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No. J-7

## OPENING DISCUSSION

Dr. Arthur Casagrande, Graduate School of Engineering, Harvard University

Until a few years ago the classical earth pressure theories were the only available tool for earth pressure computations. These theories are based on the assumption that earth is a perfectly plastic material. In other words it is assumed that an abrupt transition takes place from a condition of equilibrium to a condition of movement or failure. On the basis of this assumption one could determine the stresses only for the condition when failure takes place in every point of the mass.

Experience has rapidly shown flagrant contradictions between the classical earth pressure theories and observations, particularly in cuts. The reason could only reside in a fundamental difference between assumptions and reality. The nature of this difference is disclosed in several contributions to the Proceedings. The content of these papers eliminates these contradictions, shows the statically indeterminate character of the problem, and points out that one of the most essential variables in the analysis depends on the method of construction. At present we have no means of evaluating this factor by pure theory. Therefore we have to resort to actual measurement of earth pressure, particularly in cuts, and to a study and classification of our experience.

A second important recent development is the recognition of the effects of rain and of freezing ground water on earth pressure against retaining walls which is analyzed in a contribution by Professor Terzaghi. In this paper it is demonstrated, in agreement with experience, that the variations in pressure are much greater than what can be accounted for by periodic changes in the weight of the soil. With our understanding of the influence of seepage on the magnitude of the earth pressure we also recognize the paramount importance of drainage. In one contribution a graphical method is suggested for determining the effect of rain storms on the earth pressure.

Of great importance is the conclusive evidence that hydrostatic uplift is fully active in every soil including silts and clays, which is advanced in another contribution by Professor Terzaghi. This eliminates one of the major uncertainties in earth pressure computations.

In another contribution the influence of elastic properties on the intensity of earth pressure is analyzed. The classical theories give fairly correct values for the total pressure when the shearing resistance is fully mobilized. When the backfill consists of a soil which does not deform much, such as sand, this condition is always satisfied. However, in clay the condition is very seldom satisfied because the capacity of supports for lateral yield is in general too limited to permit the development of the entire shearing resistance of the soil. Here, our increased knowledge leaves a gap which can only be filled by experience and measurements on structures.

One contribution deals with the increase of earth pressure due to a single load on the surface of the backfill. Similar investigations were made many years ago in Zuerich, by Gerber, in which the similarity to Boussinesq's stress distribution was also recognized.

Studies on the problem of earth pressure against tunnel linings reveal valuable data. Terzaghi shows in one of his contributions that, in contrast to what was assumed until now, the ratio between the principal stresses above tunnels is approximately equal to unity; that means that passive pressure is never fully mobilized. In clay considerable yield is required to mobilize its shearing resistance. It seems that at present we are not yet able to solve this problem theoretically, and we must resort to the accumulation of further knowledge by direct measurements.

The question of earth pressure against sheet pile bulkheads is one of the most complex problems of soil mechanics. A discussion of this question is contained in a contribution by Legget.

The study of the papers in this section leaves the distinct impression that further advancement in our knowledge on the action of earth pressure depends essentially on extensive and accurate observations. I wish, therefore, to take this opportunity to impress on the members of the Conference the necessity of such observations and to close this discussion with the request to make available any information which may exist in their files.

No. J-8

## DISCUSSION

Dr. Karl von Terzaghi, Professor at the Technische Hochschule, Vienna, Austria

Two of the most important sets of observations regarding earth pressure phenomena were made by Mr. Langer in Paris. Since Mr. Langer had no time to prepare a report for this Conference I wish to present a brief abstract of his findings.

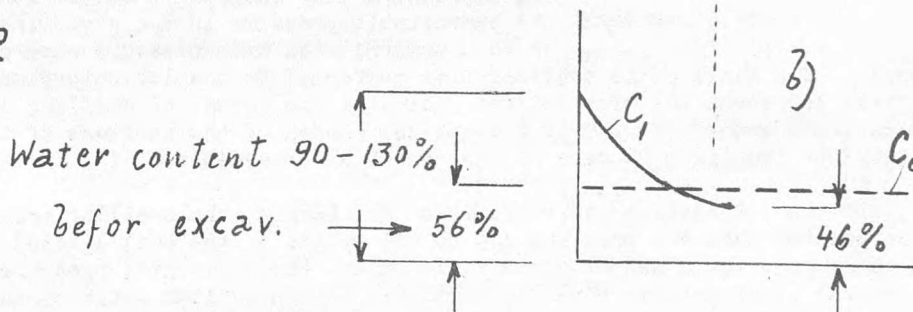
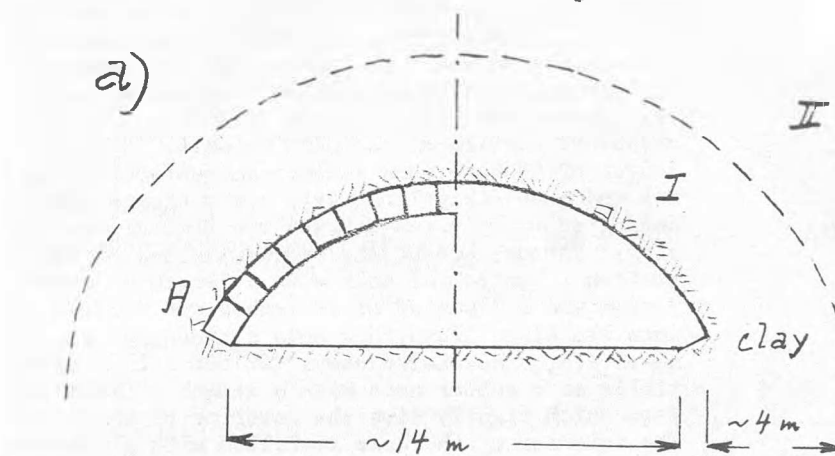
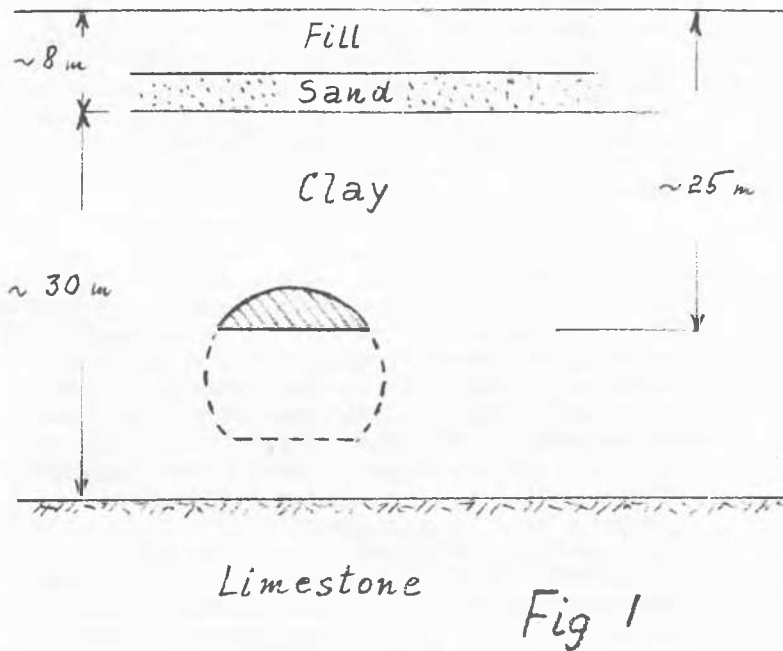
One of the two sets refers to a tunnel located at a shallow depth beneath the surface. Fig. 1 shows a section through this tunnel. The clay in which the excavation was made has a bluish-green color. It is very compact and homogenous, and 65% of its particles are smaller than 0.002 mm. In an undisturbed state its water content is 56%, its voids ratio 1.45 and its coefficient of permeability  $1.10^{-7}$  cm/min. During consolidation tests on undisturbed samples it was noticed that the voids ratio did not decrease until the pressure exceeded about 4 kg per sq cm. At lower pressures the admission of water to the sample produced a gradual and very important volume expansion.

