No. E-12

OPENING DISCUSSION
PRESSURE DISTRIBUTION

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We have to consider principally the vertical pressure, as it is the most important one. The horizontal stresses will be treated at the end.

1. The old theory of pressure distribution assumes that in each plane and in each depth the stress is distributed equally. Fig. 1.

2. The real and exact pressure distribution. The load P is also distributed along the total width just as in Fig. 1; but in a plane which lies in the depth z underneath the load, P, there is no uniform distribution as in Fig. 1, but a stress concentration is developed in the center. (Fig. 2) The stress is largest in a vertical direction under the center of the load and decreases to both sides down to p = 0. This actual stress distribution is of great importance for many questions and explains a large number of phenomena as we shall see.

3. This kind of vertical stress-distribution in a horizontal plane is a result of the general fact, that the stress distribution radiates from the load center. Therefore one gets for instance the following pictures as in Fig. 3 and Fig. 4.

The lines of equal stress, the isobars, either as the radial main stresses, or also as their vertical components, give a picture like Fig. 4.

4. The results of paragraph 2 and paragraph 3 are found theoretically and confirmed by careful investigations.

a. Theory.

1. The elements are given by the equations of Boussinesq, 1885. He derived the stress distribution within an elastic body, but with tensile and shearing strength as in solid bodies. Accordingly stresses arise everywhere and also in the zone III, which does not get any stresses in fills and loose material. (Fig. 5)

The following further publications on this subject are given in Vol II of the Proceedings of this Conference, in the paper by Mr. Gray, No. E-10. I mention only the book of Fröhlich, Druckverteilung im Baugrund 1924. In this book one finds besides an authentic treatment of the subject, also a detailed bibliography.

2. In the formulas of Boussinesq the fact is not taken into consideration, that in earth material and fills the zone III (Fig. 5) gets no stress. This fact is taken into consideration in the formulas of Strohschneider, 1912, which he derived from tests performed on a small scale.

3. For linear loads corresponding formulas have been derived, as for instance by Melan, 1929, Beton and Eisen, Heft 7/8, and also according to the ideas of Strohschneider by Kögler, Bautechnik, 1929. This too is treated thoroughly in the book of Fröhlich on stress distribution.

b. Tests

Many tests have shown that the stress distribution actually exists as given by these formulas. Experiments have been carried out by: Kick and Steiner, Prague 1879; Strohschneider, Graz 1909-11; University of Illinois 1910-13; Pennsylvania State College 1913-14; Goldbeck, Arlington, Washington 1917; Kögler-Scheidig, Freiberg 1925-27; Hugi, Zürich 1927.

Although the formulas cannot be applied for the zone directly underneath the load, they agree very satisfactorily with the test results for greater depths. A careful checking of this, that means a comparison of the formulas with the tests, is to be found in the previously mentioned book of Fröhlich.

In any case one may say that this part of the task, the computation of stresses under single and linear loads in greater depths underneath the load, is solved by theory and checked satisfactorily by experiments. Therefore further work in this line can only confirm the results already known.

5. Loads applied on large areas. The fundamental formulas of Boussinesq and others mentioned in paragraph 4 are in the first place only valid for single point or linear loads, respectively for very small load surfaces (circle or square) and for narrow load strips.

a. Theoretical computation.

If the load surface has a larger dimension, one has to use the method of superposition or addition. The large load surface is divided into several parts, each small enough to be replaced by a single point or linear load. From these partial loads one can compute the stress distribution underneath the load at any depth where one desires to know the distribution, and can finally add those partial stresses to make a total. See Fig. 5a.

However, for this way of computation one must make an assumption about how the pressure is distributed directly underneath the load surface, if: (a) uniform, (b) parabolic, (c) bellshaped. (Fig. 6)

There are many investigations and publications about this superposition and formulas and tables have been computed by the help of which one can find the vertical stresses below circular, square, rectangular, and strip loading surfaces.

b. Tests.

As far as I know, no tests have been made on large loaded areas to verify the law of superposition.

