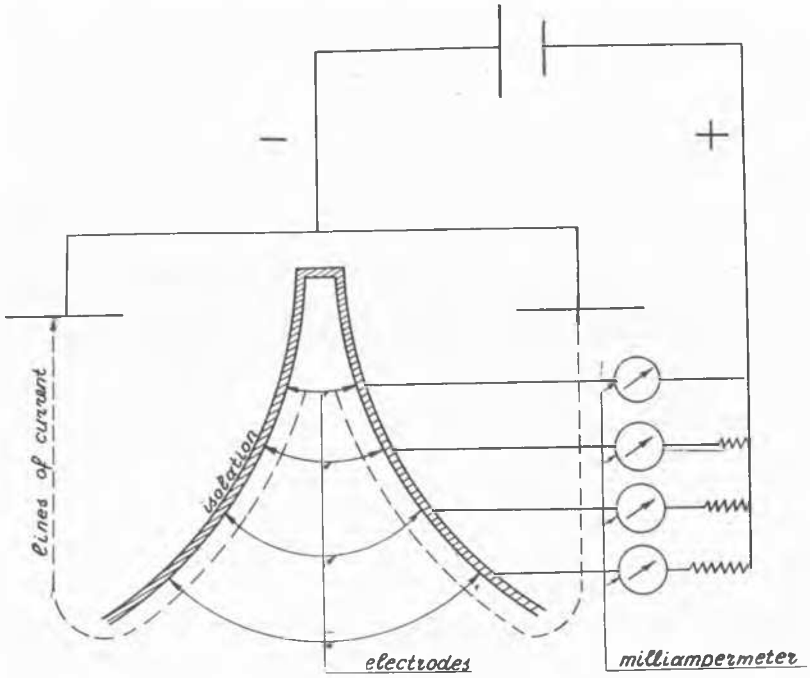

Fig 1.

$$\frac{dv}{dt} = -Kc\rho\iiint\varphi\, dx\,dy\,dz \qquad (7)$$

If on the experimental set up, the electrodes which are feeding the system with current are broken up into certain sections, then we can with a certain approximation determine the value of φ in various parts of an electrically conductive medium.

If we determine the value φ for a series of separate volumes, we can then substitute equation (7) going over to the method of final differences, with the approximate formula:

$$\frac{dv}{dt} = c\sum\varphi \qquad (8)$$

Here:

$$C = -Ko\rho v_o$$

(v_o = the value of a separate discreet element into which the volume under investigation is divided, during experiment).

If the question resolves itself into the determination of the settlement of the centre of a symetrically loaded symetrical surface, then the summation can be carried out along the central, vertically situated current path, which may be separated out by means of insulated walls running parallel to the lines of the current and without reproducing all the rest of the medium. (See Fig. 1)

We can also under any experiments whatsoever instead of deducting the value c determine it by means of experiments with such a configuration of the deformed medium and with such a distribution of stresses, by which it would be possible to carry out an experiment, parallel with the theoretical calculation of the value dv/dt.

From the preceding it is plain, that the herein described method makes possible the further development of the method of hydroelectrodynamical analogy as suggested by Pavlovski (N. N. Pavlovski, The Theory of the Movement of Subsoil Water under Hydraulic Structures, Petrograd, 1922) for the investigation into the movement of subsoil waters. What is new about this is the construction of the voluminous feeding of the whole system with current.

This may be more simply accomplished by distributing the electrodes along the surfaces of equal pressure previously determined by one means or other. The more the electrodes and the closer they are placed inside of the conducting medium, (electrolytic liquid) the nearer will the laboratory test correspond to the theoretical demands.

No. Z-10 THE CONSOLIDATION OF MARINE CLAY DEPOSITS DURING AND AFTER THE SEDIMENTATION PERIOD

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The problem of the determination of the excess hydrostatic pressures existing in clay or mud deposits during their sedimentation period and thereafter has been dealt with by K. Terzaghi (1), A. Ortenblad (2) and the writer (3) on the following assumptions:

1) The sedimentation takes place on an impervious base.
2) The excess water contained in the deposit escapes only in a vertical upward direction.
3) The average coefficient c of consolidation may be considered constant over the whole thickness of the deposited layer.
4) The speed of sedimentation v is constant during the whole period of sedimentation.

