Practical Soil Mechanics at Muskingum—III

Resisting Strength of Foundations

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AS WAS BROUGHT OUT in preceding articles (*ENR,* March 26, 1936, p. 453, and April 9, p. 532), the foundations were of questionable character at the sites of several of the Muskingum dams. The valley fills on which it was proposed to locate many of the embankments consisted of poorly consolidated clay and silt. Where these conditions existed, tests were undertaken to determine definitely the behavior to be expected under the proposed loadings. The plan deserves some emphasis.

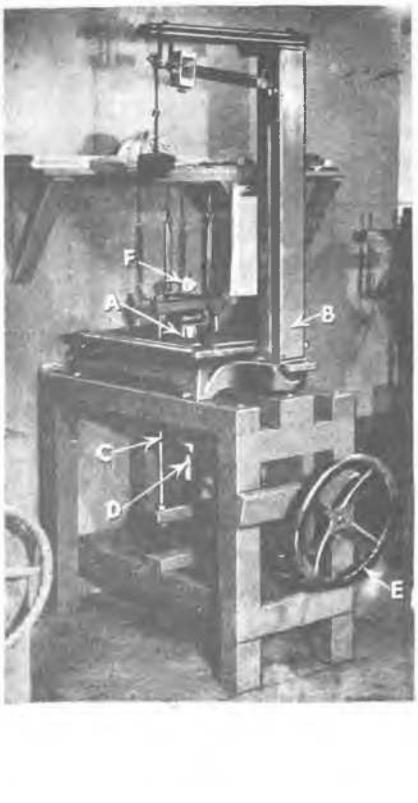
In the discussion of permeability and seepage in the previous article, the intimate relation between the laboratory tests and the design was pointed out. The same objective was held to in the foundation-strength design studies. Here the aim has been to put the structural design of the foundations on a rational basis, utilizing the excellent work of previous research and developing a technique for applying its results to the solution of practical problems. The work has gone further and adopted from the field of modern structural design the photo-elastic method of stress analysis and applied it to foundation problems. The assumption has been throughout that there is a rational or experimental method for answering every foundation problem. It is realized that only a beginning in this work has been made, and that much that is being done will be greatly altered and improved as the technique is taken up and applied in general practice. There are problems in embankment and foundation design for which as yet there are no solutions; some of these are being worked on here, and others are being attacked in other laboratories, but to the fundamental problems the solutions are now at hand.

Settlement is first problem

Consolidation is the process of increasing the density of a soil under applied loads. Volume tends to decrease as load is applied and as water in the voids is squeezed out. Where materials are saturated, which is the usual case in foundations, the rapidity with which consolidation takes place depends on the drainage characteristics of the material and the foundation in general. Ultimate consolidation on this project is considered practically instantaneous for some foundations, and for others it is estimated in periods of years; in one case the estimate is 99 years.

It is desirable to know the total amount of foundation consolidation that will take place under the proposed load and the amount of this that will take place during construction, as the difference between these two figures should be added to the height of the embankment at the time of construction so that, when the eventful consolidation has occurred the embankment will not be below the designed crown grade. It is necessary to know the rate of consolidation during construction to determine in advance what shearing strength the foundation material will have developed at the time any particular load is applied. Ordinarily we need not consider any granular material, such as sand or gravel, as the strains of these materials are comparatively small, and consequently the consolidation is also small and takes place rapidly during the construction period. We are interested, therefore, only in materials of the silt and clay type.

FIG. 11—CONSOLIDATION test machine made up of purchased parts proved highly superior in operation and has been widely copied. The loading device consists of an ordinary 1,000-lb. platform scale. The consolidation device (A) is set on the platform scales B. The floor of the scale is bored with two holes through which pass the two arms of a yoke C, which applies the load to the consolidation device. The load is applied to the bottom of the yoke by the jack D, which is a 2½-ton screw-type high-grade truck jack. The jack is actuated by the hand wheel E. The lever arm of the scale is counterbalanced for the load which is to be applied. The hand wheel D is manipulated to get the balance arm of the scale in horizontal position. The stop is removed from the balance arm of the scale, so that as deformation of the specimen takes place under the load the applied load will not change but the difference will be taken up in the movement of the scale platform as reflected in the movement of the arm. The amount of consolidation of the sample is measured by the dial gage F.