FIG 1

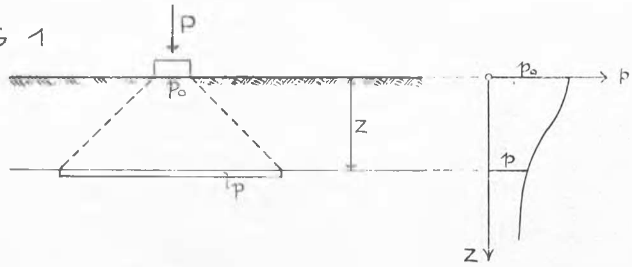


FIG. 2

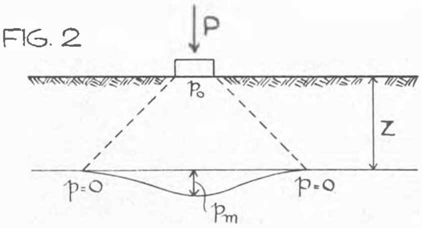


FIG. 3

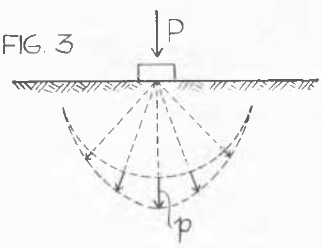


FIG. 4

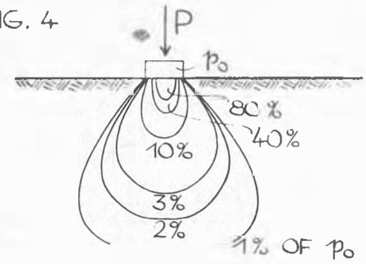


FIG. 5

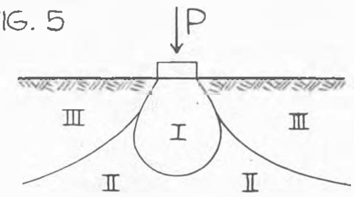


FIG. 5a

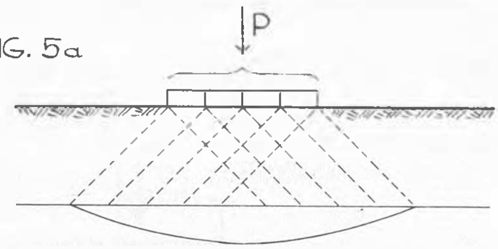


FIG. 6

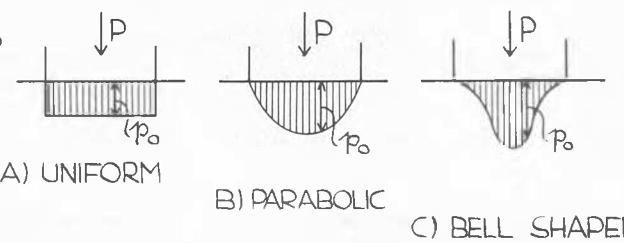


FIG. 6a

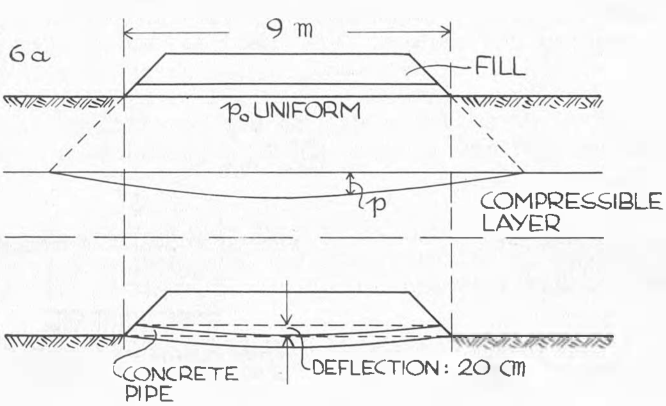


FIG. 7

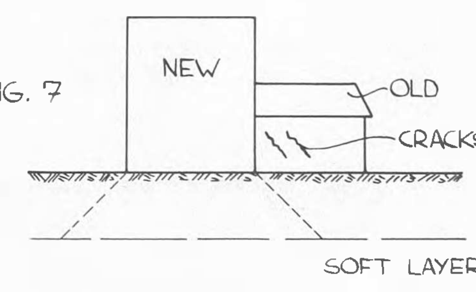


FIG. 8

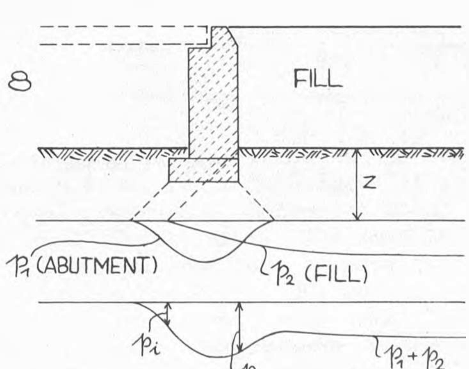


FIG. 9

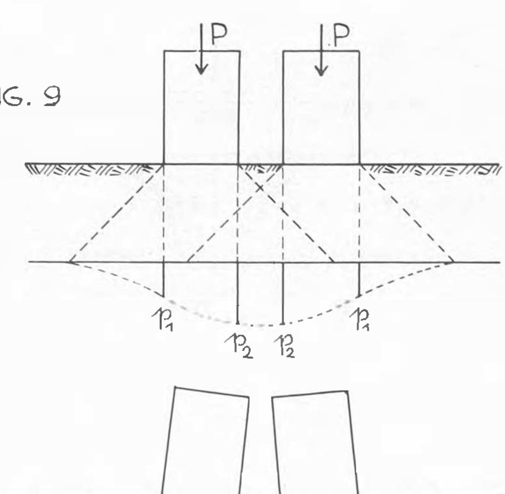


FIG. 10

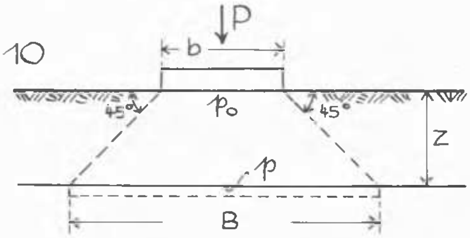


FIG. 11

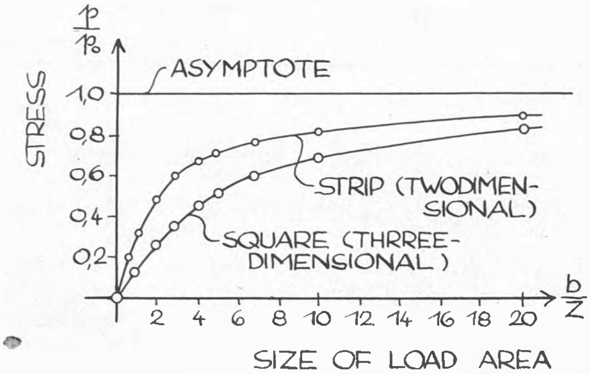


FIG. 12

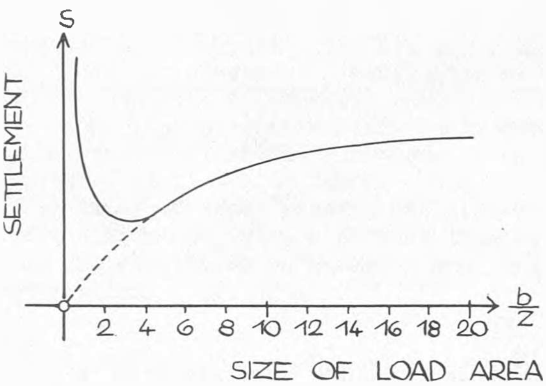


FIG. 12a

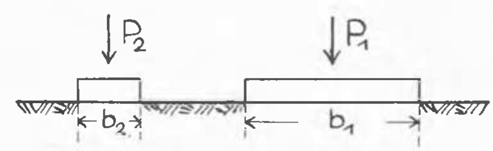


FIG. 13

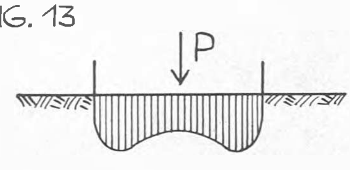


FIG. 14

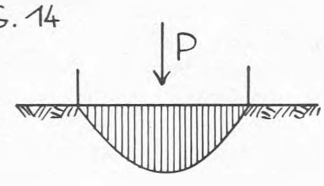


FIG. 15

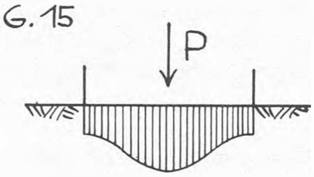


FIG. 16

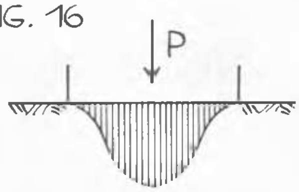


FIG. 17

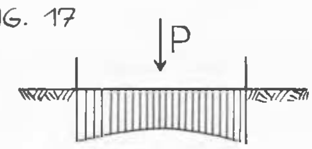


FIG. 18

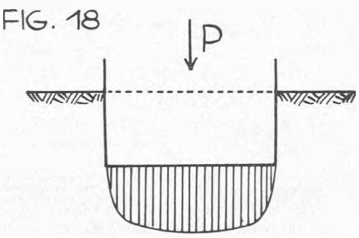
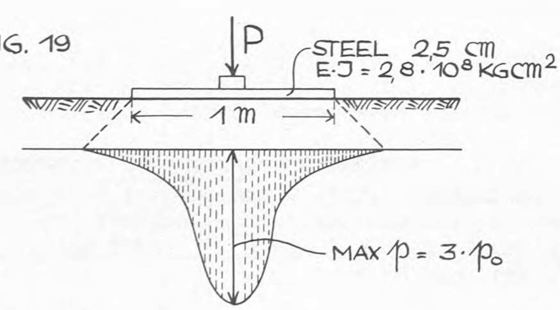


FIG. 19



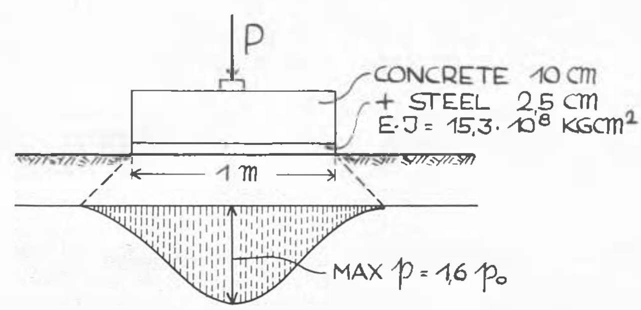
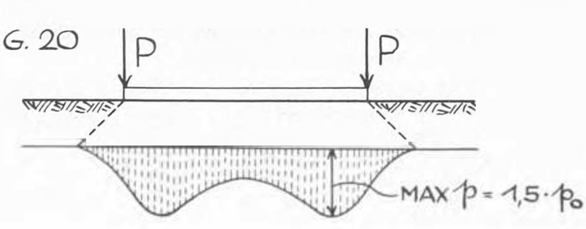


FIG. 20



It seems they are not necessary since by the previously described method one may compute closely enough the magnitude and the distribution of pressure, especially because one knows the limits within which the stresses must lie:

1. Lower limit, smallest stress, to determine from the assumption of stress distribution according to chapter I, Fig. 1.

2. Upper limit, largest pressure below the center of load, to determine from the assumption of stress distribution according to chapter II, Fig. 2.