This problem is governed by a partial differential equation, which is formally identical with the Fourier equation relating to the one-dimensional, non-stationary flow of heat through plan-parallel plates of homogeneous materials. (4) Let be:

w .... the excess hydrostatic pressure at a point of the layer characterized by
z .... the depth under its surface, and
t .... the time, elapsed since the beginning of the flow phenomena;
ε .... the voids-ratio at t = 0,
k .... the coefficient of permeability,
a .... the coefficient of compression,
s .... the specific weight of the liquid (water) filling the voids of the clay,
$c = \frac{k}{s.a}(1+\varepsilon)$ ..... the definition equation of the coefficient of consolidation,

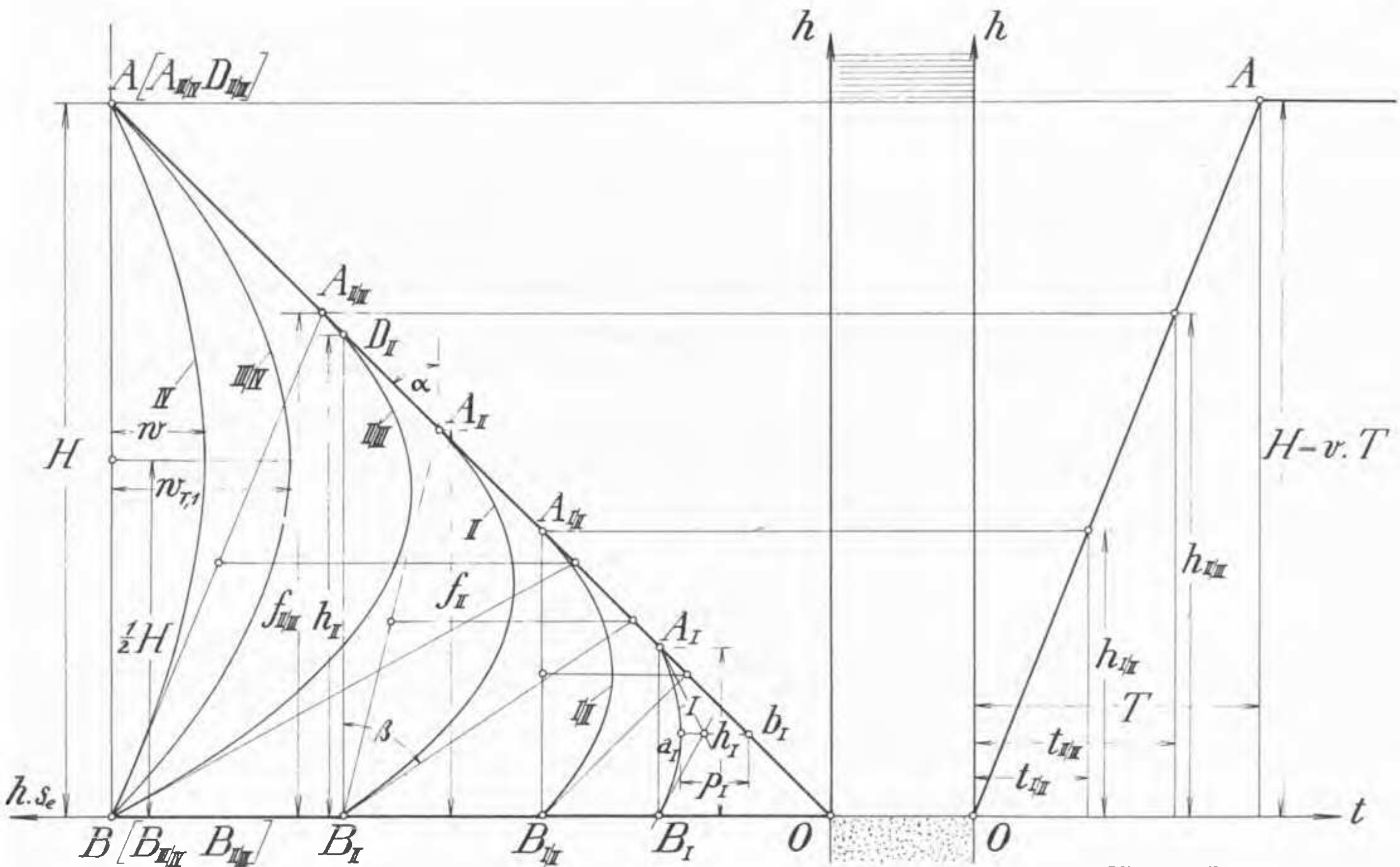
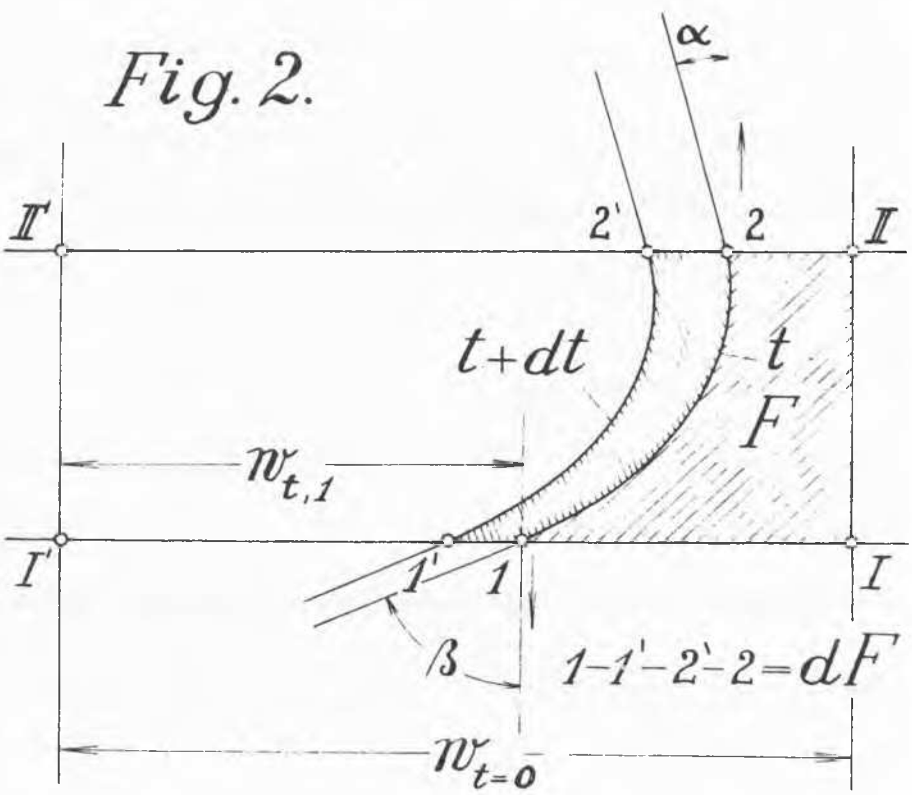


Fig. 1a.

Fig. 1b.

Fig. 2.



then the partial differential equation mentioned on the preceding page is:

$$c \cdot \frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial t} \qquad (1)$$

All efforts to find the exact solution of equation (1) for the special problem of the combined consolidation and sedimentation encountered great difficulties because of the variability of the thickness of the layer with the time, so that the authors mentioned preferred to content themselves with approximations.

Fortunately, it has been shown by comparisons of approximate and rigid solutions of consolidation problems (3) that the former satisfy all requirements from a practical point of view. Moreover, there is not much hope of establishing mathematically rigid results of consolidation problems, characterized by variable height of the deposit. For this reason, the approximate way of tackling problems of combined consolidation and sedimentation will also be accepted in this paper.