Construction was started by excavating the clay within the shaded area shown in Fig. 1. Very soon after excavation was finished the clay began to expand energetically which made it necessary to exoa-

vate between 10% and 15% in excess of the quantity required by the cross-section of the tunnel. During this process the water content of the clay adjoining the inside of the excavation increased conspicuously. A determination of the water content of samples obtained at different distances from the inside of the excavation disclosed the fact that the water content decreased from values ranging between 90 and 130% at the exposed surface to about 46% at a distance of 13 feet from this surface, which is 10% less than what it was before. These results are shown graphically in the diagram, Fig. 2b. Before the excavation was started the water-content was equal to the ordinates of the straight line  $C_0$  and after excavation it was equal to those of the curve  $C_1$ . The decrease of the water-content beyond the boundaries of the zone of swelling indicates that the water required to produce the volume expansion was drained out of the clay which surrounds the zone of swelling. These observed facts are a striking confirmation of the explanation which I published some 10 years ago regarding the mechanics of the swelling of clays in tunnels. The essence of this explanation is illustrated by Fig. 3. If one excavates a tunnel through the clay, the clay tends to expand towards the excavated space, because the pressure which acted in the clay before the excavation was made has become equal to zero

over the entire face of the excavation. Since the voids of the clay are entirely filled with water, expansion can occur only if either air or water enters the voids of the clay located within the zone of expansion. The air cannot enter, because the surface tension of the water prevents such a process. Hence the only possibility for swelling consists in an increase of the water content which in turn requires a flow of water towards the zone of expansion. The hydraulic gradient required to produce this flow is automatically established as soon as the surface tension is mobilized. Before the excavation was made the hydrostatic excess pressure in the water-content of the clay was the same everywhere and practically equal to zero. After the excavation was made the tendency of the clay to expand mobilizes the surface tension over the entire face of the excavation which in turn produces a tensile stress in the water throughout the zone adjoining the excavation. In Fig. 3 this tensile stress is indicated by (-) signs. Hence the water starts to drain from the zone of zero excess hydrostatic pressure towards the zone of tension which, in turn, increases the water-content of the clay adjoining the excavation at the expense of the water-content of the remoter parts of the clay deposit. This conclusion is fully confirmed by the results of direct observation, shown in Fig. 2b.

Theoretically it would be possible for part of the water required to produce the swelling to be drawn out of the air by a process of condensation. In order to investigate the practical possibilities for a migration of water from



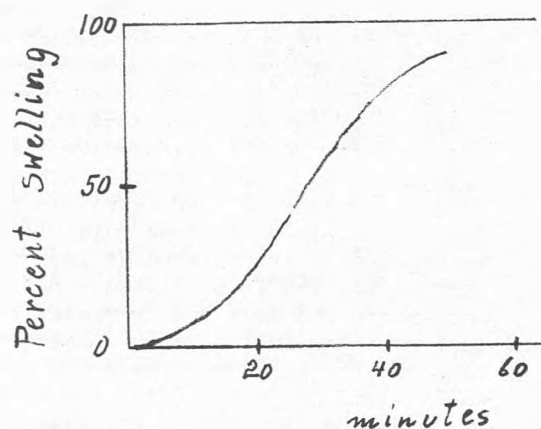
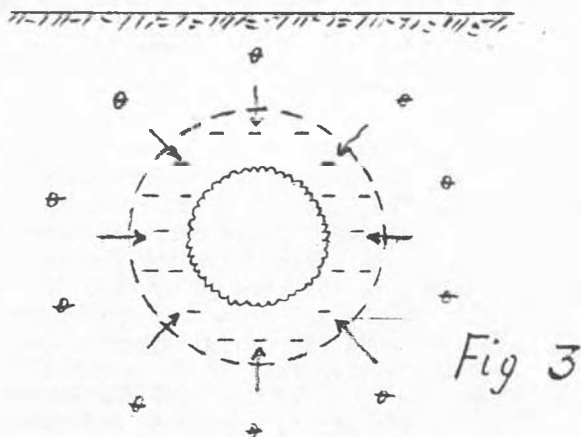