The consolidation characteristics of a foundation may be determined by taking undisturbed samples of the various strata of the materials occurring in the foundation and testing them. The tests are carried out in consolidation machines (Fig. 11) designed and built in the laboratory; these machines were assembled from platform scales, truck jacks and dial gages, which permitted their construction at a very low cost yet gave machines that are sufficiently accurate for all practical purposes. The purpose of the machine is to apply uniform and unvarying loads to a specimen for any desired length of time, often for several days, and to be able to change the intensity of the load at will. The specimen is placed in a bronze ring, 1¼ in. in depth and 4¼ in. in diameter; it is carved out of an undisturbed sample of the material to the exact size of the inside of the ring (the ring being used as a templet). Porous stone disks of slightly smaller diameter than the diameter of the ring provide drainage at the top and bottom surfaces.

To determine the consolidation characteristics of the specimen in the consolidation service, it is necessary to apply and maintain any desired vertical load on the sample. The desired load is set on the arm of the 1,000-lb. platform scale used, and the load is applied to the sample by a 2½-ton screw-type high-grade truck jack through a yoke. The stop is removed from the scale arm, permitting it to move as deformation of the specimen takes place, thereby maintaining a uniform load. The scale arm is brought back to the horizontal from time to time by taking up on the jack. The amount of consolidation of the sample at any time is measured by the reading of the dial gage.

The general results of these tests fix the time rate of consolidation and the stress-strain relationship of the particular sample considered. To illustrate, if the specimen from a stratum consolidated 5 per cent under a load equal to the proposed load over the particular stratum which was 10 ft. thick, then the stratum would ultimately consolidate 5 per cent of 10 ft. or 0.5 ft. due to the embankment load. By determining the consolidation for each stratum in this way and adding them together, the total ultimate consolidation of the foundations under the embankment may be determined. Further, since rate of consolidation varies within limits as the square of the distance between drainage boundaries and assuming that the 10-ft. stratum is underlain by a free draining material, consolidation of the stratum will take $\frac{120^2}{1.25^2} = 9210$ times as long as it did on the sample. If the consolidation of the sample took ten hours, that of the stratum would take 92,100 hours, or 3,840 days, or 10½ years. By similar methods the amount of consolidation at any time may be determined. Thus, from these results the amounts of settlement anticipated during construction and after construction is completed may be fixed with reasonable accuracy.

Stress-strength relationship

The next problem to be handled is to determine the stress-strength relationship in the foundation of the proposed embankment. First, the shearing strength of the foundation materials is determined. After that is done, the embankment must be so designed as to keep the stresses well below the strength of the foundation material. In studying foundation sections where silt or clay type material containing a high moisture content are found, tests should be run to determine the shearing strength of the various foundation materials. For these tests, as for the consolidation tests, it is necessary to have undisturbed samples of material. Ordinarily specimens from the same sample are used for both tests. Sand and gravel materials generally are of no interest in this connection, as they consolidate as the load is applied and develop high strength. In silt and clay type materials it is comparatively easy to obtain undisturbed samples, as these materials are naturally impervious and test pits can be put down through them with very little pumping. Specimens for making the test to determine the shearing strength of these materials may be cut from the undisturbed samples obtained from the foundation.

A special shearing machine (Fig. 12) was designed in the laboratory to make these tests. Two things are desired in the shearing machine: one is means of applying the vertical load uniformly during tests, and the other is a means of applying and measuring the shearing load. For the construction of this machine, as for the consolidation machine, standard manufactured articles were utilized. The shearing box that holds the sample consists essentially of top and bottom sections that are kept from contact with each other by the sample. The box is set on a 1,000-lb. platform scale, and the vertical load is applied as in the consolidation machine. The shearing force is applied to the bottom section of the box by a second jack set horizontally. The total shearing stress induced in the specimen is transferred from the top half of the box through a yoke to a bellows. The pressure on the bellows is measured by a manometer. The desired vertical load is applied by setting the scale arm, and equal horizontal strains are applied by the horizontal jack in increments of 15 sec. so that a complete shear will occur in 4 min. This rapid rate of load application tends to eliminate consolidation during the time of test in so far as it is possible to do so.

Several specimens, 4x4x$\frac{9}{16}$ in., carved from each undisturbed sample are tested at different vertical loads, so that the variation in shearing strength with the vertical load applied can be determined. Fig. 13 shows the shear curve determined by tests on specimens from one sample. The strength at zero vertical load can be defined as the cohesion, and the slope of the line gives the apparent angle of internal friction.

