Simple Tests Determine Hydrostatic Uplift

A study of hydrostatic uplift in clay and concrete with an analysis of simple tests for determining values

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THIS ARTICLE is a description of three independent methods for determining the value of the reduction factor x employed in calculations of hydrostatic uplift. Each method is simple enough to be used in any materials or soil testing laboratory. In the discussion a uniform notation given in the accompanying panel is used throughout. The meaning of other symbols is explained in the text as the discussion proceeds.

Manifestations of uplift

The best known manifestation of hydrostatic uplift is that it reduces the pressure of engineering structures on their bases, provided that the base is located below the free-water level. In the case illustrated by Fig. 1, the weight which acts on the soil beneath the base of the pier is equal to $G - FHs_ox$ wherein G is the total weight of the pier. Fig. 2 shows another case of a similar type. It represents a section through a low concrete dam founded on a bed of fissured shales with thin, horizontal seams of clay. Failure would occur by sliding along the uppermost layer of clay, AB, in Fig. 2. The frictional resistance against sliding depends on the effective weight which acts on AB. This effective weight, G_1, is equal to the difference between the total weight G of the masonry, the shale and the water located above AB and the hydrostatic uplift, or

$$G_1 = G - 1/2\ (h_1 + h_2).\ Fs_ox$$

The values h_1 and h_2 represent respectively the height to which the water would rise from the fissures in the shale into standpipes located at the points A and B, Fig. 2. For $x = 1$ the value of G_1 might be less than one-half of the corresponding value for $x = 0$. Therefore the value of x has a very important influence on the factor of safety of the structure against sliding.

Symbols Used

F = area of a section or of the base of a structure
xF = effective area or that part of an area over which the hydrostatic uplift acts
x = reduction factor
H = depth of a point below the free water level
h = height to which the water rises in a standpipe
s_o = unit weight of the water
n_w = hs_o = pressure in the water
n'_w = total normal pressure per unit of area of a section
n = $xn' - xn_w$ = effective normal pressure per unit of area of a section

The meaning of the other symbols used is explained in the text.

Another important manifestation of the hydrostatic uplift is illustrated by Fig. 3a which shows a section through a concrete dam of the gravity type on a perfectly impermeable base. If the dam consists of well-prepared concrete the quantity of water which percolates through the body of the dam towards the downstream face is equal to the quantity of water which evaporates along the face. Therefore the downstream face will appear to be dry. Nevertheless there will be a continuous though imperceptible flow of seepage water from the reservoir toward the downstream face as indicated in Fig. 3a by arrows. If the permeability of the concrete is uniform throughout the dam the hydrostatic pressure along the base of the dam will increase from H at the upstream edge of the dam to zero at the downstream edge as shown by the ordinates of the dash-dotted line a_1b in Fig. 3a. The hydrostatic uplift along the base of the dam is about equal to $\frac{1}{2}Hs_ox$. To illustrate the influence of the value x on the state of stress in the dam two extreme cases will be considered, namely $x = 0$ and $x = 1$. The corresponding distribution of the normal pressure over the base is shown in Fig. 3b and c. The values shown in the diagrams b and c of Fig. 3 were computed on the assumption of a straight line distribution of the pressures over the base of the dam. A more accurate method of computation was published by the author in the journal "Die Bautechnik" 1934, H.45.) Comparing these two diagrams one realizes that no reliable information as to the true state of stress in a concrete dam can be obtained unless the value x is known.