Fig 4

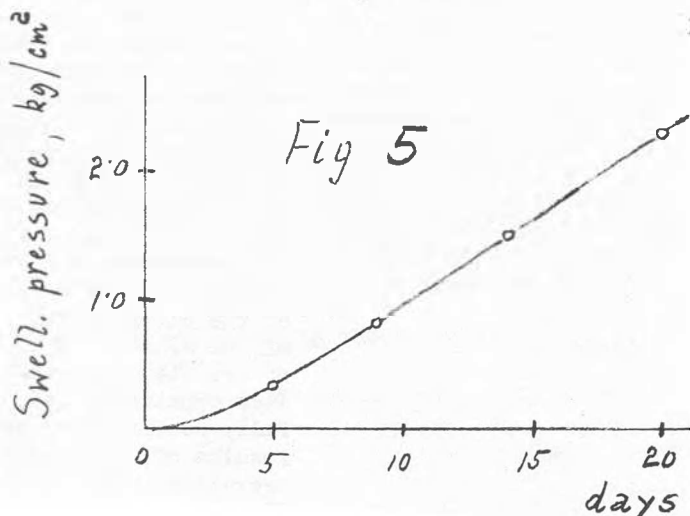


Fig 5

Fig. 4 shows the results of a small scale swelling test performed in the laboratory on an undisturbed sample. The abscissae represent the time and the ordinates the amount of swelling at zero-pressure in per cent of the ultimate value. Fig. 5 is a graphical record of the increase of the pressure on the cell in the gallery for the first 20 days of the period of observation. The test is still going on.

According to the results of the interesting observations of Mr. Langer, the swelling pressure of the clay can be several times greater than the pressure due to the weight of the soil located above the zone of swelling. This phenomenon would not be conceivable unless the horizontal pressure in the untouched bed of clay were several times greater than the vertical. At an earlier stage of their history the clays to which Mr. Langer's observations refer, have been compressed under loads up to 60

the air into the clay, Langer kept some undisturbed specimens of the clay in a closed container. In spite of his efforts to keep the water content of the air as close to the point of saturation as possible, he could not prevent a slow decrease of the water content of the specimen. This seems to disprove the hypothesis that an appreciable part of the surplus water in the expanded clay was derived from the air.

After the excavation corresponding to the shaded area of Fig. 1 was completed, the top arch of the tunnel was constructed. This top arch is shown in Fig. 2a on the left-hand side. During and after construction the swelling pressure increased and finally became so intense that it gradually forced the ends, A, of the arch in Fig. 2a through a distance of 8 inches into the supporting clay. The process of swelling weakened the clay to an increasing height above the crown of the arch. As a consequence the soil located above the ground yielded under its own weight and produced on the surface a trough-like depression with a depth ranging between 12 and 20 inches.

The second set of observations was made in a clay mine in Provins, East of Paris. The clay has a shearing strength between 1.6 and 1.8 kg/sq cm and the line of rupture in the shearing diagram for undisturbed samples obtained by a fairly slow application of the shearing force rises at an angle of 7° to 10° to the horizontal. In the mine the working galleries are located at a depth of 120 to 130 feet below the surface. They have a square cross-section of about 6 feet by 6 feet. The timbering consists of round timbers with a diameter of 10 inches placed side by side without any spacing. Immediately after excavation the clay stands without any support. As time goes on the pressure on the timbering gradually increases. After about three months the pressure assumes a value of the order of 25 tons/sq ft which suffices to crush the timbering. In order to get accurate information on the time rate of the increase of the pressure and on its ultimate values a gallery was constructed with a length of 300 feet in an untouched section of the clay deposit. Over a length of 53 feet this gallery was carefully dressed and completely lined with heavy square timbers, having no empty space between the timbers and the clay. Through one of the vertical sides of this section a horizontal hole with a length of about 7 feet and a diameter of 12 inches was drilled into the clay. Into this hole a pressure cell of novel design was introduced. It consists essentially of a rubber hose with a length of about 4 feet which tightly fits the interior of the hole. The interior of the hose is filled with glycerine. The increase of the swelling pressure increases the hydrostatic pressure in the glycerine which in turn communicates with pressure recording de-

tons/sq ft. At that time the horizontal pressure in the clay bed was of the order of 40 or 50 tons/sq ft. The observations of Mr. Langer suggest that the subsequent removal of the vertical pressure failed to reduce the horizontal pressure by more than a fraction of its previous maximum value, leaving the clay for ever in a state of excess horizontal stress. An experimental demonstration of the existence of such residual horizontal stresses in laterally confined sands was obtained a few years ago by A. F. Samsioe, the author of the Paper No. D-3, Vol I.

No. J-9

## DISCUSSION

## PRESSURE DISTRIBUTIONS ON RETAINING WALLS

Dr. Raymond D. Mindlin, *Inst. in Civ. Eng., Columbia University, N.Y.*

In Paper No. J-1, Vol I, Mr. Spangler observes that the experimental pressure distribution curves for lateral pressure against a retaining wall due to concentrated surcharges are similar in shape to those obtained from the Boussinesq solution. He also notes that the Boussinesq solution gives values which are far too low and correctly attributes this discrepancy to the presence of the unyielding retaining wall for which no provision is made in the Boussinesq analysis. It was suggested by S. D. Carothers (Engineering, London, (1924) pp. 1, 156) that the action of a smooth rigid retaining wall may be taken into account by applying the method of images to the Boussinesq solution. It is the purpose of this discussion to compare Mr. Spangler's experimental data with the results obtained by the method of images and to point out how the method may be extended to include retaining walls of finite height and also walls with sloping back faces.