This question too is solved and has been treated satisfactorily; for practical purposes many aids (formulas and tables) are at our disposal, giving the stress distribution easily and quickly and in addition with an accuracy which is fully sufficient for practical cases. For the variety of subsoils causes a much larger spread of results, than the different formulas or the assumption about the distribution of pressure according to (a), (b), and (c).

Even if the pressure is distributed uniformly on the surface of the ground one gets a stress concentration in the middle underneath the load, for instance, under a dam section. That means that if one has a soft compressible layer below the dam, the settlement will be larger in the middle than under the edges. Fig. 6a. In two cases I observed additional settlement of about 15 to 20 cm in the center under the dam and consequently also a downward deflection of a concrete discharge pipe embedded in the dam. Large buildings show the same result.

6. Some practical conclusions and applications of the pressure distribution: From the fact that the pressure distribution is radiating from the applied load it follows, that the building load spreads also under adjoining buildings, no matter if they already exist or are only under construction.

In case the subsoil of the adjoining building contains layers which are compressible under the additional load, settlements occur.

a. House adjoining an existing building. (Fig. 7)

Settlement of the old building takes place, because the additional pressure resulting from the new building compresses the soft layer of soil below the old building to a higher degree than has been caused by its own weight. Either the old building inclines as a whole, or it gets cracks in the described way. This phenomenon occurs frequently, without any mistakes having been made while excavating for the new building.

b. Fill adjoining a bridge abutment. (Fig. 8)

The weight of the fill is considerably higher and is applied upon a very large area; it therefore has also influence upon the soil layers below the abutment. The addition of both stresses below the inside edge of the abutment will produce the maximum pressure. The inclination of the abutment toward the fill is unavoidable, if the subsoil contains a soft compressible layer. The inclination of the abutment is often rather large, as many examples have shown.

Besides this we have to consider that the active earth pressure of the fill does not effect the back of the abutment, probably because the fill moves away from the abutment on account of its settlement.

c. Tanks neighbouring. (Fig. 9)

As a consequence of the concentration of pressures and of settlements we observed in many cases an inclination of tanks, when erected one near to the other.

d. Influence of the size of the loaded area.

1. From the fact that the load pressure is radiating in all directions we get with the simple assumptions in the following formulas: (Fig. 10)

Strip-load : $p = \dfrac{p_o}{1 + \frac{2z}{b}}$ (two dimensional) $= \dfrac{p_o b}{b + 2z} = \dfrac{P}{b + 2z}$

Square-load : $p = \dfrac{p_o}{(1 + \frac{2z}{b})^2}$ (three dimensional)

From this follows that the stress in the depth z does not only depend upon z, but also upon the width b of the loaded area. This dependence is shown by the curves in Fig. 11.

With equal pressure p_o below the load, large load areas produce larger stresses in a certain depth, than small loaded area. The same thing happens also with the settlements.

2. This has been confirmed by many tests which have been made for the first time by Goldbeck and later by Goerner at Freiberg. The results are sketched in the diagram Fig. 12. As can be seen from these curves they turn upward with small loaded areas. This can be explained by the fact that with small areas the subsoil tends to be squeezed out under the load, while with larger areas this phenomenon has almost no influence upon the settlement. In this connection one can compare a small load area with the point of a pile.

e. Practical application.

1. Suppose that the columns of a building are carrying different loads but ought to have the same value of settlement. In many cases even today the foundation of the columns are designed in a way in order to exert equal pressure p_o. But for equal settlements the size of foundation should be computed from formulas similar to the following ones: (Fig. 12a)

$$\text{Strips}: \quad b_2 + 2z = P_2/P_1\ (b_1 + 2z) \quad \rightarrow p = \frac{P_1}{b_1 + 2z} = \frac{P_2}{b_2 + 2z}$$

$$\text{Squares}: \quad b_2 + 2z = \sqrt{P_2/P_1}\ (b_1 + 2z)$$

2. Furthermore the curve in Fig. 12 shows that one should be careful in using loading tests because its areas are relatively small compared with building foundations. The danger of loading tests consists in the prediction of much smaller settlements than will result actually from the building load. In case very small areas are used for loading tests they give no picture whatsoever about the actual settlement of a building.

7. Stress distribution underneath a foundation. The question arises whether the stress distribution below a rigid foundation is uniform, parabolic, or bellshaped according to (a), (b), or (c) Fig. 6.

a. Rigid foundation slab.

1. According to the theory of Boussinesq on elastic bodies we get a stress distribution as shown in Fig. 13, because the body has tensile and shearing strength. The large stress along the edges of the loaded area can be explained by the fact that the soil outside of the loaded area (a) supports the soil within the area (b) by shearing strength between zone a and b.

2. Tests have contradicted the theory if applied to cohesionless material, as for instance gravel or sand. Careful investigations have been carried out by Enger 1916, Scheidig 1926, Faber 1933, Giesecke-Badgett-Eddy 1933, and results are shown by Fig. 14. This result seems evident, since the soil has no shearing strength and moves laterally near the edges.

If this lateral movement is prevented we get for sand, according to careful tests of Faber, a distribution as shown in Fig. 15.

On the other hand we get for a soil like soft clay which squeezes out easily, a distribution as shown in Fig. 16. For soils with large shearing and tensile strength the tests of Faber give a picture like Fig. 17, that means similar to Fig. 13, which follows from the theory. Those tests were carried out on hard blue clay.

At any rate one must conclude that the stress distribution directly below a rigid foundation slab depends to a large extent upon the character of the subsoil. Foundations at greater depths will result in more even stress distribution as in Fig. 18.

It would be of great value if more tests on this subject would be carried out, not only as laboratory tests, but especially in connection with actual foundations. In order to obtain good results, the subsoil should be homogeneous and the apparatus must work very exactly and reliably.

b. Flexible foundation slabs. Many theoretical papers have been published to discuss the distribution of stresses under a flexible slab foundation. Among the authors of these papers I wish to mention particularly the names of Zimmermann and Schleicher. In order to make possible a mathematical solution it is necessary to make very simple assumptions. These assumptions often do not agree with the real behavior of the soil. However, the nature of soil is so complex that I doubt whether all characteristics can be included in any one set of assumptions which will permit a mathematically correct derivation. Therefore I have little hope for further progress in this direction.

There are in existence too few test results upon this matter. What results are available show exactly what theory also teaches us, that is that the distribution of pressure depends to a large extent upon the rigidity or flexibility of the load slab itself. We find for very flexible slabs a distribution as shown in Curve A of Fig. 19. For a stiffer slab the distribution becomes like that in Curve B of Fig. 19.

When the load is acting around the edge of a slab, we find a distribution underneath a flexible slab as shown in Fig. 20.

8. Stress in the foundation slab. At this point it would be very logical to take up the question of stress in the slab itself caused by the reaction of the soil. I regret, however, that the time alloted to me now makes it impossible to present to you the results of my investigation in this matter.

9. Horizontal pressure. The horizontal pressures produce a lateral movement or a squeezing out of the soil from underneath a foundation when the soil has little internal resistance to shear. The general equations of Boussinesq and his successors give not only the vertical pressure but also the horizontal. This question is given particular study by Krynine in his Paper No. E-4 of Vol I of the Proceedings. Likewise Fröhlich in his book Druckverteilung gives extensive consideration to the same point.

