Constitutive modeling for frozen soil considering freezing temperature and cell pressure

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ABSTRACT: The artificial ground freezing can be a highly potential, reliable and environmental friendly alternative to grouting for water-proof and reinforcement under difficult geologic conditions. After freezing, the pore water is converted to the ice phase and acts as a boding agent to increase the strength of frozen soil. Therefore, an additional tensile strength should be considered when dealing with frozen soils. However, a suitable constitutive model to describe the stress-strain relationship considering the tensile strength because the freezing phenomenon is not fully addressed yet. In the present laboratory experiment, a series of frozen soil triaxial tests was conducted. The soil samples were artificially frozen in various freezing temperatures and tested under different confining pressures. Also, in this paper, the novel constitutive model that can take into account the temperature effect on the stress-strain relationship is proposed. The proposed model is validated by the frozen soil triaxial tests results.

KEYWORDS: frozen soil, constitutive model, artificial freezing technique, subsea tunnel

1 INTRODUCTION

The artificial ground freezing technique can be used for underground construction adjacent to existing structures or through highly-fractured fault zones. This technique is a highly potential, reliable and environmental friendly alternative to grouting for water-proof and reinforcement under difficult geologic conditions.

In order to design the artificial freezing technique, understanding of the thermal and mechanical behavior of frozen soil should be preceded. The experimental studies on the mechanical properties of frozen soil started in the beginning of the last century (Zhu, 1988). Comparing with unfrozen soil, the mechanical property of frozen soil is more complex because of its complex components and sensibility to temperature. The strength and deformation characteristics of frozen soils are the most important mechanical property for engineering construction. In order to meet the needs for the application of engineering activities, a series of experimental studies on the strength and deformation characteristics of frozen soil has been conducted (Chamberlain et al., 1972; Ladanyi and Johnston, 1973; Parameswaran and Jones, 1981; Fish, 1991; Li et al., 1993; Yu et al., 1993). Despite these many studies, further experimental studies are needed to standardize the strength and deformation characteristics of frozen soil.

To describe the behavior of the frozen material, several studies have been performed. Some authors (Fredlund and Morgenstern, 1976, Sloan and Abbo, 1999, Fredlund and Xing, 1994, and Gens et al., 2006) introduced an alternative two-stress variable constitutive relationship using a single Bishop-type stress variable (Bishop and Blight, 1963). Nishimura et al., 2009 extended the Barcelona Basic Model (Alonso et al., 1990) to develop a new constitutive model for frozen behavior. However, a suitable constitutive model to describe the stress-strain relationship of frozen soil is not fully addressed yet.

A simple model such as the Modified Cam-Clay (Roscoe and Burland, 1968) model involving only five input parameters has been reasonably applied in simulating the deformability and failure analysis of numerous geotechnical problems. Nerveless, the Modified Cam-Clay model has the intrinsic limitation neglecting the nature of tensile strength, which influences significantly to the deformation of soils. Therefore, the conventional Modified Cam-Clay model cannot describe the stress-strain relationship of frozen soil since after freezing, the frozen soil shows the tensile strength because pore water is converted to the ice phase and acts as a boding agent between soil particles.

In order to propose an adoptable constitutive model to describe the stress-strain relationship of frozen soil based on the Modified Cam-Clay model, the temperature effects associated with the tensile strength should be considered.

In this paper, a promising alternative to the conventional Modified Cam-Clay constitutive model, a tension Modified Cam-Clay constitutive model, which considers tensile strength of soils composed of the ice phases along with the temperature effect, is developed. In addition, a series of freezing triaxial tests were conducted to verify the proposed constitutive model. Results of numerical implementation show a good agreement with the experimental data and suggest to represent well the behavior of the frozen materials

2 TENSION MODIFIED CAM-CLAY CONSTITUTIVE MODEL

When dealing with frozen soils, an additional tensile strength should be considered because frozen pore water acts as a bonding agent combining the soil grains together. These bonds are deteriorated during applying a load to the frozen soil specimen during triaxial tests.

