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Influence of irregular loading and frequency on the cumulative damage and dissipated energy of dry loose sand under cyclic simple shear

Influence du chargement irrégulier et de la fréquence sur les dommages cumulés et l'énergie dissipée d'un sable meuble sec sous cisaillement simple cyclique

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ABSTRACT: This paper presents an experimental study involving cyclic simple shear (CSS) tests performed on dry loose Ottawa sand at an initial normal stress of 200 kPa and sheared under a constant volume condition. Horizontal shear loading involved 3 frequencies, and both uniform sinusoidal and irregular loading. The CSS tests involving uniform sinusoidal loading showed that for the 3 frequencies considered, the cycles to failure plots did not vary significantly with frequency. The study of the influence of irregular loading on the developed cumulative damage and dissipated energy was carried out by comparing these quantities with measured values in CSS tests on similar samples tested under uniform loading. The results show that the predictions of equivalent uniform cycles, done using the P-M cumulative damage hypothesis, are quite variable and that the accumulation of damage from irregular loading in most cases was observed to develop faster than predicted using equivalent uniform cycles. In contrast, the dissipated energy per unit volume until failure measured in all CSS tests (uniform and irregular loading, and 3 frequencies), was found to be relatively constant, and supports the use of energy-based approaches to study cyclic behavior of loose dry sands.

RÉSUMÉ: Cet article présente une étude expérimentale portant sur des essais de cisaillement simple cyclique (CSS) effectués sur du sable meuble sec d'Ottawa avec une contrainte normale initiale de 200 kPa et un cisaillement sous condition de volume constant. Trois fréquences ont été utilisées pour la charge de cisaillement horizontal, testée à la fois de manière uniforme sinusoïdale et irrégulière. Les tests de CSS impliquant une charge sinusoïdale uniforme ont montré que pour les trois fréquences considérées, l'évolution du nombre de cycles jusqu'à la rupture ne variait pas significativement avec la fréquence. L'influence de la charge irrégulière sur le développement des dommages cumulés et l'énergie dissipée a été réalisée en comparant ces quantités avec les valeurs mesurées dans les essais de CSS sur des échantillons similaires testés sous charge uniforme. Les résultats montrent que les prédictions de cycles uniformes équivalents—effectuées en utilisant l'hypothèse P-M de dommages cumulatifs—sont très variables. Par ailleurs, l'accumulation de dommages causés par une charge irrégulière a été observée dans la plupart des cas plus rapidement que prévu en utilisant des cycles uniformes équivalents. En revanche, tous les tests de CSS (charge uniforme et irrégulière et trois fréquences) montrent que l'énergie dissipée par unité de volume jusqu'à la rupture est relativement constante, ce qui conforte l'utilisation d'approches basées sur l'énergie pour étudier le comportement cyclique des sables meubles secs.

KEYWORDS: irregular loading, cyclic simple shear, cumulative damage, dissipated energy, liquefaction, ADVDCSS.

1 INTRODUCTION

In recent years, most of the laboratory studies on cyclic behavior of sands have involved uniform sinusoidal loading signals applied with geotechnical test devices such as the cyclic simple shear or the cyclic triaxial. However, it is well known that during earthquakes the sand layers experience complex stress histories that involve varying amplitudes and frequencies. Early generations of geotechnical testing devices were primarily geared to apply uniform sinusoidal load, or displacement, controlled cycles and this may have motivated early research, carried out in the 1970s, that looked for ways to convert random loading signals into equivalent uniform signals with a selected amplitude and frequency. This also led to the determination of an equivalent number of uniform cycles that could be used to represent the effects of certain earthquake loading in soils. Nowadays with recent advances in testing equipment it is

possible to apply complex loading to soil samples, including non-uniform shear stresses with variable amplitude and frequency. Taking advantage of these advances this paper presents results of an experimental program carried out to investigate the influence of frequency and irregular loading on the cumulative damage and dissipated energy during cyclic simple shear (CSS) of dry loose sands.

2 BACKGROUND AND LITERATURE REVIEW

The following section provides some background on CSS testing on dry sands carried out at constant volume. These tests are commonly reported as being equivalent to undrained CSS carried out on saturated sands at the same initial relative density and effective stress level. The literature review subsection describes the cumulative damage concept and the approach based on this quantity that is often used to compute number of equivalent uniform cycles for an irregular load. Additionally this

subsection provides a brief review of dissipated energy of dry sands tested under uniform CSS tests.