It is obvious that the uplift acts only on that portion of the basal area which is in direct contact with water. Yet any attempt to estimate the effective frac-

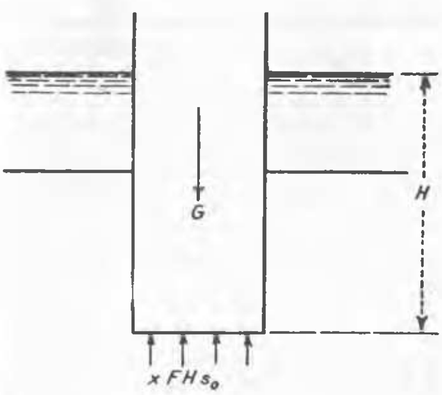
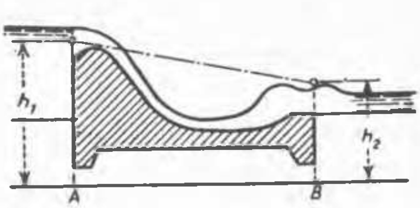


FIG. 1—MASONRY pier subject to hydrostatic uplift.

FIG. 2—HYDROSTATIC uplift in a clay seam AB located beneath the base of a low concrete weir.



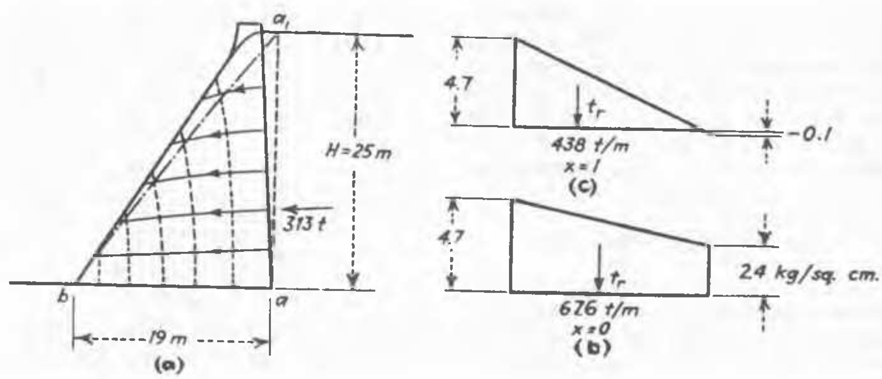


FIG. 3—(a) CONCRETE gravity dam on an impermeable base; (b) pressure in concrete above base for $x = 1$, and (c) for $x = 0$.

tion x on the basis of this vague and general statement must be considered pure speculation unfit for practical use. Reliable information regarding the reduction factor x can only be obtained from direct measurements of the uplifting force itself or of its mechanical effects. The following methods are available:

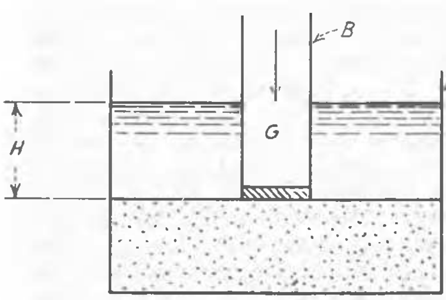
Floating test has limited value

The fundamental principle of the floating test is shown in Fig. 4, which represents a section through a vessel C whose bottom is covered with a layer of saturated sand. The sand carries a cylindrical vessel B with a known weight G and a plane bottom, which covers an area F. The test is performed by gradually filling the space between the surcharge B and the wall of the vessel C with water and by determining the depth H of the water at which the surcharge G begins to float. At this instant the hydrostatic uplift FHs_0x is equal to the weight G. Hence

$$x = \frac{G}{FHs_0}$$

wherein all the quantities are known. During the last fifty years many investigators in different countries used this method with variations in the details of the test arrangement. All of them arrived at the conclusion that the

FIG. 4—APPARATUS for floating test; S = layer of saturated sand, B = bearing block with weight G; the water level is raised until B begins to float.



reduction factor x for sands is almost equal to unity.

The method is open to the objection that the results are valid only for the state immediately before the effective surface pressure becomes equal to zero and the method fails when applied to materials with an appreciable cohesion such as plastic clays or concrete. In order to determine the value x for these two materials different methods must be used. The following paragraphs contain a description of the procedures employed by the author.