We consider the semi-infinite solid bounded by the plane  $z = 0$  and apply to the free surface a concentrated load  $P$  at point  $x = -a$ ,  $y = 0$  and an equal load at  $x = +a$ ,  $y = 0$  (Fig. 1). Considerations of symmetry lead at once to the conclusion that, under this system of loading, there is no displacement in the  $x$  direction on plane  $x = 0$ . Also there is no shearing stress on this plane. The plane  $x = 0$  may therefore be replaced by the back face of a smooth, rigid retaining wall. The lateral pressure against the wall is then simply double the Boussinesq pressures. If we assume incompressibility (Poisson's ratio =  $1/2$ ), the pressure against the wall, along the  $z$  axis, is given by

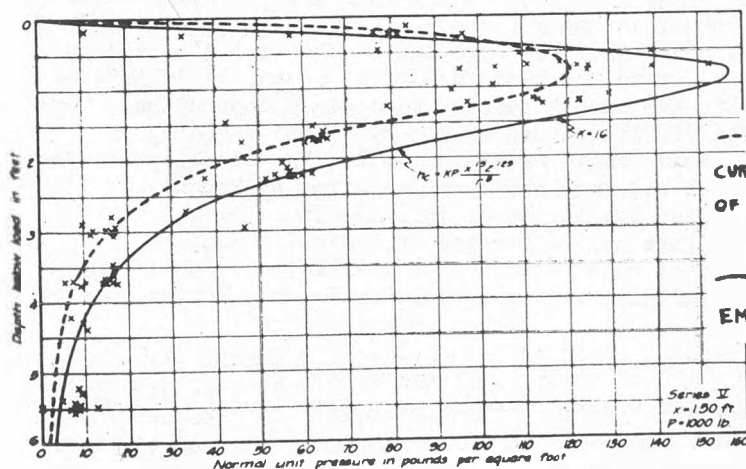
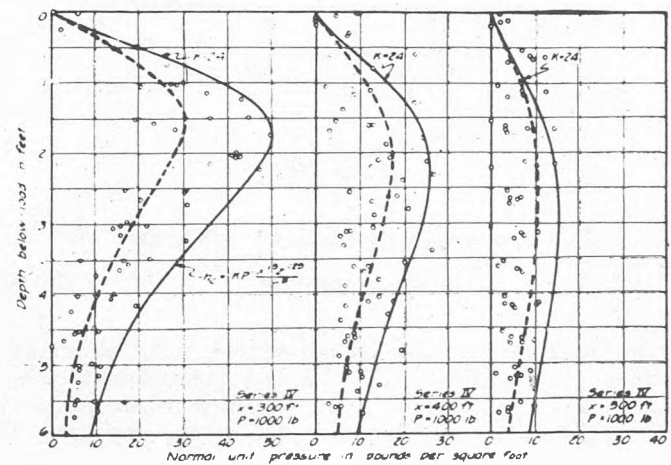
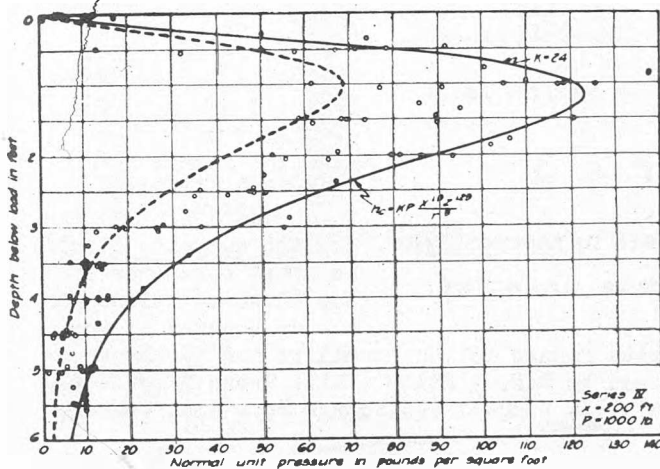
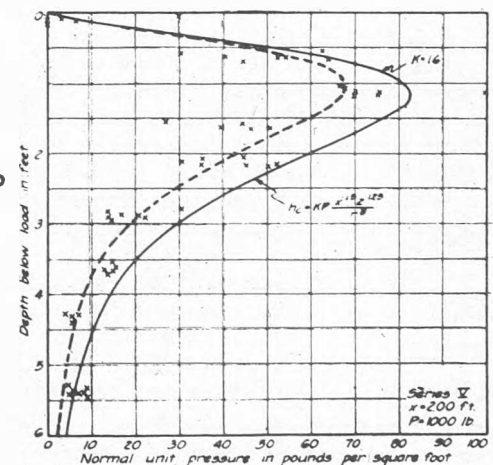


FIG. 2

--- THEORETICAL  
CURVE BASED ON METHOD  
OF IMAGES.

— SPANGLER'S  
EMPIRICAL CURVE.



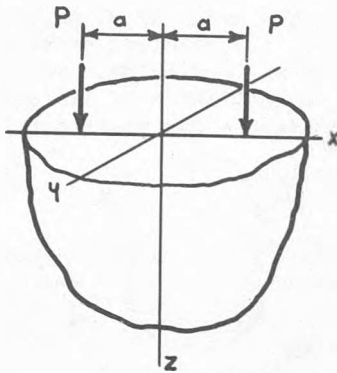


FIG. 1

METHOD OF IMAGES FOR A  
SMOOTH RIGID RETAINING WALL OF  
INFINITE DEPTH

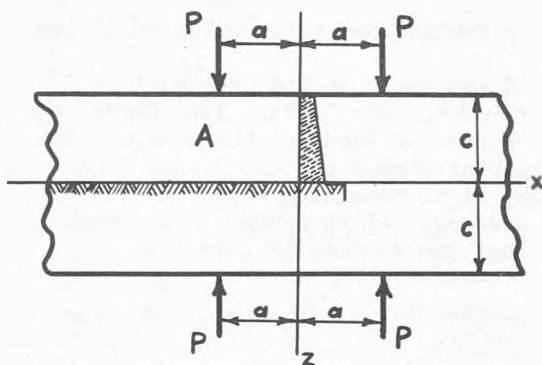


FIG. 4

SMOOTH RIGID RETAINING WALL OF  
FINITE DEPTH

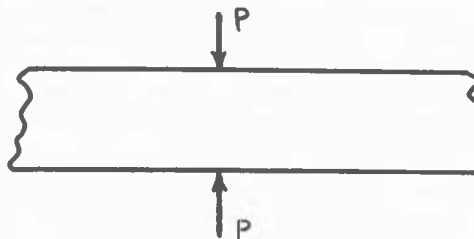


FIG. 3

BODY BOUNDED BY TWO PARALLEL  
PLANES UNDER TWO EQUAL AND  
OPPOSITE FORCES

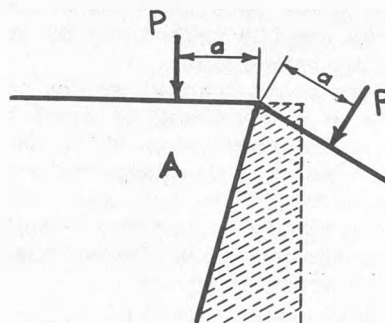


FIG. 5

SMOOTH RIGID RETAINING WALL  
WITH SLOPING BACK FACE.