2.1. Background- Constant volume CSS testing:

Constant volume CSS testing on dry sands has been used by several researchers to study liquefaction and cyclic behavior of sands (e.g., Finn et al. 1979, Lasley et al. 2016). This testing approach involves maintaining the height of a dry CSS specimen constant by using an active control of the applied vertical normal stress during the cyclic simple shearing. At any instance during the CSS test the difference between this variable applied vertical normal stress, required to maintain constant the sample height, and the initial applied vertical normal stress has been approximated by many researchers as equivalent to the excess pore water pressure that would have been developed in an undrained CSS test carried out on saturated sand samples prepared to the same initial relative density and with the same initial effective stress state. A comparison of pseudo-drained and truly undrained CSS tests was performed by Pickering (1973) using Ottawa sand samples at an average initial void ratio of about 0.65 and at a shear loading frequency of 2 Hz. The results reported in this study suggest these two test approaches are similar but the pseudo-drained tests were on the conservative side in terms of number of cycles required to reach failure. The equivalency between the constant volume CSS test on a dry sand sample and a truly undrained CSS test on a saturated sand was further reported by W.D.L. Finn (Pickering's advisor) and co-workers at the University of British Columbia (Finn 1985, Finn and Vaid 1977, Finn et al. 1979). Figure 1, from Finn & Vaid (1977), shows the number of cycles to liquefaction from dry constant volume CSS are lower compared to the number of cycles recorded from undrained CSS on saturated samples of uniform medium dense sands. This figure, often cited in references that have used dry, constant volume CSS, suggests that constant volume CSS on dry samples is on the conservative side compared to equivalent undrained CSS on saturated samples. This experimental evidence although useful, seems to be scarce (and in Fig. 1 involves different sands), and warrants further evaluation of the validity of this equivalency of these two CSS tests types. However, this was outside the scope of this paper.

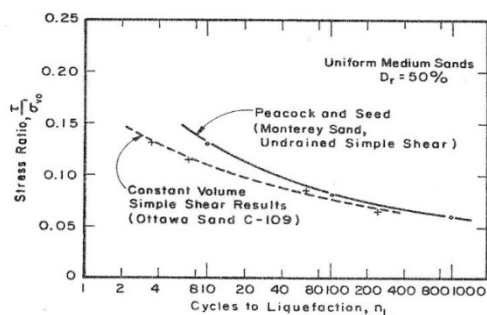


Figure 1: Comparison of Dry, Constant Volume CSS ($\sigma'_{no}=196$ kPa) and Undrained, Saturated, CSS test results for liquefaction resistance of sand (from Finn & Vaid, 1977) [fair use]

2.2. Literature review

2.2.1. Cumulative damage hypothesis and equivalent number of uniform cycles

Converting or finding equivalence between a non-uniform load, like an earthquake, and a uniform sinusoidal load applied for an equivalent number of cycles has been studied since the 1970s (e.g., Seed et al. 1975, Annaki and Lee 1977). Researchers have been looking at this problem in an effort to be able to perform an adequate comparison between field soil behavior under complex irregular earthquake loading and laboratory soil behavior, originally, and still typically, tested under uniform

cyclic stresses. The approach of converting a random signal to an equivalent uniform signal has several advantages, such as being able to analyze the effect of different earthquake on a soil by using a single set of data. Converting random stresses to an equivalent damaging stress has long been used to study metal fatigue. For example the Palmgren-Miner (P-M) (Miner 1945, Palmgren 1924) cumulative damage hypothesis, commonly used to study metal fatigue, has been applied to find this equivalency between irregular and uniform loading. This model assumes that the damage leading to failure accumulates linearly with the number of cycles of loading. Cumulative damage (D) was defined by Miner as the ratio of the absorbed work (w_i) after n_i cycles and the total absorbed work at failure (W). He also alternatively defined damage (D) as the ratio of the number of cycles (n_i) in the load history having an amplitude S_i and the total number of cycles (N_i) of this same amplitude S_i required to cause failure. Both these definitions of D, are as follows:

$$D = \sum_{i=1}^m \frac{w_i}{W} = \sum_{i=1}^m \frac{n_i}{N_i} \quad (1)$$

In the above expression, $D=0$ corresponds to the initial state of the sample before cyclic loads have been applied, $D=1$ corresponds to the damage state of the sample at failure, and D values between 0 and 1 correspond to the damage state of a sample that has been subjected to a certain loading but has not yet failed. An equivalent number of uniform cycles at any stress level (S_i) would be the value that causes the same amount of damage. A detailed description of the P-M hypothesis can be found in Green and Terri (2005).