Buoyancy compression test

This test serves to estimate the reduction factor x from the compression produced by an increase of the hydrostatic pressure in the liquid above the surface of a layer of plastic clay. The sample occupies the lowest part of a cylindrical vessel C, Fig. 5a, with a height of about 250 cm., a diameter of 10 cm. and a cross sectional area $F = 79$ sq.cm. The upper surface of the sample Cl is covered with filter paper and a layer of fine sand which in turn carries a perforated brass pipe B with a massive bottom plate. The total weight of this surcharge is G and the pressure exerted by this surcharge on the surface of the clay is $q = G/F$. Fig. 5b shows the relation between the load q per unit of area and the corresponding thickness d of the sample. By increasing the surcharge from $q_1 = G_1/F$ to $q_2 = G_2/F$ with the water standing in the vessel C, Fig. 5a at the level of the base of the surcharge the thickness of the sample decreases by Δd as shown in Fig. 5b. If the same increase of the pressure on the sample from q_1 to q_2 is performed only by filling the vessel C with water until hs_0 becomes equal to $q_2 - q_1$, the resulting decrease Δd_1 of the thickness of the sample will be smaller than Δd because the water pressure acts only on the difference $F(1 - x)$ between the total area F and the effective area xF of the surface of the sample. Assuming a straight line relation between q and d for the pressure interval q_1 to q_2 we obtain

$$\Delta d_1 = \Delta d \frac{(q_2 - q_1)(1 - x)}{q_2 - q_1} = \Delta d(1 - x)$$

$$\text{or } x = 1 - \frac{\Delta d_1}{\Delta d} \qquad (1)$$

In the tests which were performed in the laboratory of the author, the initial thickness of the sample was equal to 1.5 cm. and the values of Δd varied between 0.1 and 0.5 mm. or between 40 and 200 division lines on the Ames dials. However, in no case was the value of Δd_1 important enough to be measured which indicates a value of x, equation (1), close to unity.

To obtain the value of x for concrete, cylindrical specimens were used with a height of 20 cm. and a diameter of about 7 cm. One set of tests was made on specimens in a dry state whose entire surface was covered with a water-tight jacket. The specimens were introduced into a container C, Fig. 6a, and subjected to hydrostatic pressures up to 200 kg. per sq.cm. and their axial compression Δd was measured for different external pressures by means of the Ames dial A, Fig. 6a. A second set of tests was made, identical with the first except that the water-tight jacket was omitted. The specimens were introduced into the container in a saturated state and the liquid under pressure communicated freely with the liquid in the voids of the concrete. As the hydrostatic pressure in the first series of tests was increased from zero to 200 kg. per sq.cm., the axial compression increased up to 0.5 mm., or up to 200 units on the dial. In the second series the axial compression produced by the hydrostatic pressure was too small to be measured.

These test results demonstrate that the compression of the tested materials depends exclusively on the difference

FIG. 5—(a) APPARATUS for buoyancy compression test on clay Cl; (b) shows the relation between vertical pressure q due to weight G and the corresponding thickness d of the sample; in a second series of tests the vertical pressure q is increased by filling the vessel C with water at constant weight G.