It may be observed that during construction of an embankment a certain

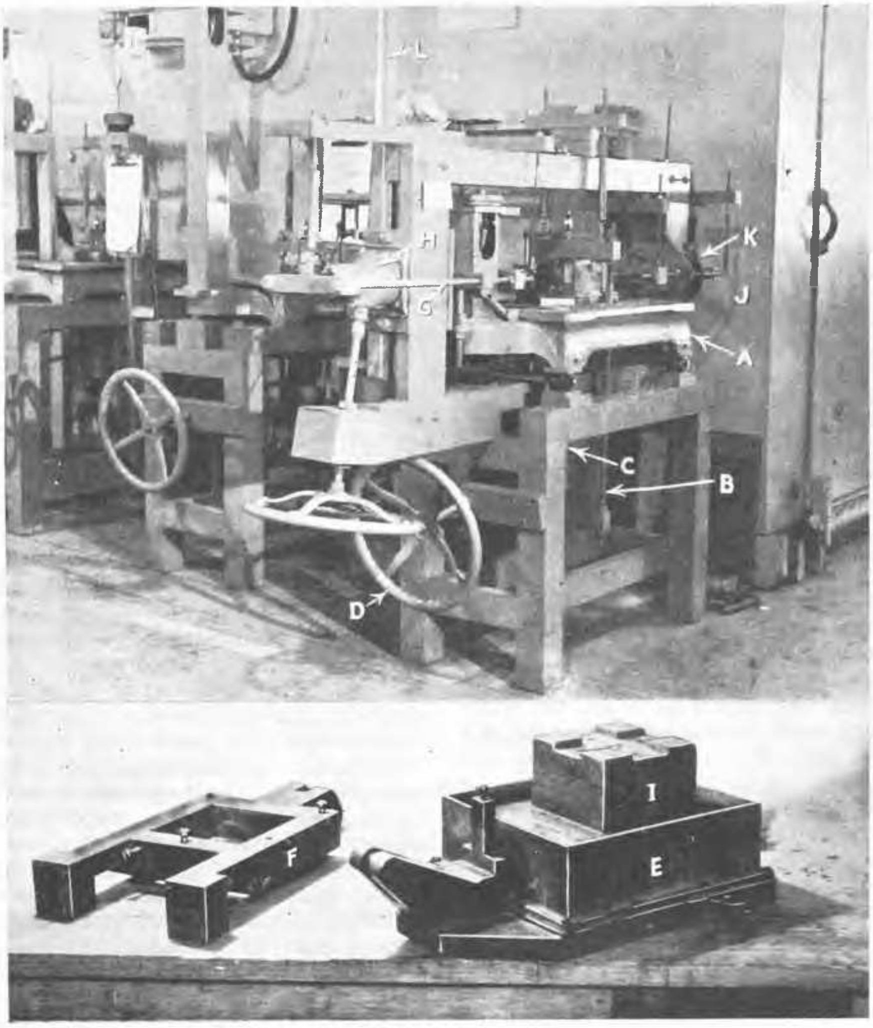


FIG. 12—SHEARING TEST MACHINE with enlarged view of shear box and specimen after test; laboratory was constructed at a cost of $400. The shear box is set on platform scales H, and the vertical load is applied through yoke B motivated by jack, operated by hand wheel D. A shearing load is applied to the bottom half of the shearing box E by means of yoke G, motivated by jack H. The top half of the box is connected by means of yoke J to bellows K, which is connected in turn to manometer L, which measures the intensity of the shearing stress in the specimen. The top section F of the shearing box has a 4-in. square opening through it, and the bottom section E has a similar opening recessed 9/32 in. into it. In this recess at right angles to the direction of shearing strain is a series of parallel ribs used to grip the bottom of the test specimen. The undisturbed test specimen is cut from the sample 9/16 in. thick to fit exactly the opening in the shear box. The top half of the shear box F is set down over the sample, but contact between the two halves of the box is prevented by set screws. Through the opening in the upper half F a bronze piston, which is corrugated on its bottom surface in the same manner as the recess in E, is pressed into the sample and locked in place by set screws. The clearance screws holding the two sections of the box apart are then released, as the sample now serves that purpose. The box is now ready to insert in the machine.

amount of consolidation does take place in the foundation, which in turn would tend to increase the shearing strength of any particular sample. However, no testing method so far devised has been able to take this factor fully into account, and the only approach to the problem is to test under the assumption of no consolidation during construction and reproduce the effect of consolidation as effective load applied in a given time. This factor induced into the shear relationship as an increased vertical load will give the minimum increased strength due to consolidation and therefore is a conservative analysis. Hence, with the shear relationship fixed it becomes necessary at any given time to estimate the vertical load effective on the material under consideration.