One of the first applications of the P-M hypothesis to attempt to represent irregular earthquake time histories as equivalent stress series for liquefaction evaluations was done by Seed et al. (1975). This study used data from DeAlba et al. (1975), who performed large scale liquefaction tests using a shaking table involving saturated sand under a constant normal stress. The authors developed S-N curves, which plot number of cycles to cause initial liquefaction versus cyclic stress ratio ($CSR = \tau_{max}/\sigma'_{no}$) used in the tests, for Monterey #0 Sand at four different relative densities.

Annaki and Lee (1977) performed an experimental program to check the validity of Seed's implementation of the P-M hypothesis using triaxial tests under uniform and irregular deviatoric load cycles. The authors stated that their data generally confirm the validity of the equivalent uniform cycles, or the cumulative damage method of dealing with irregular loading effects on soil, and support the analysis presented by Seed et al. (1975).

Tatsuoka and Silver (1981) formulated a relationship between double amplitude (DA) shear strain, stress ratio for uniform loading, number of loading cycles and relative density, finding that the equations they developed were a better representation of their test results than the linear relationship proposed by Seed et al. (1975) when plotted in a log-log scale.

Recently Lasley et al. (2016) proposed the use of a modified Richart-Newmark non-linear cumulative damage hypothesis that they report as a substantial improvement with respect to P-M based approaches as the computed damage is load-dependent. This paper did not look at this new approach.

2.2.2. Dissipated energy approach

Another approach commonly used to study the cyclic behavior of sands under uniform and irregular loading is the energy dissipated by the soil during the loading event until failure. Several researchers have used energy dissipation to evaluate liquefaction potential (e.g., Davis and Berrill 1982, Figueroa et al. 1994, Green and Terri 2005). In CSS tests the dissipated energy per unit volume is computed by the cumulative area of the hysteresis loops in the shear stress-strain curves. The dissipated energy has been found to explain densification of drained tests

and liquefaction of sands under undrained loading. Figueroa et al. (1994) and Liang et al. (1995) used the energy concept to define liquefaction potential, and validated it using undrained hollow cylinder torsional shear tests on saturated sand specimens. According to these authors, the energy per unit volume needed to induce liquefaction is not dependent on the loading form and can thus be used to evaluate the liquefaction potential under general earthquake loads.

Green and Terri (2005) and Lasley et al. (2016) emphasized the fact that the P-M hypothesis applies to high-cycle fatigue conditions (low amplitude, large number of cycles), which is usually the case in metal fatigue applications, but is not directly applicable to low-cycle fatigue conditions (large amplitude and low number of cycles). These authors also point out that damage predictions using the P-M hypothesis are load-path independent. Thus propose an alternative implementation to the P-M hypothesis using energy principles that can be used to calculate an equivalent number of cycles at a uniform stress and to predict liquefaction.

3 EXPERIMENTAL PROGRAM

The test program reported in this paper involved samples prepared with Ottawa 20/30 silica sand. This sand is a poorly graded subrounded to rounded sand, with a $D_{50} = 0.71$ mm, and 95% of the material retained between the standard sieves #20 and #30. The maximum and minimum void ratios were measured to be 0.644 and 0.302, respectively. Tests were conducted using an Advanced Dynamic Cyclic Simple Shear machine (ADVDCSS) manufactured by GDS Instruments, that is capable of running CSS tests with non-uniform loading. The samples were prepared dry on a loose state with relative density values ranging between 25% and 36%. The cylindrical CSS sample had a diameter of approximately 70 mm and an approximate height of 20 mm. Loose samples were prepared using a short pipe of 50 mm of diameter that was placed in the sample box and filled with a predetermined weight of sand (130 g). The pipe was quickly pulled up leaving the sample in a loose state. Next the sample was carefully leveled and the initial sample height measured before normal stress application.