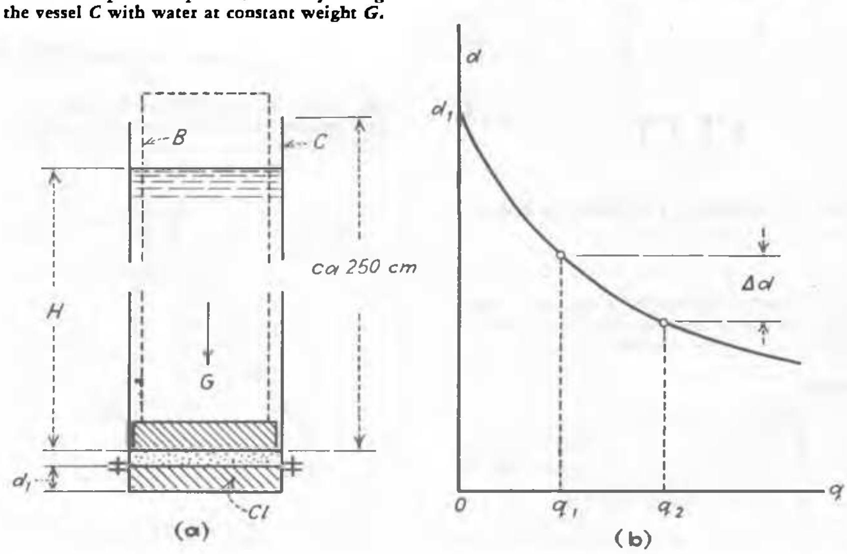
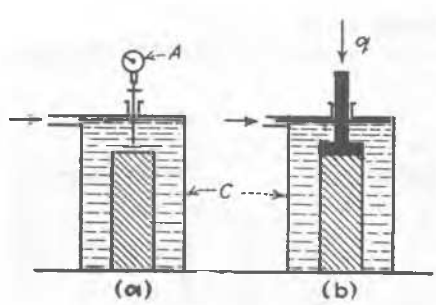


FIG. 6—HYDRAULIC chamber, (a) for measuring compression produced by increase of hydrostatic pressure in the chamber and (b) for combining hydrostatic pressure with axial excess pressure q.



between the total pressure and the hydrostatic pressure in the interstitial liquid regardless of how small the voids may be.

In each of the tests illustrated by Figs. 4, 5 and 6 an increase of the pressure n_w in the liquid automatically produced an increase of the pressure of equal intensity in the solid part of the testing material. In the following discussions any pressure of this type will be called a *neutral pressure.* The test results cited disclose the important fact that an increase of a neutral pressure produces no measurable compression of a clay or of a concrete regardless of the compressibility of the aggregate.

Buoyancy expansion test

The buoyancy expansion test is used to determine the effective fraction x for the surface of contact between a plastic clay and the smooth base of a brass piston, while the clay stands under pressure. The test arrangement is shown in Fig. 7a. The sample of clay, Cl, is inserted between the upper surface of a saturated porous stone St and the bottom of the brass piston P whose weight G can be changed at will. The piston fits the inside of a cylindrical vessel C with a cross-sectional area F. The water in the voids of the stone St communicates with the content of a stand-pipe S, 600 cm. high. In one series of tests the water level in the standpipe is kept at the elevation of the bottom of P. In this series an increase of the pressure $q = G/F$ from zero to q_o causes a decrease of the thickness of the sample represented by the ordinates of the curve K_o in Fig. 7b. A subsequent decrease of the pressure from q_o to zero produces an expansion represented by the curve K_1. For a decrease of the pressure from q_1 to q_2 the expansion of the clay is equal to Δd. In a second series of tests the water level in the standpipe is kept at the level of the base of P during the entire increase of the pressure up to q_o and the subsequent decrease from q_o to q_1. Then the weight of the piston is kept constant at $G = G_1$ and water is poured into the standpipe S until the condition $hs_o = q_1 - q_2$ is satisfied. If the reduction factor x for the base of the piston is equal to unity there should be no difference between the expansion Δd of the clay produced by reducing the weight of the piston from G_1 to G_2 at an unaltered standpipe level and the expansion produced by filling the standpipe to elevation h, Fig. 7a, because the test was made in such a way as to satisfy the condition $G_1 - G_2 = Fhs_o$. In reality x is smaller than unity. Hence we obtain

$$\Delta d_1 = x \Delta d$$

or

$$x = \frac{\Delta d_1}{\Delta d}$$

The tests were made on a very fat tertiary clay from Paris in a stiff plastic state. The other data pertaining to the tests are: Diameter of the vessel $C = 10$ cm., $d_o = 1.5$ *cm.*, $q_o = 5.0$, $q_1 = 1.0$ and $q_2 = 0.4$ kg. per sq.cm. The value x was found to about equal 0.95, in spite of the extremely low permeability of the clay and the relatively high pressure which acted on its surface.