Very few tests have been made to measure the magnitude of horizontal pressures; I can mention only Gerber, Zurich 1929.

No. E-13

DISCUSSION

ON THE DISTRIBUTION OF STRESS AROUND A PILE

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There is a fundamental property of the Boussinesq solution for a load at the surface of a semi-infinite solid which prevents its direct application to the problem of determining the distribution of stress around a pile. The Boussinesq solution deals with the stresses in a semi-infinite solid which is loaded with surface forces only. On the other hand, the action of a pile is associated with the problem of a force applied at a point in the interior of a semi-infinite solid.

The Boussinesq solution may not properly be used in the manner described by Mr. Tschebotareff (Paper No. E-1, Vol I) because, when the point of application of the force is considered to be below the surface, the condition that the tractions across the plane boundary vanish is no longer satisfied by the Boussinesq stress formulas. There are, in addition certain mathematical properties of the Boussinesq solution which render it inapplicable to any but surface loading conditions.

Similarly, the calculation of pile stresses from a line source of simple nuclei of strains of the type considered by Professor Relton (Paper No. E-2, Vol I) results in equations for stresses which do not satisfy the boundary conditions at the free surface of the semi-infinite solid. These nuclei of strain are derived from a solution by Kelvin for a force operative at a point in a solid of indefinite extent (Love, Theory of Elasticity, 4th Edition, page 183). Since the Kelvin solution for a single force produces stresses on all planes z = constant, the nuclei of strain derived from it will also fail to satisfy the boundary conditions for a semi-infinite solid.

If the problem of stresses around piles is to be attacked by this method it is necessary to know the fundamental solution for a force applied at a point in the interior of a semi-infinite solid. This solution may be found in a paper in the May, 1936, issue of "Physics" (published by the American Institute of Physics). With this solution it is a routine matter to calculate nuclei of strain for the semi-infinite solid which correspond to the nuclei of strain for the solid of indefinite extent as described by Love (Theory of Elasticity, 4th Edition, page 186). The new nuclei all have the property of vanishing tractions across a plane boundary and any combination of them designed to simulate the action of a pile will also have this property.

It is theoretically possible to determine the manner in which the load is transmitted from the pile to the surrounding soil and, with this known, the distribution of stress throughout the mass could be obtained. The governing equations may be set up by equating the strains in the pile to the strains at adjacent points of the surrounding medium. This procedure leads to an integral equation for determining the distribution of shear along the pile but, unfortunately the equation involved in this case is difficult to handle.

We may, then, accept Professor Relton's suggestion of assuming a law of shear distribution. Then, with the aid of the solution for the semi-infinite solid mentioned above, we may calculate the stresses in the soil. Furthermore, the investigation need not stop here. Having the stresses in the soil adjacent to the pile, we may calculate the corresponding strains in the pile. Then, assuming uniform strain over a cross-section of the pile, we may calculate the shape of pile which would give us the shear distribution which was assumed at the start. If this turns out to be an unreasonable shape, we may try further assumptions of shear distribution until we arrive at one which will yield a suitably shaped pile.

No. E-14

DISCUSSION

A GRAPHICAL METHOD FOR DETERMINING THE DISTRIBUTION OF STRESS IN THE UNDERGROUND DUE TO FOUNDATION LOADS

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In the present state of our knowledge it is necessary to accept the stress relations defined by the Boussinesq equation, because the true law of the transmission of stress in the underground is not known. Mathematical refinements do not seem to be justified; therefore, attention is directed toward practical simplification of the process of determining the complete stress picture in the underground beneath a structure, so that the amount of tedious computation is lessened. A graphical method of integration has been developed using a set of pressure charts for depths of 10, 20, 40, and 80 feet.

The method is based on the validity of the fundamental assumption of the law of superposition of loads and upon, what might be termed, the law of reciprocal effects, (similar to Maxwell's law of reciprocal deflection and the dummy unit loading method). In Fig. 1a is given a typical pressure distribution curve on a horizontal plane at depth - z due to a unit load - p at the surface.

By the reciprocal law the pressure - p_z on a unit area - A in the underground is found on the curve directly beneath the load - p as indicated in Fig. 1b. Now by revolving the pressure curve about the point - A, at which the pressure is to be determined, the pressure due to a load area may be found, as shown in Fig. 2. To illustrate, assume the loaded area is part of a ring one foot wide, with a center on the surface above the point - A.

In revolving the pressure curve a volume is swept out directly under the loaded area, which has a constant height represented by the pressure - p_z and an area equal to the projected loaded area. The total pressure at A is proportional first to the average ordinate - p_z to the pressure curve and is directly proportional to the length of arc swept out for a ring width of one foot. This is the

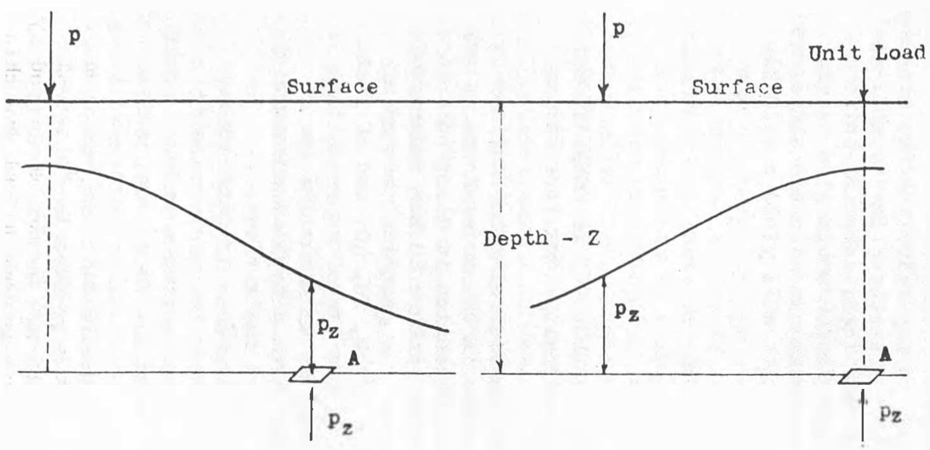


Figure 1 (a) (b)

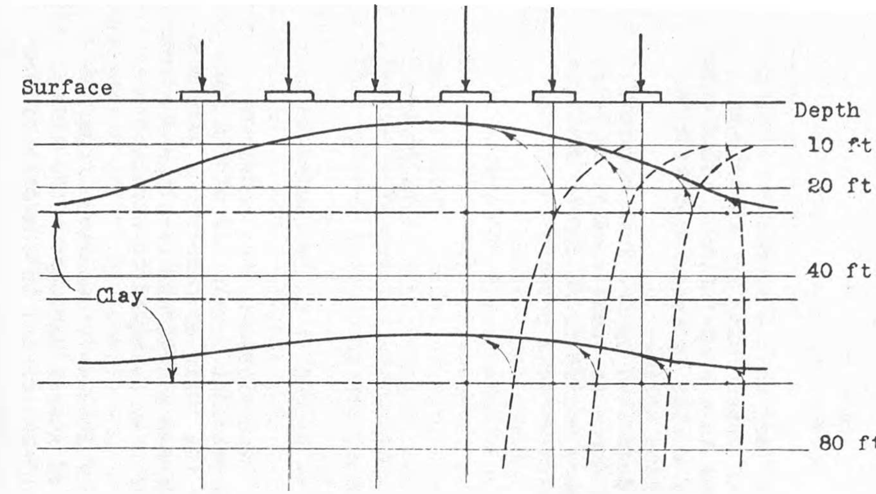


Figure 4 Pressure Distribution on a Vertical Section Under a Foundation.

