2A\ RdR - A\ (dR)^2}{R^4 + 2R^3dR + (RdR)^2} \cdot \cos\beta$$

Ignoring the infinitesmal quantities as compared with R^4 in the denominator and the infinitesmal of second order as compared with those of first order in the numerator, we get the relative compression of the element in the direction of the radius, as follows:

$$\frac{a'b' - ab}{ab} = -\frac{2\,A}{R^3}\cos\beta \qquad (3)$$

this equation being correct for points within the boundaries of the rod end out of them as well.

Designating the elasticity-modulus of the material of the rod by E and the elasticity modulus of the soil by E_o, we get the following stress values:

For the material of the piles

$$\sigma = E\,\frac{2\,A}{R^3}\cos\beta \tag{4}$$

and for the material of the soil:

$$\sigma_o = E_o\,\frac{2\,A}{R^3}\cos\beta$$

Contrary to the general rule for solid bodies, the elasticity modulus of soil has no constant value and depends on the pression which the soil is subjected to.

According to Terzaghi there exists a definite law, i.e. the modulus of normal elasticity is inversely proportional to the compacting pressure. It follows therefore that a soil experiencing the pressure of its own weight will possess variable modulus of elasticity depending on the depth of the analyzed element from earth's surface.

In accordance with formula a

$$E_o = C_s P_s$$

(P_s being the pressure on the soil), for soils not subjected to other pressures P_s is the pressure of the soil's own weight; therefore we can write

$$P_s = \gamma z = \gamma R\cos\beta$$

and consequently get for the soil

$$E_o = C_s\gamma R\cos\beta \tag{5}$$

the pressure in the soil amounting to

$$\sigma_o = C_s\gamma\,\frac{2\,A}{R^2}\cos^2\beta \tag{6}$$

Dividing the stress σ in the pile into two components σ_o and σ' so that

$$\sigma' = \sigma - \sigma_o = \frac{2\,A}{R^2}\left(\frac{E}{R} - C_s\gamma\cos^2\beta\right)$$

and taking into consideration that the ratio of the section of the pile to its length is such that β is very small and

$$\cos\beta \sim = 1.0$$

we get

$$\sigma' = \frac{2\,A}{R^2}\left(\frac{E}{R} - C_s\gamma\right)$$

Here σ' designates the excess of stress in the pile as compared with stress in the soil.

The surface of the elementary sperical zone caa_1c_1 (Fig. 3) equals

$$F = 2\pi R\sin\beta\,(R\,d\beta) = 2\pi R^2\sin\beta\,d\beta$$

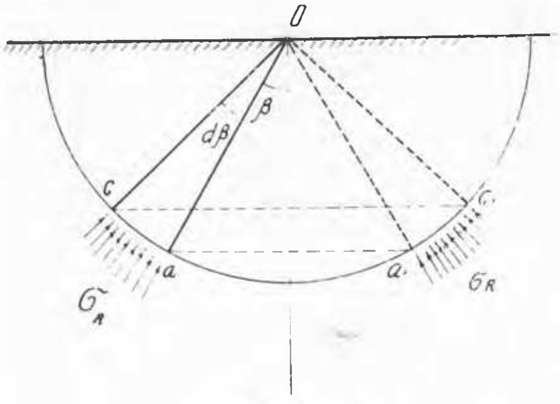


fig N3

Consequently, on the whole surface of the zone the sum of the projections onto the normal plane is:

$$\Sigma_N = \sigma_o\,F\cos\beta$$

or

$$\Sigma_N = C_s\gamma\,\frac{2\,A}{R^2}\cos^3\beta\;2\pi R^2\sin\beta\,d\beta = 4AC_s\gamma\pi\cos^3\beta\,\sin\beta\,d\beta$$

Summing up those values for the whole surface of the hemisphere, we get:

$$\int_0^{\pi/2} 4AC_s\gamma\pi\cos^3\beta\,\sin\beta\,d\beta = AC_s\gamma\pi$$

The resulting value is constant and does not depend on radius R.
The excessive forces in the rod will be

$$N = \omega\sigma' = \omega \frac{2A}{R^2}\left(\frac{E}{R} - C_s\gamma\cos^2\beta\right)$$

where Ω is the square of the section of the rod $\cos^2\beta$ we assume to be equal to 1.00.

As the inner forces must be in equilibrium with the outer ones, the latter in the analyzed type of deformation, are refused to a concentrated force applied in point "O" i.e.

$$p = AC_s\gamma\pi + \omega\frac{2A}{R^2}\left(\frac{E}{R} - C_s\gamma\right) \tag{7}$$

It follows that

$$A = \frac{p}{C_s\gamma\pi + \frac{2\omega}{R^2}\left(\frac{E}{R} - C_s\gamma\right)} \tag{8}$$

The normal stress for any point on the surface of the hemisphere will amount to:

$$\sigma = \frac{p}{\frac{C_s\gamma\pi}{2E}R^3 + \frac{R\omega}{E}\left(\frac{E}{R} - C_s\gamma\right)} \tag{9}$$

for the material of the rod, and

$$\sigma_o = \frac{p\cdot\cos^2\beta}{\frac{1}{2}R^2\pi + \frac{\omega}{C_s\gamma}\left(\frac{E}{R} - C_s\gamma\right)} \tag{10}$$

for the material of the soil.