Buoyancy crushing test

The buoyancy crushing test is used to investigate the influence of a hydrostatic pressure n_w in the water content of a clay or a concrete on the shearing resistance along any section through these materials and on the stress conditions for failure.

The method is based on the fact that the shearing resistance of the water is equal to zero under any pressure. If failure occurs by shear along any section through a cohesive, porous, saturated material, one part of the surface of rupture is located in the water and another part cuts across the solid. The frictional resistance has its seat exclusively in the solid. If the pressure in the water represents a negligible item, the shearing resistance t per unit of area of a section under a normal

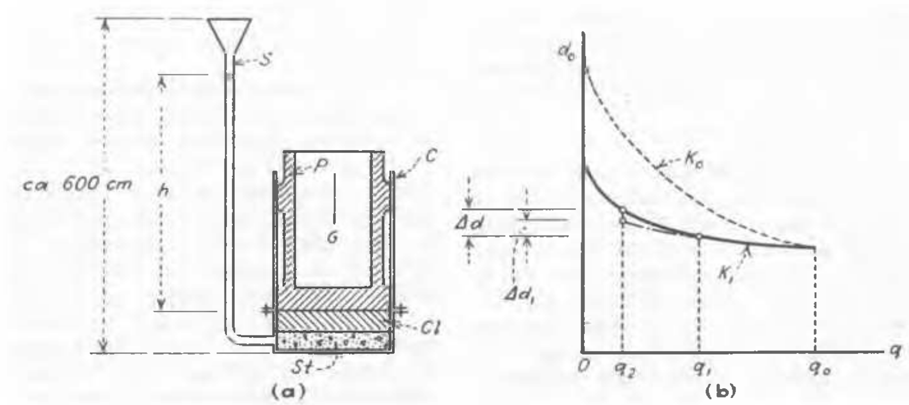


FIG. 7—(a) APPARATUS for buoyancy expansion test; (b) Curve K_1 shows the relation between the pressure q produced by the weight G and the corresponding expansion of the sample of clay, Cl in a. In a second series of tests the decrease of q is produced by filling the standpipe with water without changing the weight G of B.

pressure n is equal to

$$t = c + n \tan \phi$$

wherein c = shearing resistance under zero pressure and $\tan \phi$ = coefficient of internal friction.

On the other hand, if the water in the voids of the material is under an appreciable pressure n_w, one part, xn_w of the normal pressure n' will not produce any friction, because this part is taken up by a pressure in the water. Hence the shearing resistance will be

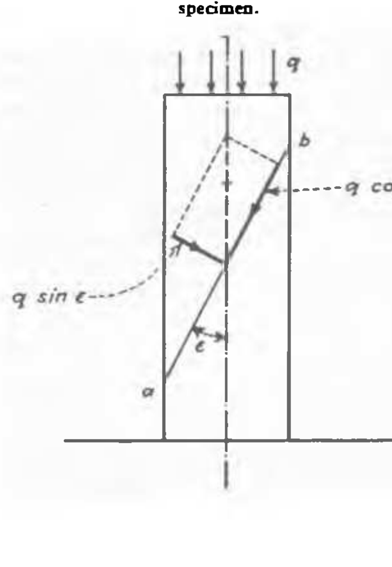
$$t = c + (n' - xn_w) \tan \phi \qquad (2)$$

To obtain the empirical data required to compute the value x by means of this equation we take advantage of the fact that the failure of cylindrical specimens subject to an axial pressure always occurs by shear along rough, yet fairly plane surfaces at an angle ϵ to the direction of the axial force. Experiments have shown that the angle ϵ is nearly equal to $45° - \frac{0.9\phi}{2}$. The roughness of the planes of rupture is due to the fact that failure occurs along a section of minimum shearing resistance. Such sections are always uneven in detail.