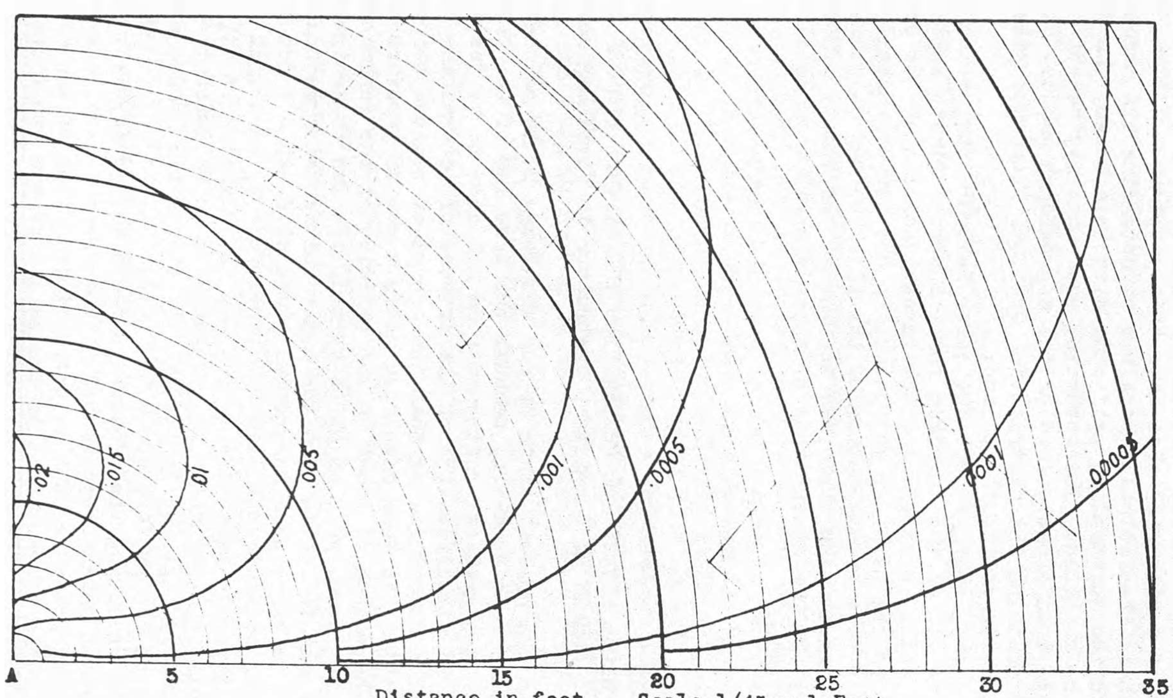


Figure 3 Pressure Chart for z = 10 feet Pressures Obtained at Point - A

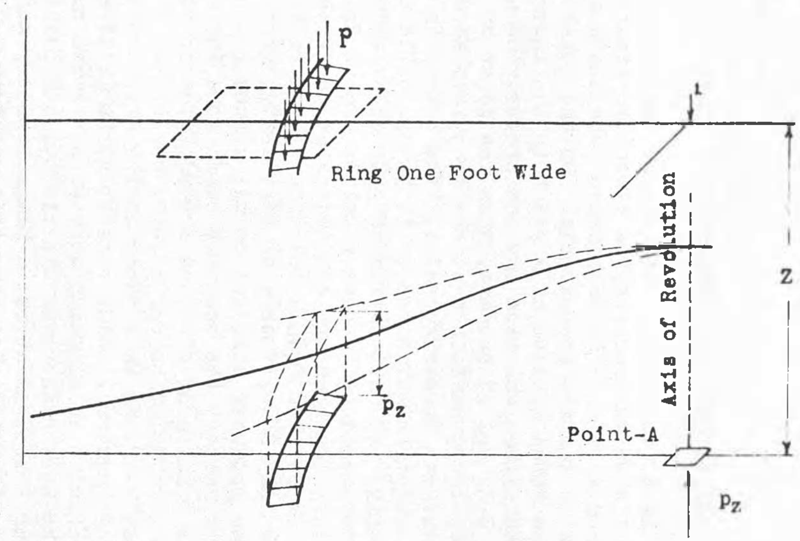


Figure 2

basis for the construction of the charts. The chart for z = 10 feet is given in Fig. 3.

The charts give a series of contours of equal pressure. It is important to note that the pressure on any ring is directly proportional to the length of arc. This greatly simplified the construction of the chart, since it is only necessary to compute the pressure for a single pressure contour, then all of the other contours may be readily interpolated by proportional dividers.

The footing superposed on the chart encloses a certain area, which represents the pressure at point - A exerted by the footing for a unit loading of 1 ton per square foot. The point at which the pressure is to be determined for all footings within the circle of influence is pinned down at the origin of the chart - point A.

To obtain the total pressure requires merely summing successively the interpolated pressures for the rings enclosed by the loaded area. This is repeated for all footings which have an appreciable influence on the pressure at A. The only computation involved is the multiplication by the actual unit loading and a summation for all footings within the circle of influence. The circle of influence is greater at greater depths.

The complete picture is obtained by determining the stress on the horizontal sections under consideration as indicated in Fig. 4.

E-15 DISCUSSION

D. P. Krynine, Research Associate in Soil Mechanics, Yale University, New Haven, Connecticut

About fifteen years ago the President of our Conference founded this science. At the present time it has become so developed that two rather separate branches may be distinguished in it: (a) study of stresses in earth masses; (b) study of resistance of the soil to those stresses. As far as the former part of soil mechanics is concerned, we have been informed this morning by Dr. Kögler about what we know in this province. I permit myself to complete Dr. Kögler's encyclopedic report with some hints on what we do not know, but what we ought to know about stress distribution in earth masses.

First of all, in studying the stress distribution in earth masses, it is assumed that in the case of a semi-infinite elastically isotropic mass, stresses propogate in straight lines from the source, or from the point of application of the concentrated force, F, at the boundary (Fig. 1) This statement has never been proven, but as it has a certain analogy to light propagation, it seems that it is close enough to reality in the neighborhood of the force F. Furthermore, if stresses propogate along straight lines in an isotropic mass, it should be concluded that they deviate from straight lines where that mass loses its isotropical properties. Thus stress trajectories close to an obstacle, for instance close to a retain-wall or to a boulder and also in all aeolotropic masses, should be curved. There is a certain reason to believe that close to all obstacles within an earth mass and close to foundation bases there are "constraint zones" or "zones of local perturbations" where the general stress distribution assumption does not hold, Dr. Terzaghi advances a theory of excessive sand arching close to a retaining wall; and Dr. Fröhlich studied what he calls "plastic" deformations under a loaded disk. These are examples of constraint zones. It is my belief that the solution of the problem of "constraint zones" should be started by using models; but for this purpose an adequate theory