The normal force acting in the cross section of the rod will be:-

$$N = \sigma\omega = \frac{p}{\frac{\frac{1}{2}C_s\gamma\pi}{E\ \ \omega}R^3 + 1 - \frac{C_s\gamma}{E}R} \tag{11}$$

The influence of the third term in the denominator $\frac{C_s\gamma R}{E}$ is very insignificant as compared with the other two terms. This influence affects only those figures which in rounding up, are cast away. Therefore, we may assume:

$$N = \frac{p}{\frac{\frac{1}{2}C_s\gamma\pi}{E\omega}R^3 + 1} = \frac{p}{M} \tag{12}$$

$$\frac{dN}{dR} = -\frac{\frac{3}{2}\frac{\pi C_s\gamma}{E\omega}\cdot R^2 p}{\left(\frac{1}{2}\frac{C_s\gamma\pi}{E\omega}R^3 + 1\right)^2} = -\frac{\frac{3}{2}\frac{\pi C_s\gamma}{E\omega}\cdot R^2 p}{M^2} \tag{13}$$

The tangents of stress on the surface of contact between soil & pile:

$$\tau = \frac{1}{u}\cdot\frac{dN}{dR} \tag{14}$$

u being the parameter of the pile.

$$\tau = \frac{1}{u}\cdot\frac{\frac{3}{2}\frac{\pi C_s\gamma}{E\ \omega}\cdot R^2}{\left(\frac{1}{2}\frac{\pi C_s\gamma}{E\ \omega}R^3+1\right)^2} = \frac{1}{u}\cdot\frac{\frac{3}{2}\frac{\pi C_s\gamma}{E\ \omega}.R^3}{M^2} \tag{15}$$

As condition for the equilibrium of the rod we must adopt the following:

a) that the tangential stress arising on the surface of the rod from an external force p should not exceed the amount of friction, and

b) that the normal force at the end of the rod should not exceed the admissible pressure on the soil in level with the end of the rod.

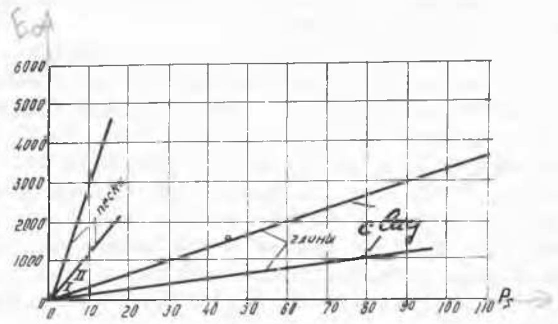
Numerical example. Fig. 4 shows the diagram of dependence of E_0 on P_{3s} (verified by Terzhagi) for various soils.

If we take, for example, sand, for which $F_3 = 10$ kg/cm^2 corresponds, as per diagram, to $E_0 =$ 3000 kg/cm^2 then $C_s = \dfrac{E_0}{P_s} = 300$.

With a volume weight of the sand $\gamma = 1.8$ ton/cm^3, we get $C_s \gamma = 0.54$ kg/cm^3.

Fig. 5 shows the curves of change of the longitudinal force in the pile for the sand we are analyzing, these having been calculated according to formula 12, with values E & R as shown on the curves.

Fig. 6 shows the curves of distribution of the tangential stresses in the plane of contact between rods and sand.



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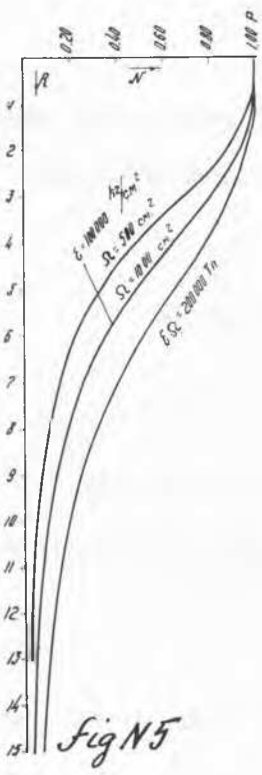

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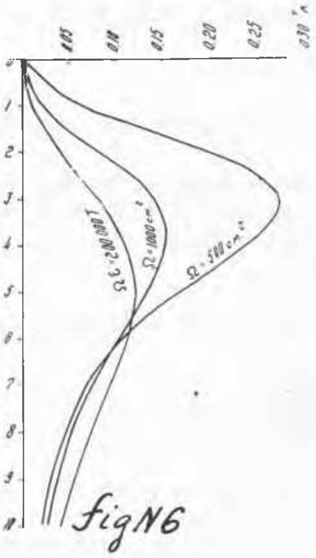

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For a wooden rod with values $\omega = 1000$ cm^2 and E = 100000 kg/cm. there results

$$\tau_{\max} = 0.152\, p$$

at a depth

$$z = 4.00 \text{ m.}$$

According to Dorry the resistance of friction on the lateral surface, is determined by the formula

$$p = \beta z = \gamma z (1 + \tan^2 \phi) \tan \phi$$

and the admissible pressure in level with the end of the pile

$$q = \alpha l = \omega \gamma \tan^2 (45 + \tfrac{\phi}{2})\, l$$

If the sand analyzed by us should possess $\phi = 35°$, then

$$p = 1.8\,(1 + \tan^2 35°) \tan 35° z = 1.88\, z.$$

and $q = 1.8 \tan^2 (45 + 17°30')\, \omega l \quad = \omega\, 6.65\, l.$

From the condition of equality

$$0.152\, p = 1.88\, z$$

we find

$$p = \frac{1.88 \cdot 4}{0.152.} = 50 \text{ tons } \pm$$

The length of rod, necessary for the above, is determined by matching on condition that

$$N = q$$

In our case it proves to amount to

$$l = 9.00 \text{ m}$$

$$N = 0.139 \cdot 50 = 6.90 \text{ tons}$$

$$q = 0.10 \,.\, 6.65 = 6.00 \text{ tons.}$$