No. Z-8 FIELD INVESTIGATIONS OF THE THEORY OF VIBRATION OF MASSIVE FOUNDATIONS UNDER MACHINES

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The rational calculation for foundations under machines requires the determination of the frequencies proper and amplitudes of vibration of the foundations. From a practical point of view the greatest interest is presented by the purely vertical vibrations or the vibrations that are accompanied simultaneously with the gyratory movement of the foundations and its displacement in one of its principal planes. The equation of free vibration of foundations in that case are:

The vertical vibration:

$$m\ddot{z} + C_z F z = 0 \tag{1}$$

Vibrations, accompanied simultaneously by the gyration and displacement of the foundation

$$m\ddot{x} + C_x F(x - h\varphi) = 0$$

$$\theta\ddot{\varphi} - C_x F h x + (C\varphi I - Qh + C_x F h^2)\varphi = 0 \tag{2}$$

Here C_z - coefficient of uniform elastic compression of the soil is equal to $C_z = \frac{-P_z}{z_p - z_o}$ where P_z - normal stress on the soil; z_p - the full value of settlement of the foundation under the action of the stress P_z; z_o = the remaining settlement, i.e. settlement remaining after the complete unloading of the foundation; in which case $z_p - z_o$ - represents nothing more than a uniform elastic, settlement of the foundation.

C_φ = coefficient of elastic uneven compression of the soil; being equal to

$$C\varphi = \frac{M}{I(\varphi_p - \varphi_o)}$$

where M represents the outer gyratory statical moment acting on the foundation, I -- moment of inertia of the bottom of the foundation, relative to the axis of its gyration; φ_p, φ_o = the full and remaining turn of the foundation, consequently $\varphi_p - \varphi_o$ = represents elastic gyration of the foundation; and lastly, C_x - the coefficient of the elastic displacement of the soil is equal to

$$C_x = \frac{P_x}{X_p - X_o}$$

where P_x the displacement stress due to the application to the foundation of the horizontal force, X_p and X_o the full and remaining displacement; $X_p - X_o$ is an elastic displacement.

The significance of the remaining values that go to make up the equations (1) and (2) is given in Fig. 1. From equations (1) and (2) are derived the following expressions for the frequencies.

For vertical vibrations proper

$$\lambda_z^2 = \frac{C_z F}{m} \tag{3}$$

For the vibrations proper corresponding to equation (2) two main frequencies are obtained, corresponding to two degrees of freedom, characterizing the changing of the parameters φ and x:

$$\lambda_{1,2}^2 = \frac{\lambda_\varphi^2 + \lambda_x^2}{2\gamma} \pm \sqrt{\left(\frac{\lambda_\varphi^2 + \lambda_x^2}{2\gamma}\right)^2 - \frac{\lambda_\varphi^2\lambda_x^2}{\gamma}} \tag{4}$$

Here, λ_φ - the ultimate frequency of the vibrations proper of the foundation when it is vibrating in a rotary motion around the axis only passing through the center of gravity of the bottom; we have

$$\lambda_\varphi^2 = \frac{C_\varphi I - Qh}{\theta_o}$$

λ_x - the ultimate frequency of vibration proper of the foundation, when it is vibrating, only in the direction of the axis x, in which case.

$$\lambda_x^2 = \frac{C_x F}{m}$$

An investigation into the correctness of the indicated formulae (3) and (4) for the determination of the main frequencies proper of the machine foundation, was carried out under field conditions on various soil formations with experimental foundations weighing up to 30 tons and having an area at the bottom up to 8 m^2.

The first experiments were begun in 1933 (Experiments carried out by D. D. Barkan and A. I. Michaltchuk), on a brown porous loam, permeated with water with a depth of strata of about 4 meters; under which was a deep underlayer of sand. By means of statical experiments were determined the coefficients C_z of the elastic compression of the soil for foundations, the bottom areas of which were 2, 4 and 8 m^2. From the determined values C_z were calculated the frequencies of the vertical vibrations

Fig 1. Conventional notations.