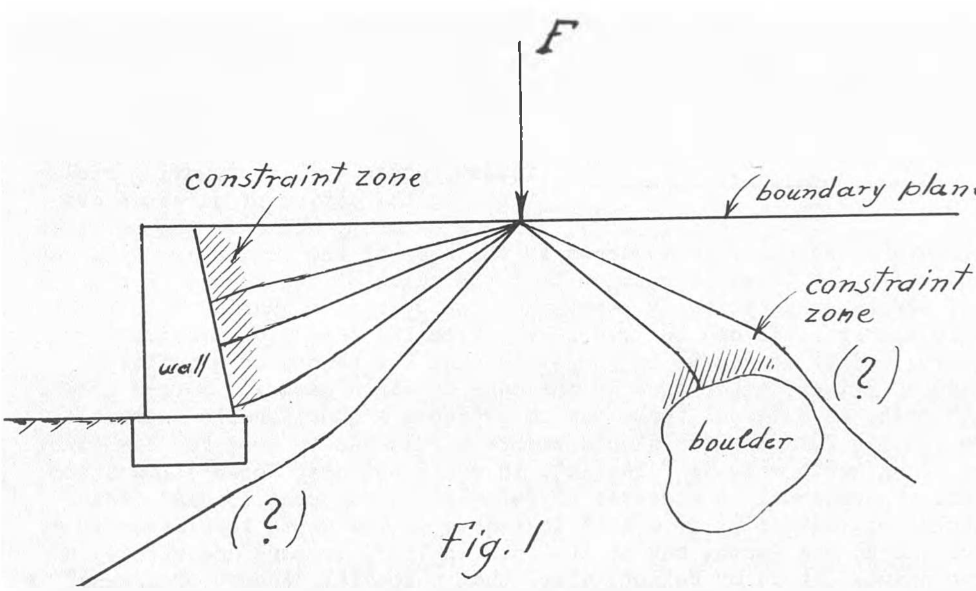


Fig. 1

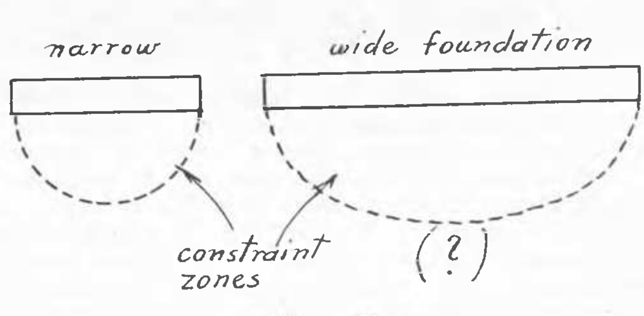
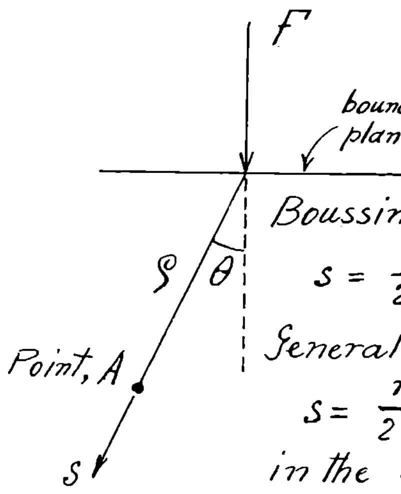


Fig. 2.



Boussinesq f-la, if $m=2$:

$$s = \frac{3F}{2\pi\rho^2} \cdot \cos\theta$$

Generalized formula:

$$s = \frac{n.F}{2\pi\rho^2} \cdot \cos^{n-2}\theta$$

in the case of sands: $n \cong 6$.

Plane Distribution:

$$s = \frac{2F}{\pi\rho} \cdot \cos\theta \text{; Michell's formula.}$$

In the case of sands:

$$s = \frac{2.95F}{\pi\rho} \cdot \cos^4\theta \cong \frac{3F}{\pi\rho} \cdot \cos^4\theta.$$

Notations:

s - stress; F - force; ρ - radius vector; θ - angular distance from the vertical; m - reciprocal of the Poisson ratio; n - concentration factor.

Fig. 3.

of similitude in soil mechanics should be developed, which practically does not exist. There is a serious doubt as to the depth of propagation of the "constraint zones" under foundations. It is again my belief that under a wide foundation the disturbed or constraint zone is not so deep relatively as under a narrow one. (Fig. 2)

To determine stresses within an earth mass most of the investigators use the Boussinesq formula which has been developed by its author for elastically isotropic bodies (Fig. 3) It is beyond question that the earth globe as a whole behaves as an elastic body; for example, the method of propagation of elastic waves proves this statement. But the upper crust of the earth globe to which our engineering activities are confined obeys the Boussinesq law but accidentally and even though approximately as for instance in the case of clays. To reconcile the discrepancies between elastic formulas and experimental data, the use of the so-called "concentration factor" has been proposed. (Fröhlich; Griffith). The physical nature of this new conception is not very clear as yet; and perhaps this is simply a temporary tool which will be put aside when the theory has been advanced in this province. One thing is clear: the "concentration factor" should decrease with the depth, and nothing is known about this decrease.

A simple survey of papers submitted to the Conference shows that most investigators work in studying resistance to the action of stresses and only a minority works in the province of stress distribution. The laboratory technique so advanced in the case of the former group is far behind in the latter case. Furthermore, it is my impression that the value of the theory of elasticity and of analytical methods of stress computation is overemphasized by soil mechanics investigators. Such conceptions as isotropy or Poisson's ratio can be transferred from the theory of elasticity to soil mechanics only with great care; and it is not to be forgotten that the theory of elasticity neglects the body forces which are of primary importance in the case of earth masses. Before producing a deformation within an earth mass, an external force has to overcome a considerable inertia of the mass in state of rest; hence elastic formulas for displacements hardly can be used for computing settlements of structures. Even if an earth mass is "elastic", it would not obey Hooke's law since displacements in this case are not proportional to stresses as revealed by numerous loading tests. There is an opinion that the modulus of elasticity of a soil increases as the depth increases; in my belief this modulus is a function not of the depth, but of the load applied, or more accurately, of the stressed condition at a given point. It is my belief, also, that a special "theory of masses" is to be gradually created to replace the theory of elasticity in soil problems. The necessity of such a theory is quite evident when the problem of stress distribution in "limited" soil masses such as earth dams or highway embankments is dealt with.

Stresses within an earth mass are to be estimated rather than computed with a great degree of accuracy; hence graphical methods are to be used for this purpose rather than tiresome analytical formulas. As an analogy it should be remembered how the design of framed structures progressed about fifty years ago when old analytical methods of determining stresses gave way to graphic statics.

Soil particles may behave either as "individuals" or as members of a "crowd" or of an "aggregate"; (or as some Russian mathematicians say, of a "collective".) This collective action of soil particles may be studied using the methods furnished by statistical mechanics. Such methods have been applied successfully in chemistry, physics, and hydrodynamics in analogous cases; for instance, the complicated phenomenon of turbulence in water movement may be studied statistically. I would like to suggest that statistical methods of stress analysis be introduced into soil mechanics as soon as possible.

No. E-16 DISCUSSION

Benjamin K. Hough, Jr., Associate Engineer, U. S. Engineer Office, Eastport, Maine

Although the subject for discussion is the question of stress distribution in soils, I have noticed that most of the previous speakers have been concerned with the determination of principal stresses, generally analyzed by Boussinesq's method, and with the action of piles. This limitation of the discussion is natural since these two topics are foremost in the minds of engineers who are concerned with the design of buildings or other structures which would be damaged by differential settlement of even relatively small amounts such as that caused by direct compression of the sub-soil.

There are however, cases in which the settlement of large structures is due to actual failure of the sub-soil in shear. Foundation analysis in these cases becomes a matter of determining the distribution and magnitude of the shearing stresses which the proposed structure will create in the soil. Therefore, this discussion is presented to draw attention to this type of problem and to point out the fact that there are instances when the topics previously discussed are of little importance.

My remarks are based on the work which is being done in the Soil Laboratory at Eastport, Maine, as a part of the investigation and design being conducted by the U. S. Corps of Engineers for the Passamaquoddy Tidal-Power Project. In that laboratory, one phase of our work is the study of gelatin models by photoelastic methods in a manner similar to that previously discussed by Messrs. Knappen and Philippe.