As a preliminary explanation, it is well to point out that while at the time of instantaneous application of load the principal stresses are carried entirely by water (assuming the material to be saturated), the resulting shear stresses must be carried by the grains themselves. As time progresses, the water being under pressure is forced out at a rate depending on the thickness and permeability of the material, and as a result the principal stresses are transferred to the soil particles themselves. As these stresses become effective the material itself increases in shearing strength. To be exact in testing, it would be necessary first to fix the consolidation and the resulting permeability relationship in the prototype and simulate that condition in the testing of individual samples. Since this procedure so far has not been developed, the accepted method is a safe approximation.

Determining foundation stresses

The characteristics of the foundation material having been experimentally determined, it remains to determine what stresses will be induced in the foundation under the proposed embankment design. Fig. 14 shows a triangular embankment load. The maximum loading on the foundation is from the crown of the embankment marked *A*, and zero load is applied at the toes *B* and *C*. The difference in load, between the load at *A* and the zero load at *B* or *C*, must be taken up as shear in the foundation. There have been two cases for which shearing stress in foundations have been worked out:

1—For an infinite depth of questionable material in the foundation: For practical purposes, infinite depth may be defined as depth equal to or greater than the base width of the embankment. For large embankments this formula rarely has any application. Dr. Jurgenson, in the *Journal* of the Boston Society of Engineers, Vol. XXI, No. 3, July, 1934, developed this from an application of Corruthers formula. The equation for maximum shearing stress is

$$c = 0.256\,p$$

where c equals the maximum shearing stress, and p equals the intensity of pressure at the center line.

2—The more usual case is where the depth of questionable material is much less than the base width of the embankment. For the case where the depth of questionable material does not exceed one-tenth of the base width of the embankment, Jurgenson has developed the following formula:

$c = \frac{pa}{L}$ where c is the stress, p is the pressure developed by the maximum height of the embankment, a is one-half the depth of the questionable material, and L is one-half the base width of the embankment.

Between the limits covered by the two equations mentioned above, no simple analytical solution has been developed for stress determinations in the foundations. Recently an involved solution has been developed by Jurgenson from the fundamental equation defined by Corruthers, but it is so unwieldy that it must be eliminated from practical use and can be best applied to check the experimental methods herewith proposed. The Jurgenson formula for the limited case does not give the location of the maximum stress zone in the foundation. To be safe in applying the formula, it must be assumed that maximum stress occurs where the questionable material is weakest. To apply this method then, it is only necessary to have the embankment dimensions for substitution in the above formula, to get the maximum stress. Knowing the shearing strength of the material, the factor of safety against failure under the worst combination of conditions is a matter of simple division.

Photo-elastic stress determination

It must be noted that the formulas given above apply in limited cases only and also apply only to triangular embankments. The analytical solution has not been developed for other than triangular embankments or for cases beyond those limits. For the purpose of checking the analytical application of Jurgenson's formula and to develop methods to work out cases not covered, a photo-elastic method of determining stress distribution experimentally was developed in the laboratory. Gelatin was used as the foundation medium, and the embankments were constructed of lead shot, as this material was found to be the only commercial material heavy enough and suitable to give the desired results.

The well-established principles of photo-elastic analysis were applied in these experiments. Fig. 17 is a photograph of a test showing the distribution of shear stress contours. With the experiment described above, it is necessary in each case to run a calibration test where a known shearing load is induced in a sample of the gelatin material cast at the same time as the model material, in order to determine the contour interval of shearing stress for the material at that particular age. Having determined the contour interval, the maximum stress may be determined by counting the contours to the point of

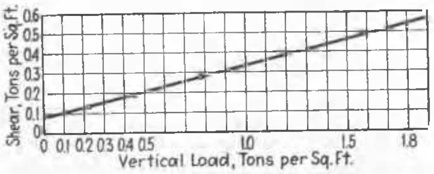


FIG. 13—SHEAR TESTS were made on each sample at several vertical loads. Curve shows results of tests on one sample.