The samples were sheared under stress control on constant volume conditions (constant height). CSS test results reported in this paper were all conducted with an initial normal stress of 200 kPa and subjected to sinusoidal horizontal shear loading that had either uniform or non-uniform amplitudes and frequencies of 0.1, 0.5, or 1.0 Hz. The uniform tests involved Cyclic Stress Ratios (CSRs) of 0.05, 0.065, 0.08, 0.09 and 0.10. Non-uniform CSS tests were carried out at 0.1, 0.5, and 1 Hz and denoted with test ID of "A", "B", and "C", respectively. The four variable amplitude wave forms involved a sequence of CSRs which are listed in Table 1 and were repeated until sample failure was reached.

Table 1: Non-uniform load types

Type	Description of shear load sequence
1	sine wave CSR = 0.05 followed by sine wave CSR = 0.065
2	sine wave CSR = 0.05 followed by sine wave CSR = 0.08
3	sine wave CSR = 0.065 followed by sine wave CSR = 0.08
4	4 sine waves CSR = 0.05 followed by a sine wave CSR = 0.08

The test program involved 27 tests, 15 uniform tests and 12 non-uniform tests (listed in Table 2). For this experimental program failure was defined when the sample reached 7.5% double-amplitude shear strain which is a failure criterion commonly used in the literature (e.g., Lasley 2015, Sivathayalan and Ha 2011, Vaid and Sivathayalan 1996).

Table 2: Non-uniform tests performed for this study

Case	1A	2A	3A	4A	1B	2B	3B	4B	1C	2C	3C	4C
Type (Table 1)	1	2	3	4	1	2	3	4	1	2	3	4
Frequency (Hz)	0.1	0.1	0.1	0.1	0.5	0.5	0.5	0.5	1	1	1	1

4 RESULTS

4.1. Uniform cycles – effect of frequency on cycles to failure

The results of the uniform tests are shown in Figure 2, along with a trendline. The results indicate that the frequency does not have significant effect on the results. The modest variability in the cycles to failure are attributable to the intrinsic variability of the tests and the samples. Furthermore this range of cycles to failure is within the variability observed from tests series carried out at a constant frequency. Thus it is concluded that frequency, between 0.1 to 1.0 Hz, did not have significant influence in the number of cycles to failure from constant volume CSS on loose dry Ottawa sand samples at $\sigma'_{no} = 200$ kPa.

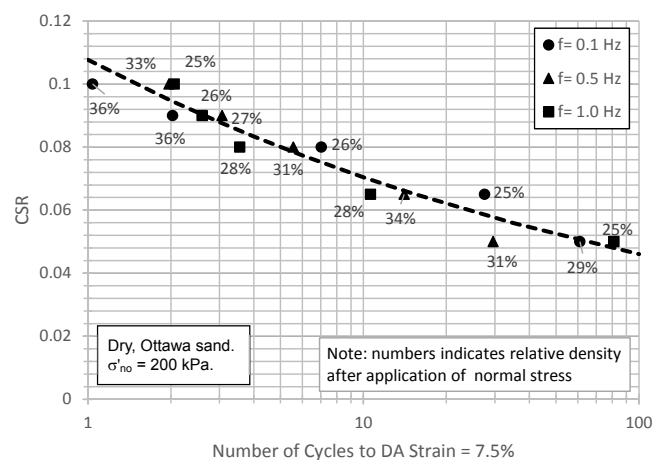


Figure 2: Influence of frequency on cycles to failure (DA Strain=7.5%) from constant volume, uniform CSS at $\sigma'_n = 200$ kPa.

4.2. Cumulative damage - number of cycles to failure

The predicted number of equivalent uniform cycles to failure computed using a simplified cumulative damage analysis based on the P-M hypothesis is presented in Figure 3.