Fig. 8 shows a section through the axis of a cylindrical specimen with a cross-sectional area equal to unity. By increasing the axial pressure q on this specimen to the breaking point failure occurs by shear along a fairly plane surface at an average angle ϵ to the direction of q with an area $F = 1/\sin \epsilon$. At the instant of failure the shearing force $q \cos \epsilon$ which acts along the plane of failure is equal to the shearing resistance. This shearing resistance is composed of two parts. One part, t per unit of area, is independent of q and the other is equal to $q \sin \epsilon \tan \phi$. Thus we obtain the equation

$$q \cos \epsilon = q \sin \epsilon \tan \phi + t \frac{1}{\sin \epsilon}$$

FIG. 8—NORMAL force and shearing force produced by an axial pressure q on the plane of failure ab in a cylindrical specimen.



$$\text{or } q = \frac{t}{\sin\epsilon\,(\cos\epsilon - \sin\epsilon\tan\phi)}$$

wherein $\epsilon = 45° + \frac{0.9\,\phi}{2}$. The denominator on the right side of this equation depends exclusively on the empirical value ϕ, which is practically the same for every test. Therefore we can replace the equation by

$$q = A_\phi . t \text{ wherein} \quad (3)$$

$$A_\phi = \frac{1}{\sin\epsilon\,(\cos\epsilon - \sin\epsilon\tan\phi)} = \text{const.} \quad (4)$$

According to equation (2) the shearing resistance t consists of the sum of the shearing strength c and the frictional resistance $(n' - xn_w)\tan\phi$. The factor $(n' - xn_w)$ includes every normal pressure which may act on the plane of failure in excess of $q \sin\epsilon$, Fig. 8. Thus we obtain

$$q = A_\phi\,[c + (n' - xn_w)\tan\phi] \quad (5)$$

In order to produce a normal pressure in excess of $q \sin\epsilon$ we introduce our specimen into the hydraulic chamber Fig. 6b and combine the application of the axial force q with the application of an all-sided pressure n. If the surface of the specimen is covered with a water-tight jacket with a discharge opening for the excess water, the hydrostatic pressure n_w in the water remains practically equal to zero for any value of n and equation (5) becomes

$$q = A\phi\,(c + n\tan\phi) \quad (6)$$

Tests of this kind are called tri-axial compression tests on drained specimens. On the other hand, if the water-tight jacket is omitted or if we test a completely saturated specimen enclosed in a water-tight jacket without any discharge opening, the all-sided pressure n produces a hydrostatic pressure of equal intensity in the water content of the specimen, whereupon the shearing resistance t then becomes equal to $c + (n' - xn_w)\tan\phi = c + n(1 - x)\tan\phi$ and the axial pressure required to produce failure is

$$q = A_\phi\,[c + n\,(1 - x)\tan\phi] \quad (7)$$

This is the equation describing the results of the tri-axial compression tests on undrained specimens. In order to determine the value ϕ, a series of tests on drained specimens is made. A second series performed on undrained specimens furnishes the data required for computing the value x.

Tri-axial compression tests on drained specimens of concrete were made by a great number of investigators, including Brandtzaeg at the University of Illinois and Ros at the Polytechnikum in Zurich. All of these tests led to the conclusion that the value of ϕ decreases slightly with increasing values of n. The average value of ϕ is about equal to 34 deg. corresponding to values of $\tan\phi = 0.675$ and $A\phi = 5$. Hence we obtain for concrete the approximate empirical equation.

