A Simple Method for Rate of Consolidation Deformation and Its Applications

Une méthode simple pour le taux de déformation de la consolidation et ses applications

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ABSTRACT: The analytical solutions of different one-dimensional consolidation theories can be transformed into an identical general solution (Zhang & Gu, 2016). Based on the general solution, a simple calculation method for the rate of consolidation deformation is proposed. The method has been applied in the analysis of consolidation settlements, the unloading swelling and the prediction of post-construction settlement. The results show that the proposed method has a good agreement with the measured.

RÉSUMÉ : Les solutions analytiques des différentes théories de consolidation unidimensionnelles peuvent être transformées en une solution générale identique (Zhang & Gu, 2016). Sur la base de la solution générale, une méthode de calcul simple pour le taux de déformation de consolidation est proposée. La méthode a été appliquée dans l'analyse des règlements de consolidation, le gonflement de déchargement et la prédiction de règlement post-construction. Les résultats montrent que la méthode proposée est en bon accord avec la mesure.

KEYWORDS: consolidation; general analytical solution; rate of deformation; variable coefficient; unloading swelling; prediction post-construction settlement.

1 INTRODUCTION

The soil settlement calculations generally include two aspects, the total settlement magnitudes and the rates at which the settlement will occur. Today, in many cases, the researchers have been able to estimate no more than 10% -20% of the total settlement amount, but the ability to estimate the time rates of settlement is still very poor (Brand & Brenner, 1981). The prediction of the settlement process according to Terzaghi theory is usually far away from the measured, and even the settlement process of the oedometer test cannot be simulated reasonably. It is generally believed that the assumption of constant consolidation coefficient lacked in rationality (Duncan, 1993; Olson, 1998). With regard to the consolidation theory of Terzaghi, the main limitations are generally considered as follows: The compressibility coefficient and the permeability coefficient are constants; the effects of large deformation and secondary consolidation are ignored. Due to the difficulties in mathematics, although many researches were carried out regarding the above issues, only a few of them achieved their analytical solutions (Davis and Raymond, 1965; Lekha et al., 2003; Gibson, 1967; Xie et al., 2002; Amir et al., 2011; Poskitt, 1969; Merchant (Christie, 1964), Gibson and Lo, 1961). Zhang et al. (2015) proposed a time-varying C_v method to calculate the time course of unloading swelling deformation according to Terzaghi's solution. Zhang and Gu (2016) have shown that the mentioned analytical solutions for various consolidation theories, including linear, nonlinear, large strain and secondary consolidation, can be represented by an identical expression in which only needs to use variable $D_v(t)$ to replace the Terzaghi constant consolidation coefficient C_v.

In this paper, the general analytical solution of consolidation theory is introduced and its applications in analysis of consolidation settlement, unloading swelling deformation and post-construction settlement prediction are shown.

2 BASIC THEORY AND CALCULATION METHOD

2.1 *General analytical solution of consolidation*

The general solution of 1D consolidation were given in Table 1 (Zhang & Gu, 2016). It can be found that: 1) the analytical solutions of all kinds of consolidation models in Table 1 can be transformed as the same expressional form. 2) Different types of $D_v(t)$ should be the outward manifestations of the different assumptions in consolidation models. When the those additional special considerations are removed, $D_v(t)=C_v$. 3) According to the principle of dimensional consistency, the dimension of $D_v(t)$ should be the same as that of consolidation coefficient C_v. In order to distinguish the consolidation coefficient C_v, the $D_v(t)$ can be defined as consolidation variable coefficient.

Table 1. The general solution of consolidation

Consolidation model	The general solution	$D_v(t)$
Terzaghi	$U_s=1-\frac{8}{\pi^2}\sum_{n=odd}^{\infty}\frac{1}{n^2}e^{-\frac{n^2\pi^2}{4H^2}D_v(t)t}$	C_v
Davis & Ragmond nonlinear theory		C_v
Lekha nonlinear theory		$C_{vi}\left\{\frac{1}{2}\left[1+\left(1+\Delta\sigma/\sigma_i\right)^{(1-C_c/M)}\right]\right\}$
Gibson nonlinear infinite theory		$c_0\left(1-\frac{\alpha}{c_0}(e_0-e)\right)$
Xie nonlinear large strains theory		C_{v0}
Amiri nonlinear theory		$C_{vi}\left(1-\frac{(\alpha-\beta)t-(1-e^{-(\alpha-\beta)t})}{(\alpha-\beta)t}\right)$
Poskitt nonlinear infinite theory		$C_{v0}\left(1-\frac{4H^2}{n^2\pi^2C_{v0}t}\ln\left(1-\alpha\left[(2n+1)^2\frac{\pi^2C_{v0}}{H^2}t-\frac{16}{\pi^2}\sum_{p=odd}\sum_{q=odd}\frac{(2n+1)^4[1-e^{-(p^2+q^2-(2n+1)^2)\frac{\pi^2C_{v0}}{H^2}t}]}{[p^2-(q-(2n+1))^2][p^2-(q+(2n+1))^2][(2n+1)^2-q^2-p^2]}\right]\right)\right)$