$$p = \frac{3P}{\pi} \cdot \frac{a^2 z}{(a^2 + z^2)^{5/2}}$$

The curves determined by this formula were calculated for all the cases of concentrated loading investigated by Mr. Spangler and were found to correspond remarkably well with the experimental results. Some typical cases are shown in Fig. 2. It is interesting to note that the maximum pressure always occurs at a distance below the surface equal to one-half the distance from the load point to the wall. It may be observed that the assumption of perfect rigidity results in higher pressures than would obtain if any degree of yielding were assigned to the wall.

The same procedure may be followed for the case of line loadings or for loadings distributed over an area.

It is frequently desirable to consider as rigid not only the retaining wall, but also the foundation below the fill. In this case a similar expedient suggests itself. We avail ourselves of the known solutions for a body bounded by two

parallel planes and loaded with a pair of equal and opposite forces acting normal to and at the boundary planes (Fig. 3). The two dimensional case is discussed by L.N.G. Filon (Phil. Trans. Roy. Soc., London, Vol. 201, A, 1903) and the three dimensional case by J. Dougall (Edinburgh Roy. Soc. Trans., Vol. 41, 1904). (See also, M. Biot, Physics, Vol. 6, No. 12, 1935).

We take the middle plane of the horizontal slice as the plane  $z = 0$ . Then, if the thickness of the slice is  $2c$ , the boundary planes are given by  $z = \pm c$ . We place one pair of loads at points  $(x = -a, y = 0, z = \pm c)$  and an equal pair at  $(x = +a, y = 0, z = \pm c)$ , Fig. 4). Considerations of symmetry again indicate that there is no displacement in the  $x$  direction on the plane  $x = 0$  and no displacement in the  $z$  direction on the plane  $z = 0$ . Furthermore, there are no shearing stresses on these planes. We may therefore replace the plane  $x = 0$  with the back face of a smooth, rigid retaining wall and we may also replace the plane  $z = 0$  with the smooth surface of a rigid foundation. Considering the quadrant marked A in Fig. 4, we see that it is bounded on top by a free surface, on bottom by a smooth rigid bed and on the right by a smooth rigid retaining wall. The normal pressures on both the wall and the foundation may then be calculated from the formulas given by Dougall or Filon, depending upon whether we wish to discuss a point loading or an infinite line load.

The cases of finite line loadings and areal loadings may be treated in a similar manner.

The pressures against retaining walls with inclined back faces may be calculated by utilizing a solution by W. M. Shepherd for stress systems in an infinite sector (Proc. Royal Soc., London, Series X, Vol. 148, 1935). See Fig. 5.

Finally, it is possible to introduce another smooth, rigid wall parallel to the plane  $y = 0$ , or a pair of rigid planes parallel to either  $x = 0$  or  $y = 0$  or both. In fact we may, if we wish, calculate the state of stress in a rectangular parallelepiped which is loaded on a free surface and restrained on four sides by smooth rigid walls.

No. J-10

## DISCUSSION OF PAPER NO. J-3

Jeremiah E. B. Jennings, Massachusetts Institute of Technology, Cambridge, Mass.

The following discussion was prompted as the outcome of certain retaining-wall tests conducted by the writer at the Massachusetts Institute of Technology, under the direction of Professor Glennon Gilboy.

The apparatus, diagrammatically illustrated in Fig. 1 was primarily constructed to reduce the size to dimensions convenient for the normal laboratory, and yet to take account of all the anticipated sources of error.

The apparatus used employs a two-dimensional analogy for the cohesionless backfill which is achieved by substituting three sizes of steel rods 9 inches long for the sand usually employed in similar tests to date. Care was taken to use a rigid wall, and to limit movements of the wall due to measurements of pressure, to as small a value as at present practicable. Movements were controlled to one ten-thousandth of an inch.

To test the validity of the analogy the material was subjected to direct shear tests, a special box being constructed for this purpose. The results of these tests were very satisfactory, indicating a curve of stress vs. movement, similar to that of sand as shown in Fig. 2. A constant value of  $\phi$  was indicated, and a further advantage was found in the high unit weight of the material as a backfill, which led to much higher pressures than if it had had the same unit weight as sand. This it was felt led to greater percentage accuracy in measurement.

Two tests results will here be discussed: (a) Condition of maximum arching, i.e. with the wall rotating about the top point of fill. The curves of this test have been diagrammatically sketched in Fig. 3. (b) Condition of minimum arching as claimed by Dr. Terzaghi, i.e. wall rotating about its bottom. The curves of this test have been shown in Fig. 4.

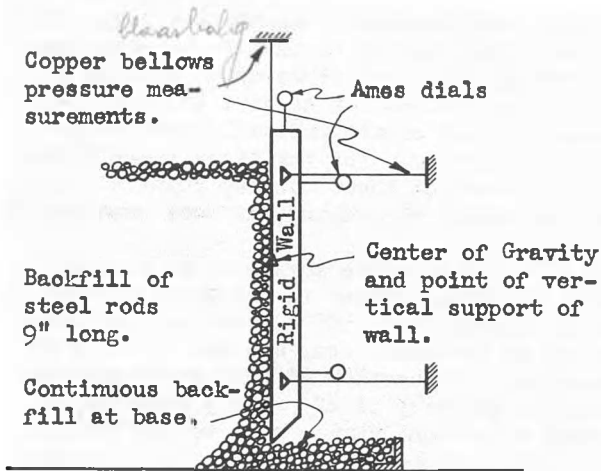


Fig. 1

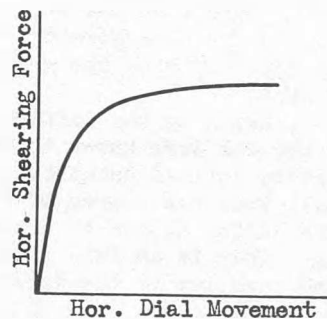


Fig. 2. Shear Test on Rods.