The behavior of frozen soil is schematically presented in Figure 1. In Figure 1, the tensile strength of ice is denoted as p'_{is}, and the consolidation stress is denoted as p'_0 . When the soil specimen begins to yield the initial circle, which is a half circle line ABC combining of p'_{is} on the left direction, and p'_0 on the other direction, shifts along with the increment of mean stress p'. The deterioration of ice phase accumulates during shearing and the tensile strength p'_{is} then moves towards 0, relatively. The slope of the critical state line (m) is supposed to change along with the breakage of ice bonds and interlocking particle, then, reaching the peak strength surface itself, before decreasing downward to the critical state, where the value of m equals to M_{cs}.

The proposed tension MCC model, which accounts for the ice strength, the nonlinear failure envelope, and the tensile strength of ice bonds, can be described by Equation 1. For plastic deformation, the associated flow rule is applied.

$$f = g = M_{cs}^2\left[p' - \left(\frac{p'_0 - p'_{is}}{2}\right)\right]^2 + q^2 - M_{cs}^2\left[\frac{p'_0 + p'_{is}}{2}\right]^2 \quad (1)$$

where $p_1' = (p_0' - p_{is}')/2$, $p_2' = (p_0' + p_{is}')/2$, and p_{is}' can be interpreted as the ice strength representing the growing of suction s.

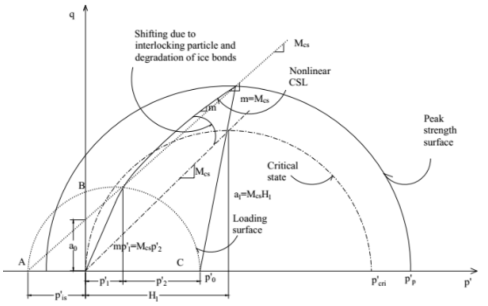

Figure 1. Yield surface for tension MCC model

The ice strength derived correspondingly with absolute temperature T was proposed by Nishimura et al. (2009) and in the early study introduced by Alonso et al. (1990)

$$p'_{is} = 0.46 M_{cs} k\left(273.15 - T\right) \quad (2)$$

The proposed model becomes the conventional MCC model when no ice strength p_{is}' is added. For the elasto-plastic framework, the total strain can be described as the combination of the elastic strain and plastic strain as follows;

$$\partial\varepsilon = \partial\varepsilon^e + \partial\varepsilon^p \quad (3)$$

where the elastic deviatoric strains and elastic mean strains are defined as;

$$\partial\varepsilon_q^e = \frac{\partial q}{3G} \quad (4)$$

$$\partial\varepsilon_p^e = \frac{\kappa}{p' v_0}\partial p' \quad (5)$$

The plastic strains are defined as follows (Schofield and Wroth, 1968);

$$\begin{bmatrix}\delta\varepsilon_p^p \\ \delta\varepsilon_p^q\end{bmatrix} = -\frac{1}{\dfrac{\partial f}{\partial p'_0}\left[\dfrac{\partial p'_0}{\partial\varepsilon_p^p}\dfrac{\partial g}{\partial p'} + \dfrac{\partial p'_0}{\partial\varepsilon_q^p}\dfrac{\partial g}{\partial q}\right]} \times \begin{bmatrix}\dfrac{\partial f}{\partial p'}\dfrac{\partial g}{\partial p'} & \dfrac{\partial f}{\partial q}\dfrac{\partial g}{\partial p'} \\ \dfrac{\partial f}{\partial p'}\dfrac{\partial g}{\partial q} & \dfrac{\partial f}{\partial q}\dfrac{\partial g}{\partial q}\end{bmatrix} \times \begin{bmatrix}\delta p' \\ \delta q\end{bmatrix} \quad (6)$$

where the derivatives are determined as follows;

$$\frac{\partial f}{\partial p'_0} = M_{cs}^2\left[p' - \left(\frac{p'_0 - p'_{is}}{2}\right)\right] - M_{cs}^2\left[\frac{p'_0 + p'_{is}}{2}\right] \quad (7)$$

$$\frac{\partial f}{\partial p'} = 2M_{cs}^2\left[p' - \left(\frac{p'_0 - p'_{is}}{2}\right)\right] \quad (8)$$

$$\frac{\partial f}{\partial q} = 2q \quad (9)$$

$$\frac{\partial p'_0}{\partial\varepsilon_p^p} = \frac{v p'_0}{\lambda - \kappa} \quad (10)$$