Gibson&Lo rheological consolidation theory	$\theta\left[1-\frac{4H^2}{n^2\pi^2\theta t}\ln\left(\frac{\frac{n^2\pi^2}{M}-x_2}{x_1-x_2}e^{-\frac{(x_1-n^2\pi^2)\theta t}{4H^2}}-\frac{\frac{n^2\pi^2}{M}-x_1}{x_1-x_2}e^{-\frac{(X_2-n^2\pi^2)\theta t}{4H^2}}\right)\right]$

2.2 Calculation method

The infinite series in Table 2 converges very fast. When the dimensionless time factor T ($D_v(t)t/H^2$) is greater than 0.3, truncation error is less than 0.2%. Regarding this feature, the first term can be used to replace the whole series as a kind of approximation, and the general solution can be written as,

$$U_s = 1 - \frac{8}{\pi^2}e^{-\frac{\pi^2}{4}\frac{D_v(t)}{H^2}t} \tag{1}$$

From Eq. (1),

$$D_v(t) = \frac{4H(t)^2}{\pi^2 t}\ln\left(\frac{8}{\pi^2(1-U_s(t))}\right) \tag{2}$$

In oedometer test, the $U_s(t)$, $H(t)$, t in Eq. (2) are known, and $U_s(t)$ can be calculated by the following,

$$U_s(t) = S(t)/S_\infty \tag{3}$$

Where, $S(t)$ is the settlement at time t, S_∞ is the final settlement. Thus the test value of $D_v(t)$ can be obtained from the conventional oedometer test, no any other special tests needed. The test value of $D_v(t)$ contains all effects including linear, nonlinear, large strains and secondary consolidation. The $D_v(t)$ can be fitted by following formula,

$$D_v(t) = D_{v\infty} + \frac{D_{v0} - D_{v\infty}}{1+(t/t_0)^n}$$

(4)

When $t=0$, $D_v(t)=D_{v0}$; $t=\infty$, $D_v(t)=D_{v\infty}$; $t=t_0$, $D_v(t)$ equals to the mid-value, n is coefficient. Parameters of D_{v0}, $D_{v\infty}$, t_0 and n can be obtained by fitting the test data.

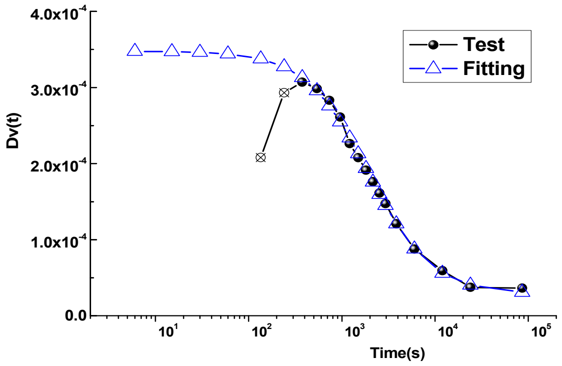

Figure 1. Variations of $D_v(t)$ with time

Considering the influences of truncation error, unstable seepage and instantaneous settlements at the beginning of loading, several disturbed test data of $D_v(t)$ at the beginning should be ignored. The highest point of the measured $D_v(t)$ was chosen as the first valid test point in this paper. The variations of $D_v(t)$ with time were given in Fig. 1.

3 APPLICATIONS

3.1 Consolidation test

Figure 2 is the results of consolidation test with an undisturbed clay sample under staged loading. It can be seen the variations of $D_v(t)$ under different consolidation pressures are similar to each other in a staged loading test, shown in Fig. 2(a). And the $D_v(t)$ behaves like a process factor which repeats itself in each loading period, takes the loading process as its cycle and decreases during each consolidation process. Fig. 2(b) is the comparison between the computed settlements and the test results.