A problem, closely akin to that described above, which, without doubt, is apt to interest both foundation engineers and geologists, consists of the determination of the excess hydrostatic pressures, produced by the effective weight of the soil during the sedimentation period and afterwards, if the base consists of a pervious stratum, for example sand, which is in steady communication with the suspension liquid, so that an excess hydrostatic pressure in this underlying sand cannot occur.

The only difference between the first problem, characterized by an impervious base and the one dealt with in this paper from a physical point of view is, that the excess water contained in the voids of the clay escapes vertically upwards in the first case, while the flow takes place in both directions, namely vertically upwards and downwards in the second case, to be treated hereafter.

The problem may be considered to be solved, if we are able to indicate the excess hydrostatic pressure at any depth z and at any time t, in other words, if the isochronic curves for any time t are computed.

Fig. 1a shows that, generally, there are four phases to be distinguished, namely two phases (I and II) during the time of sedimentation from $t = 0$ to $t = T$, and two phases (III and IV) from $t = T$ to $t = \infty$

The exact isochronic curves for these phases may be replaced by simple parabolas having horizontal axis, which, in many cases, has been proved to give satisfactory results. By this approximation the partial differential equation (1) is reduced to an ordinary differential equation between two variables, as the general shape of the curve is assumed to be known, so that only the relation between a parameter of the special parabola and the time has to be determined.

The writer has shown elsewhere (3) that equation (1) may be replaced by the following:

$$\frac{dF}{dt} = c\ (\tan\alpha + \tan\beta) \tag{2}$$

in which the symbols F, α and β have the significance explained by Fig. 2 for the case of the excess water flowing vertically up and downwards. The directions of the flow of the excess water at the points 1 and 2 are indicated by vertical arrows.

I, II is the isochronic curve (straight line) at $t = 0$

1, 2 is the isochronic curve at $t = t$

1', 2' is the isochronic curve at $t = t + dt$

I', II' is the isochronic curve at $t = \infty$

F is the area I, II, 1, 2.

$\tan\alpha$ and $\tan\beta$ are the hydraulic gradients at the points 1 and 2 respectively.

Equation (2) represents the fundamental means for determining the isochronic curves in each of the four above-mentioned phases.

Phase I. During the interval $0 \geqq t \leqq t_{I/II}$ the thickness $B_I\ A_I$ in Fig. 1a of the clay deposit is still relatively small and therefore the escape of the excess water takes place quickly. The isochronic curve I is represented by a parabola, which intersects the inclined straight line 0 A at A_I. The horizontal velocities of the points A_I and B_I in the diagram are equal. The distance 0 B_I represents the intergranular pressure at the pervious base of the clay deposit and is equal to $s_e . v . t$; s_e being the specific weight of the soil reduced by buoyancy (effective weight). With $a_I\ b_I = p$ (intergranular pressure in the middle of the deposit at the time t) the area $F = 0\ A_I\ B_I$ of the first phase may be written:

$$F = \frac{1}{6}\, s_e \,.\, h^2 + \frac{2}{3}\, p \,.\, h \tag{3}$$

Both h and p are functions of t, therefore:

$$\frac{dF}{dt} = \frac{\partial F}{\partial h}\cdot\frac{dh}{dt} + \frac{\partial F}{\partial p}\cdot\frac{dp}{dt} \tag{4}$$

With $h = v\ .\ t$ equation (4) takes the form:

$$\frac{dF}{dt} = \frac{1}{3} v^2 \,.\, t + \frac{2}{3} p \,.\, v + \frac{2}{3} v \,.\, t \,.\, \frac{dp}{dt} \tag{5}$$

The hydraulic gradient $\tan\beta$ in this phase is equal to $\tan\alpha$, because of the equal horizontal velocities of A_I and B_I.

$$\tan\beta \;=\; \tan\alpha \;=\; \frac{2\left[\frac{1}{2} h \,.\, s_e - p\right]}{\frac{1}{2} h} = 2\, s_e - \frac{4\, p}{v\,.\,t} \tag{6}$$

By substituting the values (5) and (6) into (2) the differential equation

$$\frac{dp}{dt} = \frac{6\, c \,.\, s_e}{v \,.\, t} - \tfrac{1}{2}\, s_e \,.\, v - \frac{p}{t}\left[1 + \frac{12\, c}{v^2 \,.\, t}\right] \tag{7}$$

is obtained.