The general aspects of our problem are that some of our large earth and rock fill dams are to be constructed on deposits of marine clay which reach in some places a thickness or depth of over one hundred feet. Although by some standards this depth is considerable, in our work we consider it relatively small since the base width of the dams is generally at least three times as large or larger.

In studying these conditions with the gelatin model we represent the clay which is to be stressed by the proposed structures with clear, transparent gelatin, and the embankments with lead shot. This composite model when set up in a polarimeter makes visible a pattern of color bands which indicates the distribution of the shearing stresses induced in the gelatin.

Studies of this sort indicate that an area of high shearing stress develops under each toe of the embankment on the rock surface, and another such area develops directly under the embankment at the centerline but this is generally found at the surface of the gelatin, never at the rock surface. The relative magnitude of the intensity of stress developed respectively under the toes and at the centerline changes for a given height of dam with the ratio of the base width of the dam to the depth of the gelatin. If this ratio is large, the stress at the toes is larger than that at the centerline but as the ratio decreases and the effect of the rock surface becomes less, the stress at the centerline becomes larger and concentration of stress under the toes gradually disappears. It is interesting to note that the shearing stress on the rock surface at the centerline of the dam is practically zero when the ratio of base width to depth of clay is large.

When it was discovered by this and other methods of analysis that the shearing stresses developed by the proposed structures were in excess of the strength of the clay, studies were conducted to determine the extent to which the side slopes would have to be flattened to prevent this condition of overstress. It was then discovered that slopes as flat as one on five and in some cases one on ten might be necessary for stability of the structures. Since the original design called for slopes of one on one and three-quarters, this represented a large amount of extra material. A study was then made to discover to what extent the original structures would settle into the clay by reason of the indicated failure of this material in shear so that the amount of rock fill needed on account of this settlement might be compared with the amount needed to flatten slopes to prevent settlement.

This new study was conducted by building a model in which the foundation material instead of being represented by gelatin was actually made of clay, a good deal of which was secured from samples taken from the dam sites. The use of clay rather than gelatin was adopted in order to avoid the complication of a marked change in the character of the foundation material during failure as is the case when gelatin is overstressed. Tests with this clay model indicate that the type of settlement to be expected for the structures proposed for the Eastport project is such that the originally horizontal surface of the clay will be deformed to a profile resembling an inverted heart shape, that is greatest under the toes and least under the centerline of the dams. Beyond the toes, typical mud waves may be expected. The extent of settlement indicated is considerable, in many cases being sufficient to cause complete displacement of the clay at the points of maximum settlement.

The testing and analytical work just described is not yet quite complete and therefore the extent of application of the findings noted is not determined. However this work is considered to be consistent and complete enough to serve as a basis for estimates on the particular structures studied and gives promise of being of general interest.

No. E-17

DISCUSSION (By Letter)
STRESS DISTRIBUTION AS A PROBLEM IN ATTRACTION

George W. Glick, Moran and Proctor, New York City

About fifty years ago Boussinesq showed how the direct, logarithmic, and inverse potentials could be written in such forms as would satisfy the differential equations of elastic equilibrium, and thereby be used to express the stress and distortion in an elastic body of infinite extent under the action of a force applied at a point on its horizontal surface.

With the growing interest in soil mechanics and the need for more exact knowledge of stress under foundations Boussinesq's formulae are finding a wider application. However, inasmuch as Boussinesq's solution is based upon the theory of elasticity, a question of doubt has been cast as to its validity in foundation problems, especially when large distortions are involved. It is with this thought in mind that the writer will endeavor to present the problem from a new point of view, which was brought to his attention by D. E. Moran who suggested that the only forces which could act upon a body at the surface of the earth were those of gravity.

It is the purpose of this article to show that stress distribution is the result of the distribution of the attracting matter which creates the force. It is our common observation of the action of gravity that it creates a force directed toward the center of the earth, which causes us to overlook the fact that attraction is a distributed force and that gravity is its vertical component.

It is the fundamental concept of the law of attraction that attraction is a property of every particle of matter, and hence every mass within the earth must contribute to the total force acting upon any mass at its surface.

In order to illustrate the character of stress induced by the force of attraction consider a small mass, m, located at the apex of a pyramid, a cross section of which is a square with sides equal to ax, where x is the distance from the apex to the section in question. The fundamental law of attraction is given by the formula,

$$f :: k\, m\, m' \,/\, r^2$$

where
- f :: the force of attraction
- k :: the gravitational constant
- $m\,m'$ :: the product of the masses
- r :: the distance between the masses

If we now consider a small element of the pyramid f may be replaced by dF, r replaced by x, and m' replaced by its equivalent

$$m' :: w\,(ax)^2\,dx$$

where w is the density of the matter. Rewriting our formula, we have

$$dF :: m\,w\,(ax)^2 k\,dx \,/\, x^2$$

$$dF :: k\,m\,w\,a^2\,dx$$

Integrating this expression from the section to the length of the pyramid L, we have the total force acting on the cross section.

$$F :: k\,m\,w\,a^2\,(L - x)$$

Now when we make x equal to zero, F becomes P, the total force acting at the apex of the pyramid, or

$$P :: k\,m\,w\,a^2\,L$$

Placing this value in our previously derived expression we have

$$F :: P\,(1 - x/L)$$

and the unit stress, s, takes the following form

$$s :: F/(ax)^2 :: P(1 - x/L) \,/\, (ax)^2$$

If L now is allowed to approach infinity x/L approaches zero for points in the vicinity of the force, P, and we have the case where the stress is merely the force, P, divided by the area, showing that stress caused by attractive forces are identical to those produced in any other manner. The fundamental difference in our investigation has been that we have considered only the forces created by the attraction of matter beyond the point where the stress was sought.

Now let us pass to the problem of stress due to the attraction between a mass, m, on the surface of an homogenous sphere and the sphere. Let the point where the mass, m, is located be the origin of a spherical coordinate system, the pole of which passes through the center of the sphere. We may now write the expression for any small mass, m', within the body of the sphere, it is

$$m' :: w\,r^2 \sin\theta \;\; d\varphi \;\; d\theta \;\; dr$$

and the attraction on the mass, m, may be written as

$$dF :: k\,m\,w\,r^2 \sin\theta \;\; d\varphi \;\; d\theta \;\; dr/\,r^2$$

In order to get the force of attraction at the point r we must integrate this expression between r and the surface of the sphere, $D \cos \theta$, which gives

$$F \;::\; k\,m\,w \sin\theta \; d\varphi \; d\theta \; (D \cos\theta - r)$$

We may now get the unit stress at the point r by dividing the above expression for F by the area, $r^2 \sin\theta \; d\theta \; d\varphi$, which gives the radial stress, Rr,

$$Rr \;::\; k\,m\,w\,(D \cos\theta - r)/r^2$$

Returning to the force F we note that this acts in a radial direction and that the summation vertical components of these forces, taken between the origin and the surface of the sphere, must be the attraction of the mass, m, to the sphere; this is

$$P \;::\; 2\,\pi\,k\,m\,w\,D / 3$$

or

$$k\,m\,w\,D \;::\; 3\,P / 2\,\pi$$

Substituting this in our preceding expression for Rr we have

$$Rr \;::\; 3\,P\,(\cos\theta - r/D) / r^2\, 2\pi$$

Now if we are only concerned with points in the vicinity of the load and the diameter of our sphere is large, r/D approaches zero and we may write

$$Rr \;::\; 3\,P \cos\theta \;/\; 2\,\pi\; r^2$$

This is the radial stress of attraction, and it is interesting to note that there are no other stresses acting at the point. In other words the compressive stresses $\Theta\theta$ and $\Phi\varphi$ are zero as are the shearing stresses $R\theta$, $R\varphi$, and $\Phi\theta$.
We may now resolve the above radial stress into the more familiar rectangular coordinate system

$$Xx \;::\; (3P/2\pi)\,(zx^2/r^5)$$
$$Yy \;::\; (3P/2\pi)\,(zy^2/r^5)$$
$$Zz \;::\; (3P/2\pi)\,(z^3/r^5)$$
$$Xy \;::\; (3P/2\pi)\,(zxy/r^5)$$
$$Yz \;::\; (3P/2\pi)\,(z^2y/r^5)$$
$$Zx \;::\; (3P/2\pi)\,(z^2x/r^5)$$

We immediately note that the stresses Zz, Yz and Zx are identical with those derived by Boussinesq, and further that the stresses Xx, Yy and Xy are those given by Boussinesq's formulae when Poisson's ratio is placed equal to one-half. Thus we see that under certain conditions the two derivations yield identical results.