On the other hand, the plastic strain increments are given as

$$\delta\varepsilon_p^p = \delta\lambda\frac{\partial f}{\partial p} \quad (11)$$

$$\delta\varepsilon_q^p = \delta\lambda\frac{\partial f}{\partial q} \quad (12)$$

where $\delta\lambda$ is the plastic multiplier defined by the yield function under the consistent condition, and it can be derived as;

$$\delta\lambda = \frac{\left(D\dfrac{\partial f}{\partial\sigma}\right)^T}{H + \left(\dfrac{\partial f}{\partial\sigma}\right)^T D\left(\dfrac{\partial g}{\partial\sigma}\right)}\delta\varepsilon \quad (13)$$

where D is the elastic fourth-order stress tensor

$$D = \begin{bmatrix}K & 0 \\ 0 & 3G\end{bmatrix} \quad (14)$$

In order to describe the influence of ice bonds enhancing the strength of frozen soil, the hardening rule governed by the plastic volumetric strains corresponding to the MCC model is expressed by the following equation.

$$\delta p'_0 = \frac{\partial p_0}{\partial\varepsilon_p^p}\delta\varepsilon_p^p \quad (15)$$

The hardening modulus, denoted as H in the plastic consistency in Equation (16), standing for the hardening laws of material can be expressed by Equation (17).

$$\delta f = \left(\frac{\partial f}{\partial\sigma}\right)^T\delta\sigma - H\delta\lambda = 0 \quad (16)$$

The hardening modulus can be determined as

$$H = H_1 H_2 \quad (17)$$

where H_1 controls the shape of the yield locus by means of the ice strength and internal variables, while H_2 influences the plastic deformation and both are derived as follows;

$$H_1 = -\begin{bmatrix}\dfrac{\partial f}{\partial p'_0} & \dfrac{\partial f}{\partial p'_{is}}\end{bmatrix} \quad (18)$$

$$H_2 = \begin{bmatrix}\dfrac{\partial p'_0}{\partial\lambda} \\ \dfrac{\partial p'_{is}}{\partial\lambda}\end{bmatrix} \quad (19)$$

Accounting for the ice strength of frozen soil, The Prager consistency condition is rewritten as;

$$\delta f = \frac{\partial f}{\partial p'}\delta p' + \frac{\partial f}{\partial q}\delta q + \frac{\partial f}{\partial p'_0}\delta p'_0 + \frac{\partial f}{\partial p'_{is}}\delta p'_{is} = 0 \quad (20)$$

This differential of the yield function governs the hardening functions of plastic shear strain and plastic volumetric strain.

$$\delta p'_0 = \frac{v p'_0}{\lambda - \kappa}\partial \varepsilon_p^p \quad (21)$$

$$\delta p'_{is} = -k_d \frac{a_0}{M_{cs}} e^{-k_d \varepsilon_q^p} \partial \varepsilon_q^p \quad (22)$$

The hardening modulus *H1* and *H2* can be consequently expressed as follows;

$$H_1 = -\begin{bmatrix} -M_{cs}^2\left[p' - \left(\frac{p'_0 - p'_{is}}{2}\right)\right] - M_{cs}^2\left[\frac{p'_0 + p'_{is}}{2}\right] \\ M_{cs}^2\left[p' - \left(\frac{p'_0 - p'_{is}}{2}\right)\right] - M_{cs}^2\left[\frac{p'_0 + p'_{is}}{2}\right] \end{bmatrix}^T \quad (23)$$

$$H_2 = \begin{bmatrix} \frac{v p'_0}{\lambda - \kappa}\frac{\partial g}{\partial p} \\ -k_d \frac{a_0}{M_{cs}} e^{-k_d \varepsilon_q^p} \frac{\partial g}{\partial q} \end{bmatrix} \quad (24)$$