Putting $\frac{12\, c}{v^2\,.\,t} = \omega$ the solution of (7) becomes:

$$p = \tfrac{1}{4}\, s_e \cdot v \cdot t \left\{ -1 \quad 3\omega \left[1 + \omega \cdot e^{\omega} \cdot \phi(\omega) \right] \right\} + C_I \tag{8}$$

Here, $\phi(\omega)$ denotes the so-called logarithmic integral

$$\phi(\omega) = 0.577216 + \ln\omega - \omega + \tfrac{1}{2} \cdot \frac{\omega^2}{2!} - \frac{1}{3} \cdot \frac{\omega^3}{3!} + \frac{1}{4} \cdot \frac{\omega^4}{4!} - \cdots\cdots \tag{9}$$

which is tabulated in many mathematical auxiliary works for a wide range of the argument ω.

e is the basis of the Neperian logarithms. By equation (8) the isochronic curves for phase I are fixed for any value of ω, that is of the time t.

The integration constant C_I is determined by the boundary condition, that for $t = 0$ ($\omega = \infty$) the value of p must vanish ($C_I = 0$).

The validity of (8) ceases as soon as the parabola is tangent to the inclined line 0-A; this is the case for $\omega = 3.4 = \omega_{min}$. This value, substituted into (8), delivers:

$$p = 0.25\, s_e \cdot h_{I/II} \tag{10}$$

At the corresponding moment $t = \dfrac{12\, c}{v^2 \cdot \omega_{min}} = 3.53\, \dfrac{c}{v^2} = t_{I/II}$, the first phase ends and the second phase begins.

Phase II. $\quad (t_{I/II} \gtreqless t \leqq T)$

The isochronic curve II in Fig. 1a is the lefthand boundary for the area $F = 0\text{-}A_{II}\text{-}B_{II}$, the right-hand boundaries being the inclined line $0\text{-}A_{II}$ and the horizontal $0\text{-}B_{II}$. This phase is characterised by the fact that the horizontal velocity of the point A_{II} lags behind that of the point B_{II} in the pervious base of the deposit. The isochronic curve during the period $t_{I/II} < t < t_{II/III}$ consists of a parabola $A_{II}\, B_{II}$ and a straight line $A_{II}\, D_{II}$. Whilst the velocity of B_{II} is solely influenced by the speed v of the sedimentation, the movement of A_{II} depends mainly on the consolidation process, taking place inside the deposit having the height $h_{II} = D_{II} - B_{II}$ at the time $t = t_{II}$. As the distance $0\text{-}B_{II}$ equals $s_e \cdot h_{II}$ and $h_{II} = v \cdot t_{II} = v \cdot t$, the area F may be written

$$F = \frac{1}{3}\, s_e \cdot v \cdot t \cdot f \tag{11}$$

f representing the height of the point A_{II} of tangency above the sedimentation base 0-B.

In conformity with equation (4) we may write:

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial f} \cdot \frac{df}{dt} \tag{12}$$

and obtain:

$$\frac{dF}{dt} = \frac{1}{3}\, s_e \cdot v \cdot \left(f + t \cdot \frac{df}{dt}\right) \tag{13}$$

The hydraulic gradients at A_{II} and B_{II} are $\tan\alpha = s_e$ and $\tan\beta = \dfrac{2\, s_e \cdot v \cdot t}{f} - s_e$ respectively. Substituting these values into the fundamental equation (2), we obtain:

$$\frac{1}{3}\, s_e \cdot v \cdot \left(f + t \cdot \frac{df}{dt}\right) = \frac{2\, c \cdot s_e \cdot v \cdot t}{f} \tag{14}$$

The solution of this differential equation leads to the relation:

$$f = 4\, c \cdot t - \frac{C_{II}}{t^2} \tag{15}$$

in which C_{II} is an integration constant; it is determined by the boundary condition, that for $t = t_{I,II} = \dfrac{12\ c}{v^2 \cdot \omega_{min}}$, f must equal $v \cdot t_{I,II} = \dfrac{12\ c}{v \cdot \omega_{min}}$.