X,Y,Z - the main central axis.

x,y - compon. on the axis XZ. The shifting of the centre of gravity of the foundation and the machine.

φ - the rotation of the foundation with relation to the axis Y.

Q, m - weight & mass of the foundation & the machine

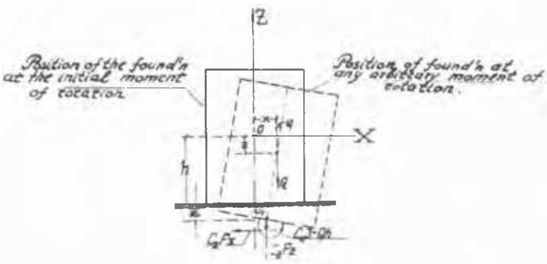
Θ - moment of inertia of the mass relative to the main central axis Y,

Θ₀ - ditto relative to the axis parallel to Y but passing through the centre of gravity of the bottom.

J - moment of inertia of the bottom of the foundation relative to the same axis,

F - Area of the bottom of the foundation,

h - distance from centre of gravity of the mass to the the bottom



Resonance curves of the forced vertical oscillation of the foundations with a bottom area of 8.0 m² and with a weight of about 30 tn.
The foudation was established upon brown porous watersoaked loam

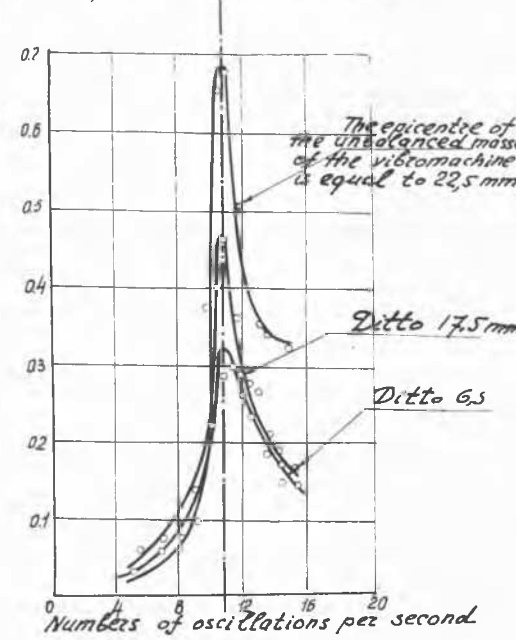
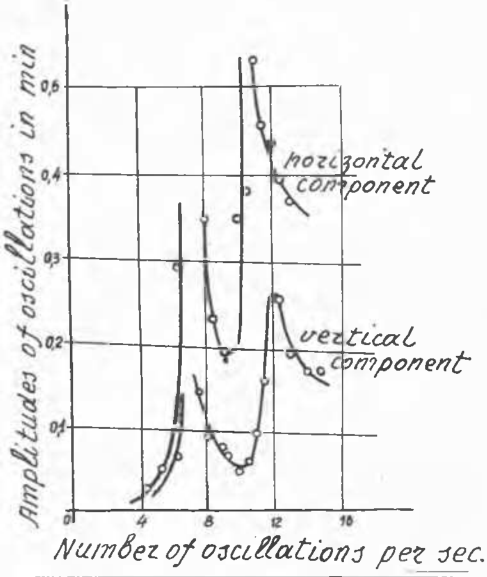


Fig. 2.

Fig. 3 Resonace curves of the compulsory oscillations of the foundations with the bottom area = 8m. accompanied by its rotation and displacement. The soil being a porous water soaked loam



Relation of the coefficient C_x of elastic displacement of the soil from the normal pressure.