$$q = 5\,[c + 0.675\,n(1 - x)] \quad (8)$$

The tests performed in the laboratory of the author in Vienna were limited to undrained specimens with a height of 20 cm. and a diameter of 7 cm. Table I contains the most important data concerning the concrete and Table II the test results. According to Table II an increase of n from zero to 400 kg. per sq.cm. acting on an undrained specimen produced no noticeable increase of the value q. For drained specimens, the increase Δq of q associated with an increase of n from zero to 400 kg. per sq.cm. can be computed from equation (6) by introducing the values $A\phi = 5$ and $\tan\phi = 0.675$. Thus we obtain

$$\Delta q = q_{n=400} - q_{n=0} = 5\,[c + 0.675 \times 400] - 5c = 1350 \text{ kg. per sq.cm.}$$

Hence the value of Δq is equal to about 1,350 kg. per sq.cm. for drained specimens against a negligible value for undrained specimens. This fact alone shows without any further computation, that the value of x was practically equal to unity for all the specimens tested.

Similar tests were made with a plastic Tertiary clay from Vienna. The value of ϕ was found to be equal to 24 deg. Table III contains the data obtained from tests on undrained specimens. Each numerical value represents the average of three individual tests. Although the clay contained some air, the value of the reduction factor x was close to unity.

The results shown in the Tables II and III lead to the conclusion that the frictional resistance produced by the neutral pressure n_w is negligible, regardless of the value $\tan\phi$ of the coefficient of shearing resistance.

TABLE I—DESCRIPTION OF THE CONCRETE USED IN THE TESTS

Series	Weight of cement in kg. per cu. m. of finished concrete	Water-Cement-Ratio in per cent	Age of Specimen of in days	Dry weight of Concrete in kg. per cu. m.	Volume of voids of Concrete in per cent	Volume of voids of the cement in per cent
I	600	44	60	2280	13.4	34.3
II	400	56	60	2265	13.9	44.8
III	236	100	65	2210	16.2	61.9

TABLE II—TRI-AXIAL COMPRESSION TESTS ON UNDRAINED CONCRETE SPECIMENS

q = Axial Excess Pressure at Failure

Water pressure n. in kg. per sq. cm.	Value of q in kg. per sq. cm.		
	Series I	Series II	Series III
0	580	532	66.3
100	630	.	67.8
200	646	534	67.0
300	404	654	66.5
400	604	454	67.1

TABLE III—TRI-AXIAL COMPRESSION TESTS ON UNDRAINED PLASTIC CLAY SPECIMENS

q = Axial Excess Pressure at Failure

Water pressure n in kg. per sq. cm.	q kg. per sq. cm.	Reduction factor x
0	0.67	.
2	0.75	0.97
6	0.85	0.985

Cause of the high values of x

The high values of x obtained from the tests seem to be due to the fact that the compressibility of the water and of the solid constituents of most porous materials including clay and concrete is extremely small compared to the compressibility of the entire aggregate. Hence the greatest part of the compression produced by a change in the state of stress is due to a reduction of the pore space associated with a deformation and a relative displacement of the solid elements. If a pressure is applied in such a fashion that it acts in the solid and in the liquid with equal intensity every particle is slightly compressed without changing its shape and its position with respect to its neighbors. Therefore the corresponding compression and all the phenomena associated with compression, such as the increase of the shearing resistance, are negligible.

Conclusions

1—The shearing resistance t per unit of area of a shearing plane in a saturated, porous material is equal to

$$t = c + (n' - xn_w)\tan\phi$$

wherein c = shearing resistance at zero pressure, n^1 = total normal pressure, x = reduction factor, n_w = pressure in the liquid and $\tan\phi$ = coefficient of shearing resistance. The paper contains a description of three independent methods for determining the value x. Each one of these methods is simple enough to be used in any materials- or soil-testing laboratory. When undrained specimens of clay or shale are tested, care should be taken to exclude air from the voids of the specimen.

2—For concrete and for plastic clay the reduction factor x was found to be practically equal to unity which indicates that the hydrostatic uplift in these materials is almost as active as it is in a sand.

3—The strain in clay and in concrete exclusively depends on the differences between the total stresses and the neutral stresses. In every point of the saturated material the neutral stresses act in every direction with equal intensity and they are equal to the pressure in the water at that point.