Phase II ends at the moment, when the horizontal velocity of B_{II} has become zero, that is at the end of the sedimentation period $t = T$. At this moment point D_{II} has just reached $D_{II/III}$ (A) and point A_{II} is at $A_{II/III}$.

Phase III. $\quad (T \gtreqless t \leqq T_1)$

The treatment of this phase is quite analogous to that of phase II, only much simpler. We just mark the different steps of the computation:

$$F = \frac{1}{3}\, s_e\, H \cdot f\ ; \quad \frac{dF}{dt} = \frac{1}{3}\, s_e \cdot H \cdot \frac{df}{dt}\ ; \quad \tan\alpha + \tan\beta = \frac{2\, s_e \cdot H}{I}$$

Differential equation: $$f \cdot \frac{df}{dt} = 6\,c \qquad (16)$$

Solution: $$f = \sqrt{12\,c\,t + C_{III}} \qquad (17)$$

The constant C_{III} is determined by the condition that equation (15) and (17) must show identical values of f for $t = t_{II/III} = T$.

Phase IV. $(T_1 \gtreqless t \leqq \infty)$

The mathematical treatment of this phase is already known. (3) At the time $t = T_1 > T$ the isochronic curve III/IV becomes tangent to the sloping line O-A. This is the beginning of phase IV. The excess hydrostatic pressure at the starting moment in the middle of the clay deposit may be called w_{T1}. From Fig. 1a and 1b is to be seen that

$$w_{T1} = \frac{1}{4}\, s_e \cdot H \qquad (18)$$

The isochronic curve during this phase is the parabola IV, characterized by the maximum value w of the excess hydrostatic pressure in the middle of the deposit, which is given by the relation:

$$w = w_{T1} \cdot e^{-3(\tau - \tau_1)} \qquad (19)$$

in which τ and τ_1 are so-called time factors, determined by:

$$\tau = \frac{4\,c\,t}{H^2}, \qquad \tau_1 = \frac{4\,c\,T_1}{H^2} \qquad (20)$$

The above computation may easily be adpated to the case, where the coefficient of consolidation c varies with the thickness of the deposit and, therefore, with the time. One method of taking into account the variability of c would be to fix 4 average values of c according to the four phases. If this is not sufficient, one may subdivide each phase in two or more sub-phases, characterized by special average values of $c = \frac{k}{s\,.\,a}(1 + \varepsilon)$.

Summary. In this paper a mathematical treatment is outlined for the determination of the excess-hydraulic pressures in mud and clay deposits, caused by their own weight during and after the period of sedimentation.

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No. Z-11

PRETEST SHORING OF RETAINING WALL

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Improper design, together with failure to provide adequate facilities for drainage, resulted in the buckling, cracking, and lateral displacement of a retaining wall in Douglaston, L. I. The wall serves to retain a bank 17 feet high; on the lower level, the wall is from 13 ft to 19 ft distant from a three story apartment house; on the upper level, 11 ft from the top of the wall is another apartment, three stories high. Failure of the wall would result in the collapse of the building on the upper level and endanger the rear of the building on the lower.

In addition to the cracks developed in it, the wall bulged approximately 12" in its length of 58 ft, and moved laterally 6". This lateral movement resulted in the main wall pulling away from the return; the space resulting from this separation of the two portions of the wall was carefully filled with concrete before repairs to the wall were started, as shown in Fig. 1. The lateral movement of the wall is also evinced by the position of the corner post of the wire fence surmounting the wall, as shown in this figure.

The material on which the wall was founded is a mixture of coarse sand and gravel; such material is an excellent foundation and accounts for the fact that apparently no settlement of the wall occurred. As shown on the return wall in Fig. 1, weep-holes were provided at the time of construction. Unfortunately