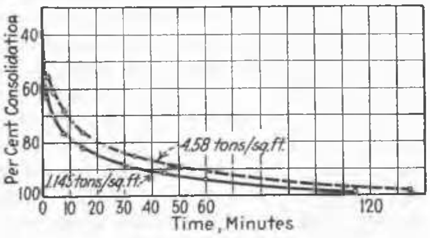


FIG. 15—TIME-CONSOLIDATION test curves at Clendening Dam, used in determining rate of construction of embankment.

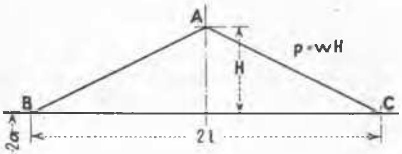
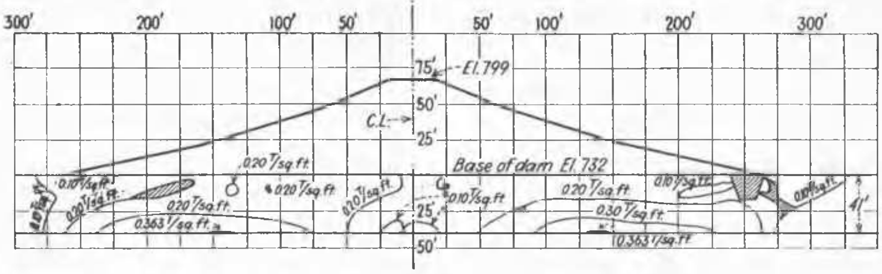


FIG. 14—DIAGRAM OF TERMS used in Jurgenson formula for stresses in foundations under embankment load.

FIG. 16—DISTRIBUTION OF SHEAR STRESSES in dam foundation (Wills Creek) established by operator's sketch from photo-elastic test shown by Fig. 17.



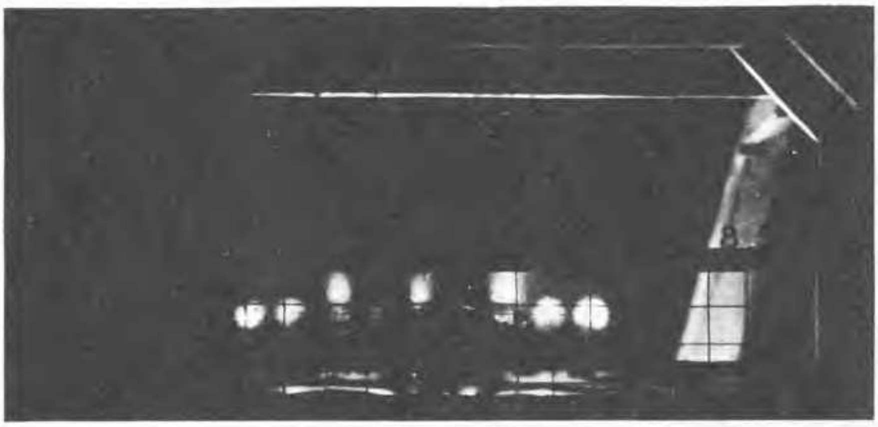


FIG. 17—DISTRIBUTION of shear stress contours in foundation of dam models (Wills Creek) disclosed by photographs of images produced by polarized light.