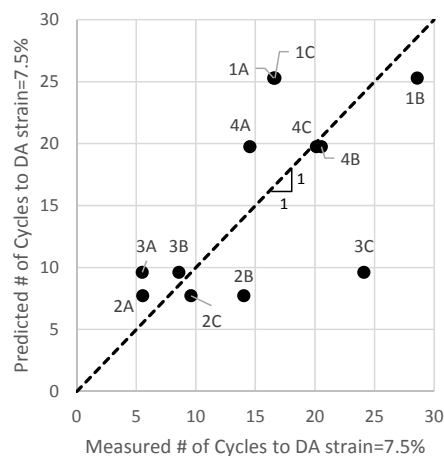


Figure 3: Measured vs predicted number of cycles to failure (DA strain=7.5%) using cumulative damage hypothesis for non uniform data

This figure highlights some of the shortcomings of cumulative damage analyses based on the load-path independent P-M hypothesis.

4.3 Dissipated energy to failure

The measured dissipated energies per unit volume at failure, without the correction for softening in the final load cycles proposed by Green and Terri (2005), are summarized in Figure 4 for all the CSS tests considered in this paper. The dashed line in this figure represents the overall mean of all energy values measured.

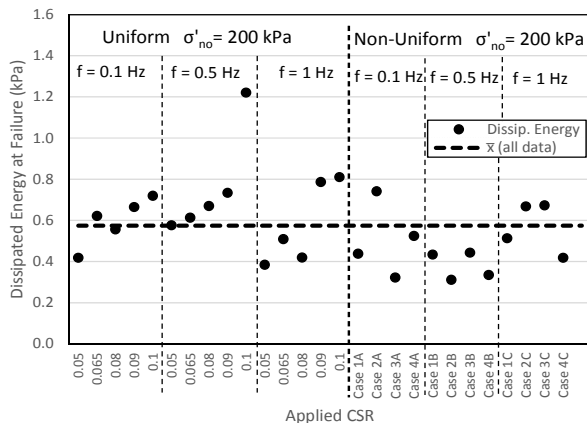


Figure 4: Summary of dissipated energy per unit volume to reach failure (DA shear strain = 7.5%) from all tests

It is important to point out that the dissipated energy per unit volume was computed to the last zero stress instance in the shear stress-strain cycle just prior to failure defined herein when a 7.5% double amplitude shear strain is reached (Lasley, 2015). A summary of the computed averages for each loading frequency considered are summarized in Table 3.

Table 3: Average Dissipated Energy per unit volume (Uniform and non-uniform tests)

Frequency	Mean (kPa)	St. Dev. (kPa)	COV
0.1 Hz	0.5566	0.1438	26%
0.5 Hz	0.5935	0.2770	47%
1.0 Hz	0.5762	0.1628	28%
All	0.5754	0.1959	34%

From the results presented in Figure 4 and in Table 3 we can see that the energy per unit volume dissipated by all samples until failure is reached was fairly uniform, regardless of the loading type, CSR level, or frequency. Variability of results is reasonably small and likely related to inherent sample variability, even though special care was taken in making samples as repeatable as possible. It is also worth noting that even with completely different number of cycles to failure, the dissipated energy to failure remained at very similar levels.

5 Summary and Conclusions

Twenty seven constant volume CSS tests were conducted at uniform and non-uniform shearing loading applied to samples of loose, dry Ottawa sand at an initial normal stress of 200 kPa. All CSS tests were conducted until failure, defined as a double amplitude shear strain of 7.5%. The tests results were used to assess the influence of irregular loading and frequency on the cumulative damage and dissipated energy per unit volume of dry loose sand. The main conclusions drawn are:

- CSS tests with uniform load cycles showed that the curve of number of cycles to failure versus CSR was not significantly influenced by frequency, at least for the 3 frequencies considered of 0.1, 0.5, and 1 Hz.
- The predictions of number of cycles to failure using a P-M based cumulative damage model for the irregular cycles were found to be imprecise. This is related to shortcomings of the P-M hypothesis that as pointed out by Green and Terri (2005) was originally meant to be used for high-cycle fatigue as in metals and is load path independent which for cyclic loading of dry sands is not realistic.
- The measured dissipated energy per unit volume to reach failure was found to be fairly uniform for the 27 CSS tests considered in this paper. Even though some variability was observed, dissipated energy values were quite uniform despite the different loading conditions considered and the great range of number of cycles to failure observed within the test program. Despite the modest variability observed in the measured dissipated energy per unit volume it is believed to be within variability reported by others in the literature and thus supports the approach of using dissipated energy to study cyclic behavior of sands and liquefaction.

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