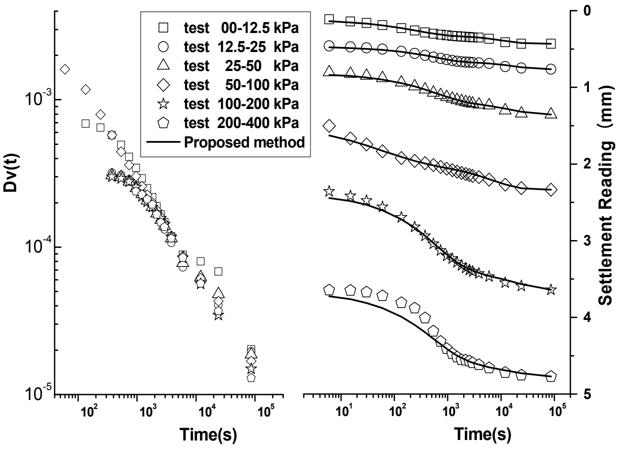

Figure 2. Settlement process of staged loading consolidation test of undisturbed clay sample, a) $D_V(t)$ process, b) settlement process

3.2 Secondary consolidation test

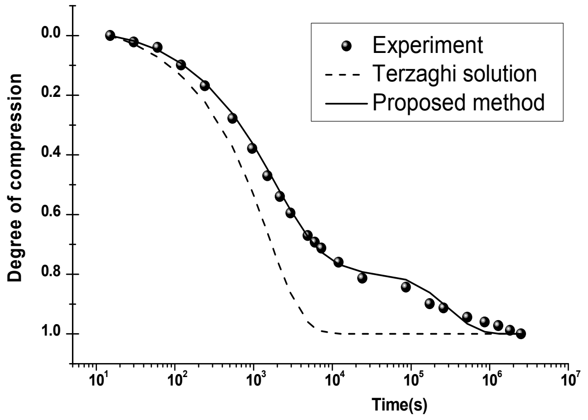

Figure 3. Settlement of secondary consolidation test

Fig. 3 is the results of secondary consolidation test (Yin, 1998), the test period as long as 30 days. It can be seen that the results by proposed method basically agree with the test value. As a contrast, results of Terzaghi method are given in the figure also.

3.3 Unloading swelling test

Based on the mechanism of consolidation the unloading swelling was regarded as the reverse process of consolidation. Therefore, the basic equations and their solutions under consolidation can all apply to the swelling process directly, and it is only need to expand the definitions of consolidation as consolidation and swelling, the excess pore pressure expanded as excess pore pressure and suction, and the compression degree expanded as compression degree and swelling degree, etc.

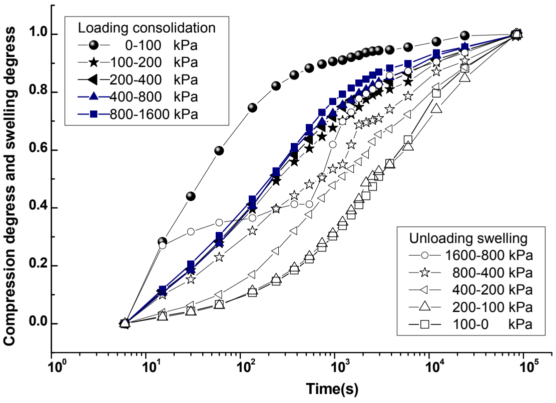

Figure 4. Comparisons between consolidation and swelling process

Figure 4 is the test results of a remolded clay sample under loading and unloading conditions. Both of the compression degree and the swelling degree with time (solid marks for consolidation, empty marks for swelling) are given in the figure. It can be seen that, the unloading swelling process is similar to the loading consolidation process, both processes of consolidation and unloading swelling are time-dependent and without any sign that the swelling can be finished faster. And most of the swelling curves are lower than the consolidation curves, which means that not only the unloading swelling cannot be finished rapidly, but even slower than the well-known consolidation process.