In nature we do not encounter such a condition as a point concentrated load, hence we must consider our force P as an infinitessimal, or equal to $p\,dx\,dy$, where p is a function of x and y which expression defines the intensity of surface load over a definite area. We therefore write

$$P \;::\; p\,dy\,dx \;::\; f(xy)\,dy\,dx$$

which is to be substituted in all the stresses as specified in the rectangular coordinate system, and which may be integrated to give the total stresses.

Now if we make the usual assumptions; that the earth is spherical, and that the density is in general uniform, we may conclude that our solution may be directly applied to foundation problems. This requires, however, that we know how the load is distributed by the footing and also whether any shearing tractions are induced at the underside of the footing. These statements do not imply that stresses cannot be induced by cavities, inclusions, or discontinuities, for we must naturally conclude that our lines of radial forces must be diverted from their natural paths to circumvent these obstacles.

No. E-18

COMMENTS ON VARIOUS PAPERS

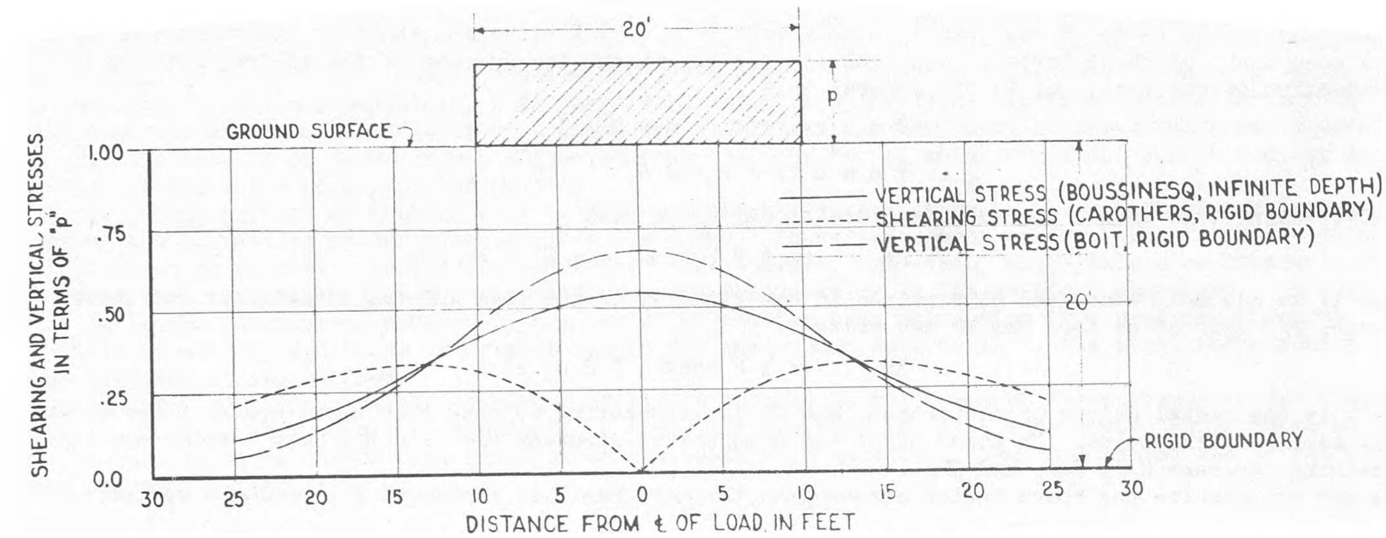
(Editorial notes abstracted from oral and written communications.)

Paper E-9: From several sources the question was raised how the angle of internal friction of 1° 55' for Hudson River Silt was determined. It is suggested that no time was allowed for consolidation during the shearing tests.

No. E-19 DISCUSSION (By Letter)

Thomas A. Middlebrooks, Associate Engineer, U. S. Engineer Office, Fort Peck, Montana

It is noted that the foundation material under all the buildings is composed of clay and sand with the latter at a shallow depth in most cases. If this sand acts as a rigid boundary, which is a logical assumption, both the vertical and shearing stresses would be increased (Fig. 1); therefore stresses computed for infinite depth should not be used.



COMPARISON OF STRESSES UNDERNEATH
A LONG FOOTING

Fig. 1

The writer is of the opinion that more consideration should be given in building design to the shearing stresses which exist under the footings. There are many instances where the foundation stresses beneath a footing, although not great enough to cause complete failure, are sufficient to cause considerable plastic deformation in the soil, with consequential settlement of the footing to an appreciable extent. Fig. 1 shows a typical example of a long footing resting on clay which is underlain by sand at a depth of 20 feet. As shown, the shearing stress is a maximum at the outer edges of the footing where the resistance to plastic flow is the least, and for a 1.0 T/□' loading its value would be approximately 0.35 T/□'. In most clays appreciable plastic deformation will occur under this shearing stress.

No. E-20 DISCUSSION OF PAPER NO. E-4 (By Letter)

Dimitri P. Krynine, Research Associate in Soil Mechanics, Yale University, New Haven, Connecticut

In Fig. 3, Paper No. E-4, Vol I, an approximate method of determining the direction of the major principal stress is shown. A more accurate method would be as follows (Fig. 3a). The auxiliary arc, MN, is supposed to be loaded with the same unit loads as the foundation, M_oN_o; and to be subdivided into equal parts as shown by dotted lines. Then the ordinates a; b; c; d at the middle of each section would be practically proportional to the respective forces acting at the arc, MN. A force polygon abcd is constructed, starting from Point O; and the direction of the resultant of the forces a; b; c; d; is that of the major principal stress. In Fig. 3a the final point of the force polygon abcd accidentally lies at the perimeter of the loading area of the foundation.

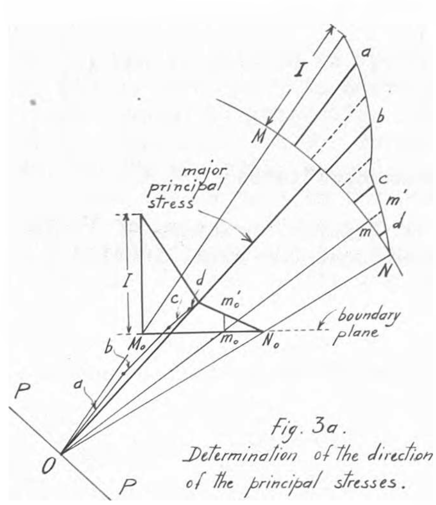


Fig. 3a.
Determination of the direction of the principal stresses.