Examination of the centre of pressure ratio curve in Fig. 3 immediately indicates a close correlation to the prediction by Dr. Terzaghi in his paper, for this condition of maximum arching. The centre of pressure starts off at 0.33 and rises to approach the value 0.50, indicating a value of  $c_1$ , the confinement index of about 3.6, which Dr. Terzaghi claims to be the practicable limit for the normal material.

Examination of curves

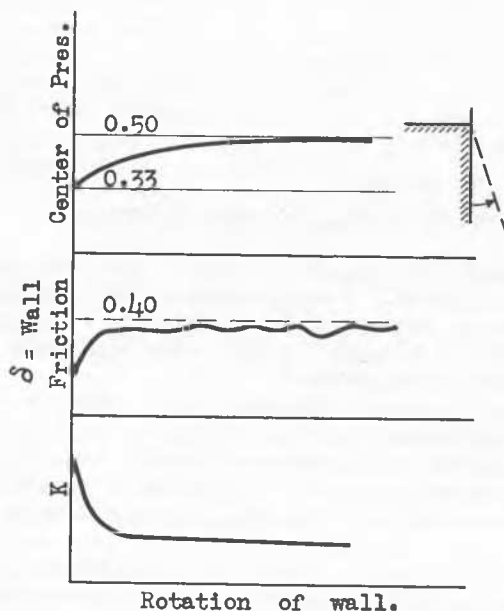


Fig. 3

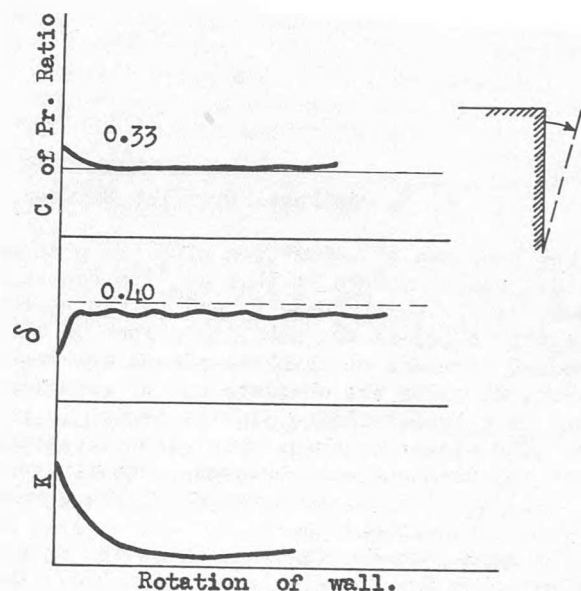
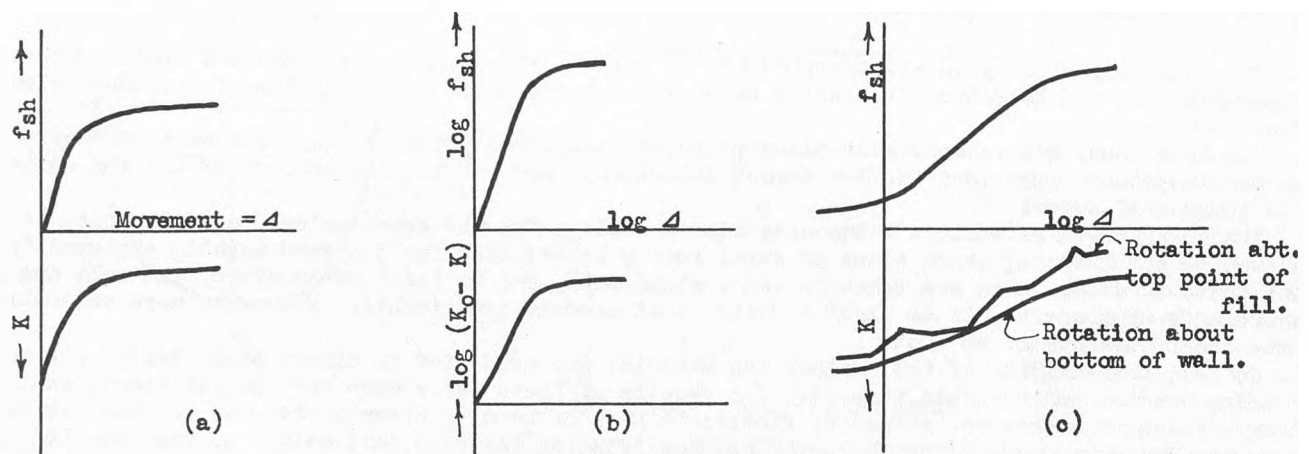


Fig. 4





Note: All upper plots are the results of horizontal shearing tests. All lower plots wall tests.

Fig. 5

of Hydrostatic Pressure Ratio,  $K$  vs. movement, in all tests run by the writer led to the thought that these bore a remarkable resemblance to the Stress vs. movement curve obtained in the shearing test. This thought was further borne out by the acknowledged fact that much greater movements are required to induce a minimum  $K$  when the material is in a loose condition, than when it is in a dense condition.

In the attempt to find some relation between these two curves, the plots represented in Fig. 5, were made, and especially in Fig. 5c. The thought is remarkably well borne out as here in every case where the arching effect is small, i.e. the centre of pressure does not depart radically from the 0.33 region, there are at least seven points on the straight line portion. For the cases where arching was more marked, and especially in the case already cited of rotation about the top point of fill, the straight line is not nearly so well defined; the greater the degree of arching the more erratically the points lie along the straight line.