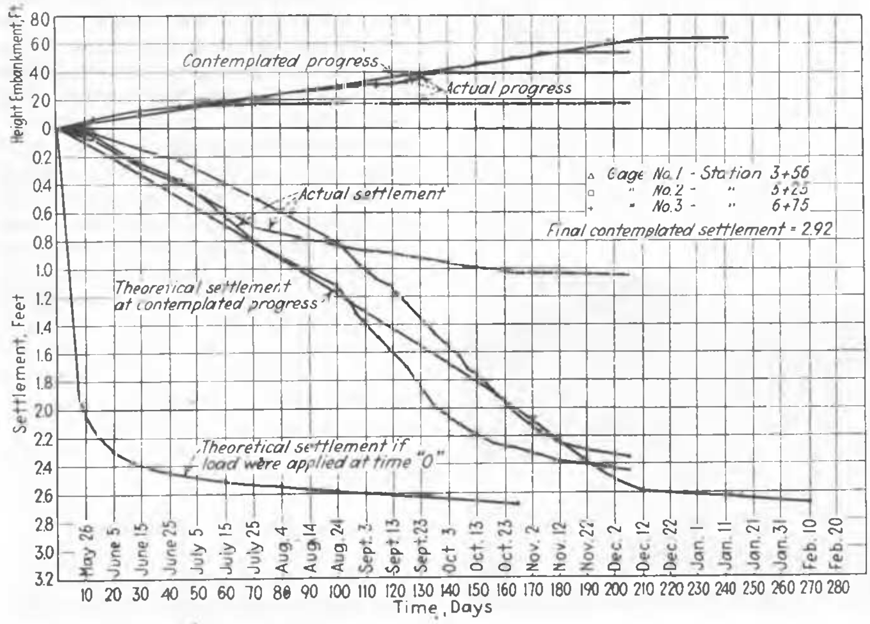
greatest stress and multiplying the number of contours by the contour interval. The results of the experiments are recorded by photographing the image of the model produced by the polarized light, Fig. 17. At the same time, a color sketch is recorded by the model operator from which the stress distribution is developed as shown by Fig. 16.

From the intensity and location of the maximum shearing stress in the model, the intensity and location of maximum shear in the prototype may be determined. To check the results of the model experiments against the analytical solution, a number of cases were worked out for triangular embankments. In about a dozen check tests the maximum variation between the results obtained by the two methods was 19 per cent, and the average variation was 6 per cent. As this variation is well within the limits of the factors of safety adopted for use, it is believed that entirely reliable results can be obtained by the experimental method where the work is carefully done by an experienced personnel.

Gelatin was the only photo-elastic material which we were able to discover that would give the proper relations for modal study. The difficulty in using any other material develops because all other photo-elastic materials which we could locate were too high in strength to develop sufficient stress contours within the limits of embankment loads that could be applied readily in the laboratory. As it was, it was necessary to go to the heaviest material obtainable in commercial quantities; that is to say, lead shot, for embankment construction in the models.

It must be noted that the methods of stress analysis are based upon the assumption of elasticity. It is not within the scope of this article to develop the validity of such an assumption. It suffices to point out that in investigating the foundations of various conditions, each a borderline case, the results of tests indicate what was known to be an actual condition—i.e., failure or stability. The conclusion drawn from these investigations was that the error of such an assumption was well within the accuracies of such work. The analysis also provides against the overstress of any portion of the foundation. When such a condition exists that at the point of maximum stress the material has just sufficient strength to balance this stress the remainder of the foundation is understressed and capable of more load than is at present exerted upon it. The addition of shear stress would cause a redistribution of stress to the understressed portions of the foundation. Only an overload of sufficient magnitude to produce stresses in excess of all the available strength of the foundation can cause a total failure. Such a failure is preceded by excessive settlements and characteristic mud waves at the toes.

FIG. 18—RELATIONSHIP between the computed foundation consolidation during construction and the consolidation that actually took place at Clendening Dam.



Final failure is characterized by a blow of the foundation and a sudden subsidence of the load.

Our practice has been to design with a factor of safety of 1.5 against overstresses and an estimated factor of safety against failure of 2.6. This factor of safety may seem small, but since a progressive increase in foundation strength takes place after completion of construction due to consolidation of the foundation, the factor of safety tends to increase with time, and if the embankment can be completed there is little fear that failure will take place in the future.

It will frequently happen that tests will show that a foundation material has a very low factor of safety where the load is instantaneously applied. However, where the strata are comparatively thin or are drained by sand or gravel layers, the material will consolidate more or less rapidly as load is applied and develop increased strength against failure in shear. The time-consolidation curve indicated in Fig. 15 may be used to determine the amount of consolidation and increase in strength under the proposed method of construction, and it may be in some cases that by limiting the proposed rate of construction, the desired factor of safety may be obtained. This is illustrated by the Clendening Dam, where the rate of construction was limited to 4 ft. per week.

For the purpose of checking the foundation consolidation during construction to be sure that the factor of safety is maintained, a system of settlement gages has been installed at several of the dams. Fig. 18 is a chart that shows the relationship between the computed consolidation during construction and the consolidation that actually took place for Clendening Dam. These charts are carefully followed during the construction period to see that the consolidation is at least as great as the computed consolidation.

In the closing article the laboratory and field methods of selecting embankment material and controlling its placement are described.