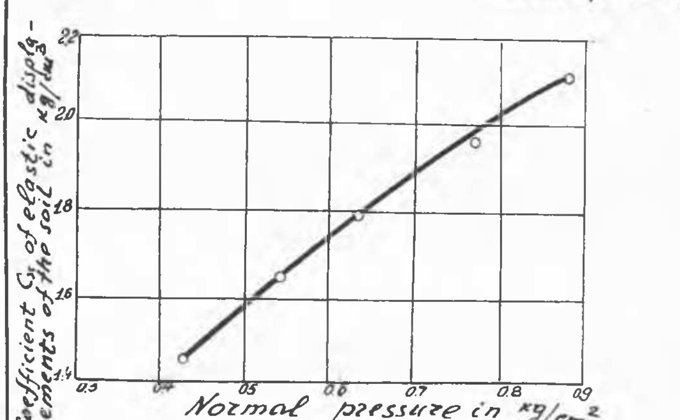


Fig 4.

Relation of the coefficient of elastic even compression of the soil from the bottom of the foundation

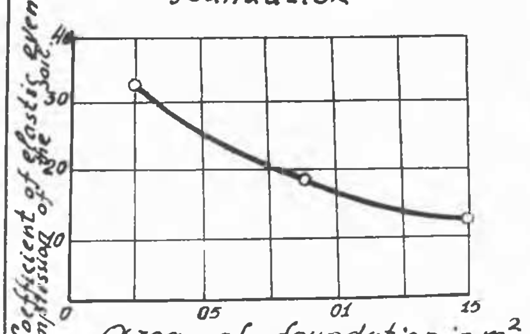


Fig. 5.

Resonance curve of the forced oscilations of the foundation accompanied simultaneously by its rotation & displacement.
(The rings represent the amplitudes as recorded by the vibrograph foundations established on Loess)