No definite solution was found, owing to the difficulty of expressing the movements as strains, but it is felt that if some Shear Modulus were known to relate the Shear Stress to the Shear Strain, and that this latter were some rigidly defined quantity one of the greatest difficulties in the problem would be solved. There is still too, the degree of arching to be considered, and the writer suggests that the Hydrostatic Pressure Ratio,  $K$ , can be expressed as some function of this shear modulus, and the degree of arching existing. That is in Fig. 3 the latter quantity is of primary importance, while in Fig. 4 the first-mentioned quantity is the determining criterion, giving rise to the straight line plot in Fig. 5c.

The above thoughts were added to this discussion, not as any new theory, but more as a basis of thought for others tackling the same problem, which the writer feels must be closely tied in with the shearing phenomena.

No. J-11

#### DISCUSSION OF PAPER NO. J-1 (By Letter)

A. E. Cummings, District Manager, Raymond Concrete Pile Co., Chicago, Illinois

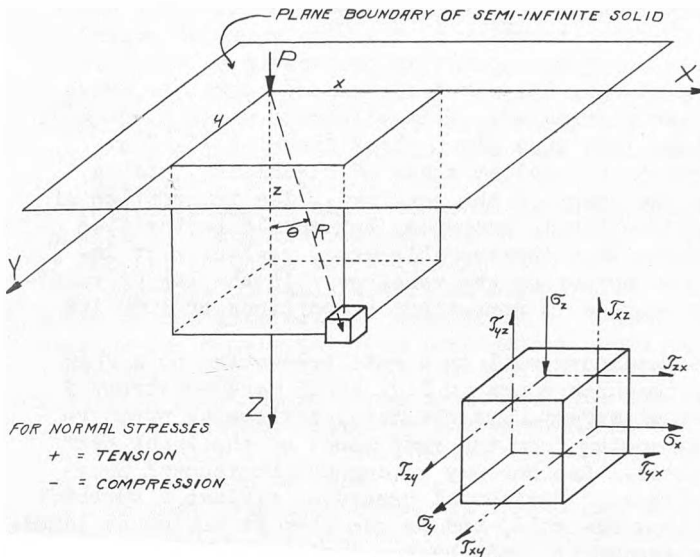
For purposes of comparison with his proposed empirical equation (1), Mr. Spangler uses his equation (2), and he refers to this as "The Boussinesq equation for normal lateral pressure on a vertical plane." With a point load,  $P$ , and in the rectangular co-ordinate system of Mr. Spangler's Fig. 6, this equation (2) is the shearing stress in the  $x$  direction on a horizontal plane. The equations for the normal stresses on vertical planes are much more complicated expressions.

Fig. 26 shows the complete set of equations for normal and shearing stresses over a small volume element in a semi-infinite elastic isotropic solid due to a point load applied to the surface of the solid. The stress equations are given in rectangular co-ordinates and equations (7), (8) and (9) represent the three normal stresses. The six shears, which are equal in pairs, are given by equations (10), (11) and (12). The notation in the equations is easily understood from the figure and  $m$  is the reciprocal of Poisson's ratio.

The normal stress in the  $x$  direction on a vertical plane is given by Equation (7) which is quite different from Mr. Spangler's equation (2). The shearing stress in the  $x$  direction on a horizontal plane is given by equation (12) which is the same as equation (2). If the assumption is made that the soil is incompressible ( $m = 2$ ), equation (7) reduces to



$$\sigma_x = - \frac{3P}{2} \frac{x^2 z}{R^5} \quad (13)$$



$$\sigma_x = \frac{P}{2\pi} \left[ \frac{R^2 z - 3x^2 z}{R^5} - \frac{2}{m} \frac{z}{R^3} - \frac{m-2}{m} \frac{(R^2 - x^2)(z+R) - Rx^2}{R^3(z+R)^2} \right] \quad (7)$$

$$\sigma_y = \frac{P}{2\pi} \left[ \frac{R^2 z - 3y^2 z}{R^5} - \frac{2}{m} \frac{z}{R^3} - \frac{m-2}{m} \frac{(R^2 - y^2)(z+R) - Ry^2}{R^3(z+R)^2} \right] \quad (8)$$

$$\sigma_z = - \frac{3P}{2\pi} \frac{z^3}{R^5} \quad (9)$$

$$\tau_{xy} = \tau_{yx} = \frac{P}{2\pi} \left[ \frac{3xyz}{R^5} - \frac{m-2}{m} \frac{xy(z+R)}{R^3(z+R)^2} \right] \quad (10)$$

$$\tau_{yz} = \tau_{zy} = \frac{3P}{2\pi} \frac{z^2 y}{R^5} \quad (11)$$

$$\tau_{zx} = \tau_{xz} = \frac{3P}{2\pi} \frac{z^2 x}{R^5} \quad (12)$$

FIG 26.

This is the normal stress in the  $x$  direction on a vertical plane for the condition of incompressibility.

It is easily seen that equations (2) and (13) differ only in the exponents of  $x$  and  $z$ . The minus sign in equation (13) simply denotes compression which is the sign convention used in Fig. 26. However, it is interesting to note that in Mr. Spangler's empirical equation (1), the exponent of  $x$  is greater than that of  $z$  which is in accordance with equation (13) rather than equation (2). In other words, it would have been more rigorous if Mr. Spangler had used equation (7) and reduced it to equation (13) for comparison with equation (1). The comparison of the normal stress with the shearing stress is not valid and the statement that equation (2) is a normal stress is incorrect.

Mr. Spangler has failed to state whether or not there was any movement of the retaining wall during the experiments. It is well known that, in a problem of this kind, wall movements have an important effect on the pressure distribution. It would be interesting to know the amount of any movement that may have occurred and also to know the nature of the movement; that is, whether it was a horizontal translation or a rotation about the base of the wall.

No. J-12

## DISCUSSION OF PAPER NO. J-1 (By Letter)

D. P. Krynine, Research Associate in Soil Mechanics, Yale University, New Haven, Connecticut

The object of this discussion is to support the opinion of Mr. Spangler that there is a similarity in the behavior of a backfill behind a retaining wall and of a loaded elastic body. A conclusion may be drawn from the experiments described that the horizontal pressure against a rigid (non yielding) retaining wall is very close to the horizontal component of the stresses in the backfill. The situation becomes quite different if the retaining wall yields; then the interrelationship between stress and strain should be considered as shown by Dr. Terzaghi in Paper No. J-5, Vol I.