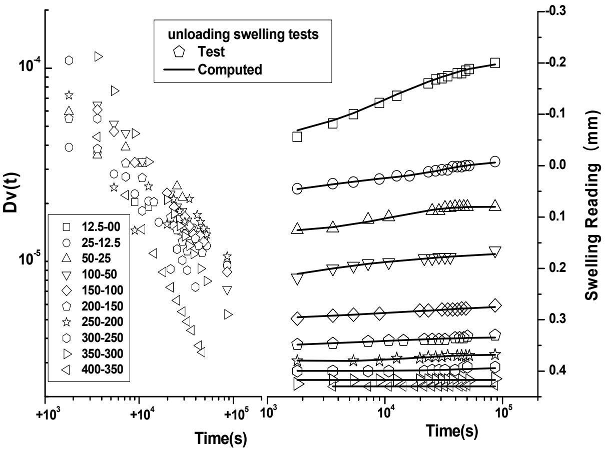

Figure 5. Swelling process of staged unloading swelling test of undisturbed clay sample, a) $D_v(t)$ process, b) swelling process

Fig. 5 is the unloading swelling test results. The test data of undisturbed clay sample are provided by Dr. Li and Professor Teng (2011). It can be seen that the proposed method corresponds well with all levels of unloading swellings.

3.4 Prediction of post-construction settlement

The soft foundation of the highway link between Hangzhou bay bridge and Ningbo is about 50km long, with several different soft soil layers, depth of 20~30m, partially reaching 40m. The typical profile of the soft foundation is shown as Fig. 6. The observations of settlement, deformation and pore water pressure are carried out during-construction and post-construction.

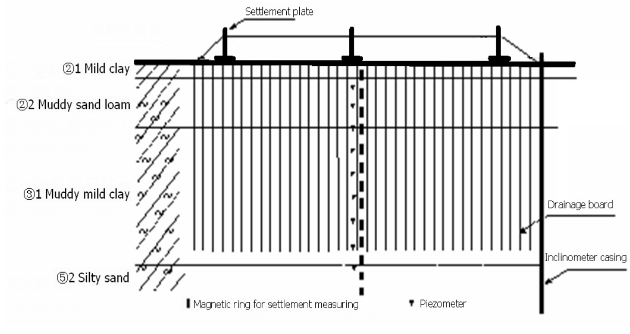

Figure 6. Diagram of the highway soft foundation

Generally the observed settlements contain all of the effects, including the effects of the multiple layers of soil and the different lengths of drainage path and etc. In the prediction analysis an equivalent single layer of soil is assumed, its drainage path is H and its settlements under the engineering loading are equal to the observed settlements. The proposed method was used to predict the post-construction settlement. The results were given in Fig. 7 and Fig. 8.

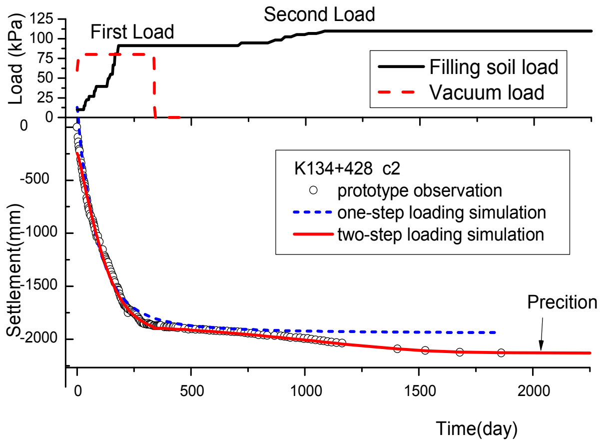

Figure 7. Prediction of post-construction settlements of section K134.

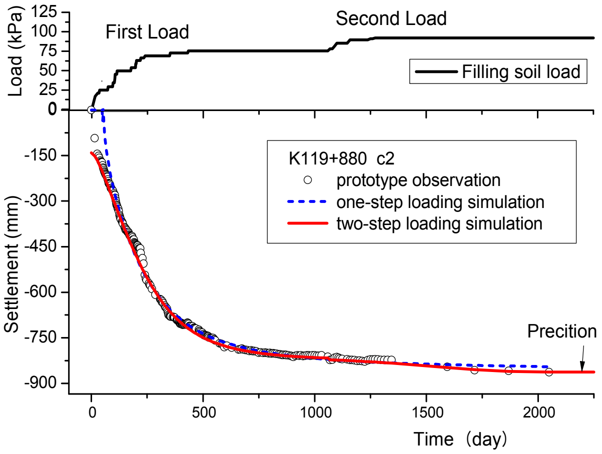

Figure 8. Prediction of post-construction settlements of section K119.

4 CONCLUSIONS

The analytical solutions of one-dimensional consolidation theory in Table 1 can be transformed into an identical general solution. The proposed calculation method based on the general solution is applicable not only to consolidation analysis, but also to the analysis of unloading swelling and the prediction analysis of post-construction settlement. The results show that the method is in good agreement with the measured.

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