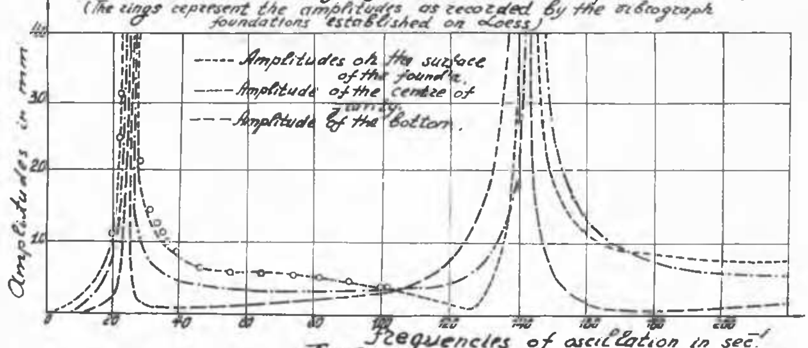


Fig 6

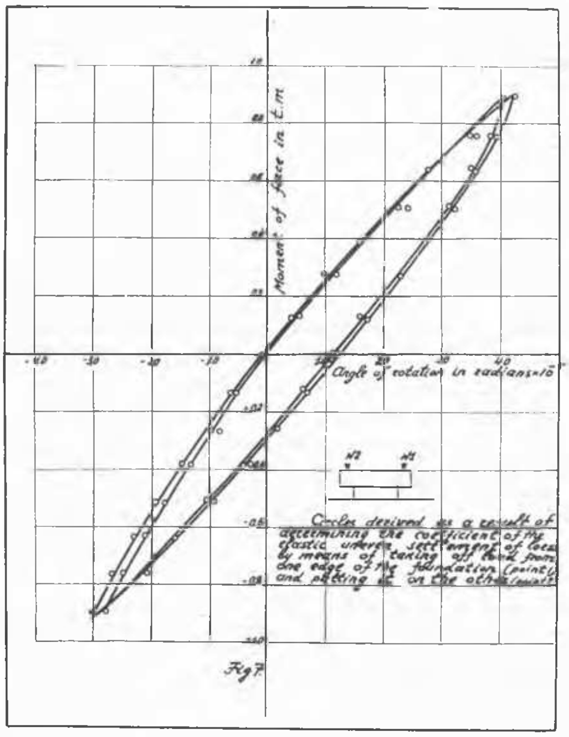


Fig.7

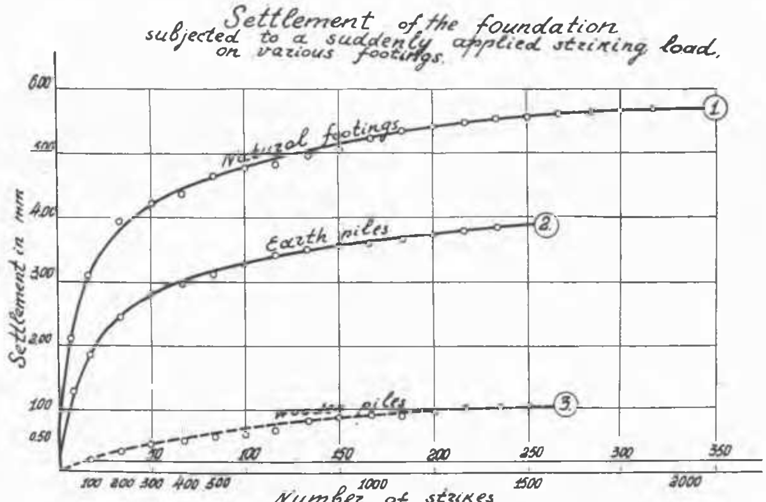
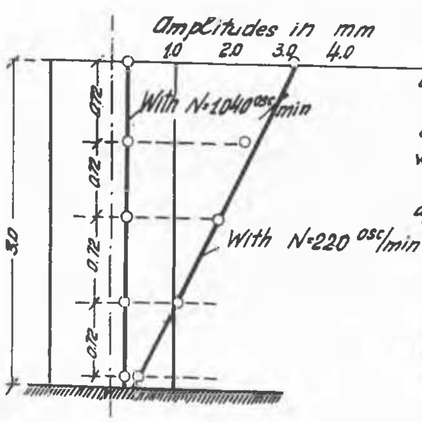


Fig. 9



Distribution of the amplitudes along the height of the foundation carried out with direct calculations with from cal tests circles signifying the aplitudes as recorded by a vibrograh at various heights

Fig.8

Resonance curves of oscillation of the foundation under the action of horizontal force. Full lines with extinguishers, dotted lines without

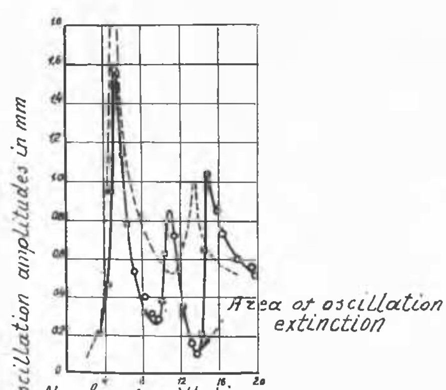


Fig. 11

Table I

Results of statical & dynamical tests carried out upon fundations built on porous water-soaked brown loam

Fundation area m²	Coefficient Cz of elastic compression as obtained through the statical test in kg per cu cm	Frequencies (in Hertz's) of vertical vibrations proper of the fundation.		The value of Cz as obtained from the rotary vibrations of the fundation
		Computed from Cz as obtained from statical test	obtained from the compulsory vibrations	
8	2.0	11.5	11.00	3.9
4	2.5	10.6	9.8	4.0
2	4.4	14.0	11.1	10.6

Table II

Results of statical & dynamical tests carried out upon fundations built on Loess.