The maximum lateral pressure in the experiments by Mr. Spangler takes place close to the ground surface, at a depth of  $1\frac{1}{2}$ —2 feet, which roughly is a half of the distance of the load to the wall. The curve of horizontal pressures at a point computed using the Boussinesq formula passes through a maximum point at the depths  $z = 0.5x$  to  $z = 0.65x$  if the value of the reciprocal of the Poisson ratio is  $m = 2$  and  $m = \infty$ , respectively. In these formulas  $x$  is the distance of the load from the wall. By dropping all the constant factors in equation (1) and equalizing its derivative to zero, we have

$$x^2 = 3z^2, \text{ or } z = \frac{1}{\sqrt{3}}x$$

Proceeding in the same way with formula (5), it may be concluded that the maximum pressure in the case of a linear live load distant  $X$  from the retaining wall, takes place at a depth,  $z$ , about  $2/3x$  (Fig. 17). These results check closely with the Boussinesq formula. Since both the latter formula and equation (1) do not contain the height of the wall, it follows that if in Mr. Spangler's experiments a low wall, for instance two feet high, were used, the maximum pressure would be at the same absolute depth from the surface of the backfill as in a high wall. It would be interesting to introduce experiments with such low walls; perhaps in this connection a problem could be solved as to the influence of the natural ground surface which serves as a base for the wall, on the stress distribution in the backfill.

The quantitative difference between formulas (1) and (2) of Mr. Spangler's paper could be partly

eliminated if instead of the "concentration factor",  $n = 3$ , in the Boussinesq formula, a higher value of that factor, namely,  $n = 6$ , corresponding to coarse sands, was used. This would raise all the results about 100 per cent. There remains still a difference of about 60 per cent which can be explained perhaps by the fact that curve (1) is rather an envelope of the experimental data and not their average as may be seen in the drawings. Another fact which calls attention is the wide range of experimental data, since readings corresponding to a depth are sometimes double in comparison with others at the same depth. The writer's belief is that the experiments described correspond to rather fresh fills in which non-elastic deformations have not been yet eliminated. This elimination can be done by successive loading and unloading of the mass with a given load at a given place (perhaps 30-40 or more loadings and unloadings). The mass is thus brought to a peculiar state of elasticity, stable under given conditions of loading, and this may narrow the range of the readings. The integration of formula (6) may be done if the law of superposition holds which is probable, but should be verified experimentally by bringing the mass to the elastic state with a considerable load, replacing it immediately by one or several successive smaller loads, and comparing the readings. If the law of superposition does not hold, the reducing of the measured pressures to correspond to portions of 1000 lbs is not justified either.

The writer acted as a consultant in the case of a retaining wall on a road conducting to a farm where only one heavy truck was moving. Curves similar to those shown in Fig. 13-25 were constructed using the Boussinesq formula with a conservative factor of safety, and the total horizontal pressure coming from the four wheels was computed. The pressure coming from the rear wheel of the truck next to the wall was found to be about 90 per cent of the total. Another way to compute horizontal pressures from a system of loads would be to construct isolines of horizontal pressures against a vertical plane caused by a unit load acting at a unit distance from the wall, and to use them as influence lines.

Mr. Spangler's experiments have destroyed a wrong assumption made by the designers as to uniform distribution of pressure along the height of a wall if that pressure is caused by a surcharge.

No. J-13

#### DISCUSSION (By Letter)

#### SOME REFERENCES TO LATERAL EARTH PRESSURE IN BRITISH TECHNICAL PUBLICATIONS

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Discussion at the Conference sessions, and informal conversations during the week of the meeting, indicated that British technical papers dealing with Lateral Earth Pressure were not well known. As the Proceedings already include some short bibliographies, it is thought that a list of the most important of these publications may be of interest and of use. The following titles and notes are therefore submitted:

1. "The Actual Lateral Pressure of Earthwork"; by Sir Benjamin Baker; Minutes of Proceedings of the Institution of Civil Engineers. (P. Inst. C.E., below) Volume 65, page 140, 1880. (A Classic Paper)
2. "On the Horizontal Thrust of a Mass of Sand"; by Sir G. H. Darwin; P. Inst. C.E. Volume 71, page 350, 1882.
3. "Some Experiments on Conjugate Pressures in Fine Sands and their Variation with the Presence of Water"; by G. Wilson; P. Inst. C.E. Vol. 149, page 208, 1902.
4. "The Lateral Pressure and Resistance of Clay and the Supporting Power of Clay Foundations"; by A.L. Bell; P. Inst. C.E. Volume 199, page 233, 1914.
5. "Experiments on Earth Pressure"; by P. M. Crosthwaite; P. Inst. C.E. Volume 203, page 124, 1916.
6. "Experiments on the Horizontal Pressure of Sand"; by P. M. Crosthwaite; P. Inst. C.E. Volume 209, page 252, 1921.
7. "The Overturning Moment on Retaining Walls"; by A. R. Fulton; P. Inst. C.E. Volume 209, page 284, 1921.
8. "Earth Pressures on Flexible Walls"; by R. N. Stroyer; P. Inst. C.E. Volume 226, page 116, 1927.
9. "The Pressure on Retaining Walls"; by C. F. Jenkin; P. Inst. C.E. Vol. 234, pages 103-223, 1932. (Unquestionably the most important British paper yet published on this subject, presenting the results of five years' research work on the lateral pressure of granular materials, conducted at the University of Oxford, and later at the Building Research Station, Watford. After repeated failures, theory and experiment were correlated. References are made to Resal's work, and also to complementary publication of the author before the Royal Society, and in "Engineering". Discussion of the paper includes further valuable data.)
10. "Earth Pressures on Flexible Walls"; by J. P. R. N. Stroyer; Journal of the Institution of Civil Engineers, Volume 1, page 94, November, 1935.