A new elastoplastic constitutive equation is defined by considering the effect of hardening works as follows;

$$\delta\sigma = \left[D - \frac{\left(D\frac{\partial g}{\partial \sigma}\right)\left(D\frac{\partial f}{\partial \sigma}\right)^T}{H + \left(\frac{\partial f}{\partial \sigma}\right)^T D\left(\frac{\partial g}{\partial \sigma}\right)} \right]\delta\varepsilon \quad (25)$$

3 FROZEN SOIL TRIAIXIAL TEST

3 .1 EXPERIMENTAL VALIDATION

The behavior of frozen soils developed by the proposed model depends on the confinement, in a similar way to the conventional MCC model for unfrozen soils. An additional feature mentioned throughout this paper is the sensitiveness of frozen soils to temperature. Because the contribution of ice strength is highly dependent on water content, keeping the same water content for all specimens prior to freezing is a prerequisite to obtain the proper experiment results. Triaxial test results under the drained condition on silica sand were conducted in order to model the characteristics of frozen soils with different temperatures at -1℃, -5℃, -10℃, and -20℃ and at different confining pressures of 300kPa and 600kPa.

3 .2 EXPERIMENTAL SET-UP

Testing of artificial frozen soils was conducted under low temperature and high pressure, as shown in Figure 2. The experimental system for frozen soils has a capacity for operating at temperatures as low as -40℃ and confining pressure up to 2MPa. Typically, the system includes a high-pressure low-temperature triaxial cell with an internal cooling system connected with a refrigerator through dual pipes, and sensors to measure temperature, load, displacement and pore-pressure. In addition, all of the assembles are connected and automatically controlled by a computer. In order to maintain the temperature of the soil sample regardless of external conditions, the external cooling system is used.

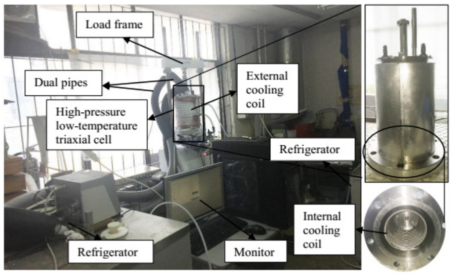

Figure 2. Freezing triaxial test equipment for frozen soil.

Cylindrical soil specimens were prepared with 38 mm in diameter and 76 mm in length. The specimen was composed of uniform fine silica sand with the dry density of approximately 1.41 g/cm^3. The effective particle size of the sample was around 0.16 mm. In order to prepare the specimen, the silica sand was carefully poured into a membrane and saturated with de-aired water. Afterward, the soil specimen was frozen at the given temperature and confined under the target confining pressure. The freezing time was around 12 hours until the sample reached the required temperature and confining pressure.

In order to determine mechanical properties of frozen soils with respect to the influence of temperature and confining pressure, a series of triaxial compression tests was carried out. The test condition is summarized in Table 1.

Table 1. Experimental conditions for frozen soil

Confining pressure (kPa)	Freezing Temp. (℃)
300	-1, -5. -10, -20
600	-1, -5. -10, -20

3 .3 EXPERIMENTAL RESULTS

The experimental results in accordance with temperature at different confining pressures are described in Figures 3 and 4. The frozen soil strength is proportional not only to the decrement of temperature, but also to the increment of confining pressure. The peak strength increases dramatically while freezing temperature was ranging from -10^{o}C to -20^{o}C.

However, the shear strength increases slightly with freezing temperature from -5^{o}C to -10^{0}C. These results show that even decreasing the freezing temperature from -5^{0}C to -10^{0}C, the temperature does not significantly affect the phase change from water to ice, which means the amount of ice in the specimen frozen at -5^{0}C and -10^{0}C are almost identical to each other. In addition, Figures 3 and 4 illustrate that the confining pressure between 300kPa and 600kPa does not have a significant effect on the strength of frozen soils.