Fundation area m²	Coefficient Cz of elastic compression as obtained through the statical test in kg per cu cm	Frequencies (in Hertz's) of vertical vibrations proper of the fundation		Coefficient Cφ of the uneven elastic settlement of Loess as obtained from the statical tests in kg per cm³	Lowest main frequency of the foundation accompanied by rotation & displacement	
		Computed from Cz as obtained from statical test	obtained from free vibrations		Calculated from Cφ as obtained from statical test	From observations at free and compulsory vibrations
0,81	14,2	25,8	25,0	25,2	10,2	10.4
1,40	10,8	25,2	23,8	18,3	14,4	11,7
2,0	10,3	21,8	18,5	16,3	12,3	11.0
4,0	8,25	22,0	18,9	15,7	17,2	12.4 (?)

proper of the foundation, after that with the aid of a vibrating machine the foundation was subjected to compulsory vertical vibration and the resonance diagrams were recorded, (Fig. 2). From the location of the maximum points of this curve were determined the frequencies of the vertical vibrations proper of the foundation. In nearly all cases they differed but little from the theoretically calculated one (Table I). Upon observation of the compulsory vibrations, accompanied at the same time by a gyration and displacement of the foundation, in accordance with the theory of these vibrations the presence of two main frequencies λ_1 and λ_2 of the foundation were discovered (Fig. 3). This indicated, that the coefficient C_φ of an elastic uneven soil compression, found from the gyratory vibrations, is approximately twice as large as the one found from the purely vertical vibrations (Table I).

In 1934 experiments were carried out (Carried out by J. N. Smolikov with the assistance of D. D. Barkan) with a foundation, having a bottom area of 0.5--1.0 and $1\frac{1}{2}$ m^2, with a sandy plastic gray loam, permeated with water, running to a depth of about 10 m. In these experiments all the main results of the former experiments were confirmed. Apart from that in these experiments particular notice was taken of the coefficient C_x of the elastic displacement of the soil, its relation to normal pressure σ_n and area. Aside from that, it was established that C_x increases almost proportionally with σ_n (Fig. 4) and decreases with the enlargement of the area.

In addition to this the well-known formula of Schleicher concerning the relation between the coefficient C_z, the elastic compression of the soil, and the area of the bottom of the foundation (Fig. 5), was confirmed. Lastly and especially on a large scale, experiments were carried out in 1935 (Carried out by D. D. Barkan, J. N. Smolikov and P. A. Saitchev) on loess, where six foundations ranging in area from 0.81 to 4 m^2 were experimented upon. The foundations were of square and rectangular shapes. Statical tests for determining the coefficients of elasticity of soils, necessary for the calculation of the vibration of foundations, as well as dynamical observations of the free and compulsory vibrations of foundations (Fig. 6), were carried out.

For the first time was carried out a determination of the coefficient C_φ of the uneven elastic settlement of soil by statical means (Fig. 7) and a comparison made of its value, found from observation of the free and compulsory gyratory vibrations of foundations. The value C_φ found by these two methods, were found to differ but little from each other (Table II).

From the solution of the problem of the uneven settlement of foundations it follows, that for square foundations $C_\varphi/C_z = 1.87$; experimentally this relation was determined to be equal to 1.76--i.e. very little different from the theoretical one.

On these experiments the theory of the changing of the form of the vibration of foundations with the changing of the frequencies, was subjected to a particular check up, in which was found a very good agreement of theory and experiments. Furthermore the experimental data relative to the amplitudes of compulsory vibration coincided very well with the calculated expressions of these values (Fig. 8).

Investigations into the coefficients of elasticity of soil were carried out not only for natural, but also for (artificia) pile foundations. It was observed (as in the other experiments, carried out by the All Union Foundation Institute on other soils), that even friction piles increase very strongly the elastic stiffness of foundations and diminish the remaining settlements of foundations (Fig. 9).

On the last two experiments, apart from the check up on the theory of vibration of massive foundations, a check up was made on the theory of the so-called dynamic extinguishers. In this a good agreement was found to exist between theory and experiment (Fig. 10, 11, 12).

Fig.10