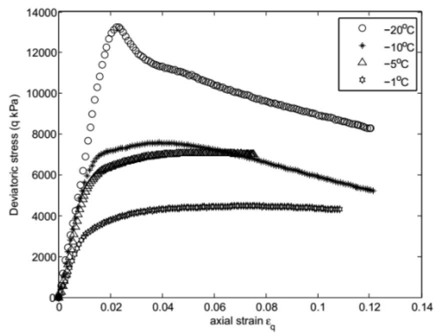

Figure 3. Experimental results under confining stress of 300 kPa at freezing temperatures of -1^{0}C, -5^{0}C, -10^{0}C, and -20^{0}C.

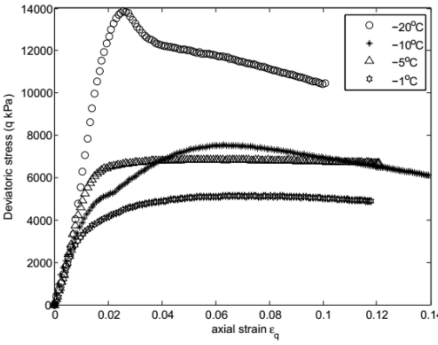

Figure 4. Experimental results under confining stress of 600 kPa at freezing temperatures of -1^{0}C, -5^{0}C, -10^{0}C, and -20^{0}C.

3 .5 MODEL VALIDATION

The simulated results of the stress-strain relationships of the proposed model for frozen soils are compared with the experimental results carried out by the triaxial test at freezing temperatures of -1^{0}C, -5^{0}C, -10^{0}C, and -20^{0}C as shown in Figures 5 and 6 at confining pressures of 300kPa and 600kPa, respectively.

Those observations demonstrate that the proposed model is capable of accurately simulating the stress-stain behavior of frozen soils.

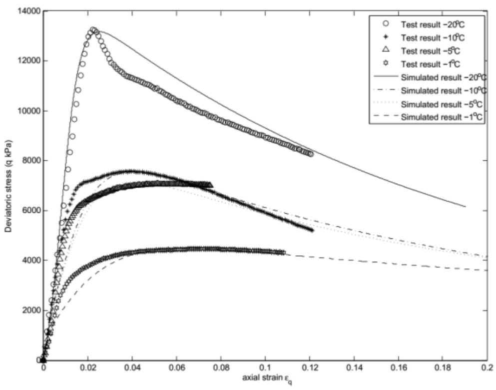

Figure 5. Comparisons between simulated and measured results of under isotropic stress of 300kPa, and subsequent triaxial compression at freezing temperatures of -1^{0}C, -5^{0}C, -10^{0}C, and -20^{0}C.

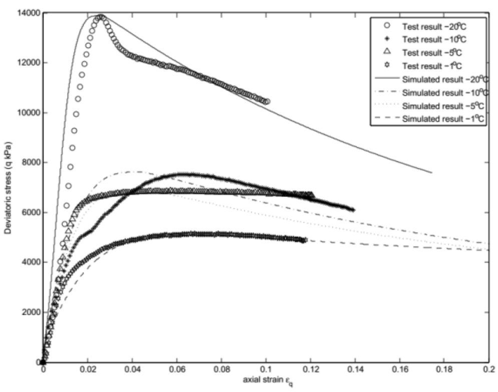

Figure 6. Comparisons between simulated and measured results of under isotropic stress of 600kPa, and subsequent triaxial compression at freezing temperatures of -1^{0}C, -5^{0}C, -10^{0}C, and -20^{0}C.

4 CONCLUSION

In this paper, a tension modified cam-clay constitutive model for frozen soils is proposed, and the model was verified by the frozen soil triaxial tests. The findings in this paper can be summarized as follows:

1. In order to apply the MCC model for frozen soils, the additional tensile strength should be considered because the ice becomes a boding agent combining adjacent soil particles together to increase their strength. These bonds are deteriorated during applying a load to the frozen soil.
2. The mechanical properties of frozen soils are influenced by temperature and confining pressure. The strength of frozen soils has a tendency to increase with a decrease in temperature, and an increase in confining pressure. However, the confining pressure does not significant effect on the strength of frozen soils because the confining pressure conditions are not high enough compared to the frozen soil strength.
3. The proposed model is verified by the experimental results carried out by the triaxial test. The proposed model is capable of simulating the stress-stain behavior of frozen soils with high accuracy.

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