

# INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING



*This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:*

<https://www.issmge.org/publications/online-library>

*This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.*

# Use of shear wave velocity to evaluate in-situ state of silty sands

## Utilisation de la vitesse des vagues de cisaillement pour évaluer l'état in situ des sables limoneux

Yi Guo, Jun Yang

*Dept. of Civil Engineering, The University of Hong Kong, Hong Kong, China, h1185007@hku.hk*

*Dept. of Civil Engineering, The Univ. of Hong Kong, Hong Kong, China, junyang@hku.hk (corresponding author)*

**ABSTRACT:** In design of large-scale earth structures such as hydraulic-placed fills for artificial islands or tailings dams, a major concern is the susceptibility of the silty sands involved to flow failure or liquefaction. Compared with uniform clean sands that have been extensively studied in the laboratory, the large-strain behavior and liquefaction potential of silty sands is not yet well understood partly due to the complex role of the fine particles. A difficult problem in the liquefaction evaluation in practice is to evaluate the in-situ state of the soil; this is because for sandy soils it is extremely difficult and costly to obtain high-quality undisturbed samples. This paper presents a framework for evaluation of the in-situ state of silty sands by using shear wave velocity measurements. The framework is built upon a comprehensive experimental program involving shear wave measurements for both clean sand and sand-fines mixtures in conjunction with interpretation and analysis in the theory of critical state soil mechanics. Since shear wave velocity can now be reliably measured both in the laboratory and in the field and since the initial state controls different aspects of the stress-strain behavior of sandy soils, the proposed framework is highly promising in a variety of geotechnical applications.

**RÉSUMÉ :** Dans la conception de structures terrestres à grande échelle telles que les remplissages hydrauliques pour îlots artificiels ou barrages à résidus, une préoccupation majeure est la susceptibilité des sables limoneux impliqués à l'écoulement ou la liquéfaction. En comparaison avec les sables propres uniformes qui ont été étudiés en profondeur en laboratoire, le comportement à grande déformation et le potentiel de liquéfaction des sables limoneux n'est pas encore bien compris en partie en raison du rôle complexe des fines particules. Un problème difficile dans l'évaluation de la liquéfaction en pratique est d'évaluer l'état in situ du sol; C'est parce que pour les sols sablonneux il est extrêmement difficile et coûteux d'obtenir des échantillons non perturbés de haute qualité. Cet article présente un cadre pour l'évaluation de l'état in situ des sables limoneux en utilisant des mesures de vitesse d'onde de cisaillement. Le cadre est construit sur un programme expérimental complet impliquant des mesures d'onde de cisaillement pour le sable propre et les mélanges sable-fines en conjonction avec l'interprétation et l'analyse dans la théorie de mécanique de sol d'état critique. Puisque la vitesse d'onde de cisaillement peut maintenant être mesurée de manière fiable en laboratoire et sur le terrain et que l'état initial contrôle différents aspects du comportement contrainte-déformation des sols sableux, le cadre proposé est très prometteur dans diverses applications géotechniques.

**KEYWORDS:** dynamics; laboratory tests; liquefaction; sand with fines; shear wave velocity.

## 1 INTRODUCTION

Flow liquefaction of sand is characterized by a sudden loss of strength and a rapid development of deformation. Over the last several decades, considerable efforts have been made to study flow liquefaction and several procedures have been proposed for evaluating liquefaction potential (e.g. Ishihara, 1993). Generally, these procedures are based on extensive laboratory tests on clean uniform sands along with the steady state or critical state concepts. Steady state or critical state (Casagrande, 1975), defined as the soil deforms continuously under constant stress and constant volume. There is now a general agreement that the behavior of a sand is more closely related to the proximity of its initial state to the critical state locus, which can be described by a state parameter ( $\Psi$ ) as defined by Been and Jefferies (1985). If the initial state of a saturated sand lies above or to the right of the critical state locus with a positive value, it tends to contract when sheared undrained, accompanied by strain softening and a buildup of high pore pressures. The initial state defined by the void ratio and mean effective stress with reference to the critical state locus is therefore a meaningful index that can be used to identify.

In practice, it is very difficult and expensive to obtain undisturbed sand samples, in order to determine the in-situ state. Several attempts have been made to use shear wave velocity ( $V_s$ ) to estimate the state of sand (e.g. Robertson et al., 1995; Cunning et al., 1995; Wride et al., 2000). These  $V_s$ -based methods rely on the empirical correlation among void ratio, effective confining pressure and shear wave velocity, which derive from laboratory testing on  $V_s$  in clean sand samples. A

classic empirical correlation proposed by Hardin & Richart (1963) on Ottawa sand:

$$V_s = (a_1 - a_2 e)(P')^{0.25} \quad (1)$$

where  $e$  is void ratio,  $P'$  is mean effective stress,  $a_1$  and  $a_2$  are material constants.

These previous studies mainly focusing on clean uniform sands. Often natural sand deposits or fills are not clean, but contain a certain amount of fines (referred to as silty sand in practice). Even inside a single location, the fines content may vary appreciably. Application of existing  $V_s$ -based methods developed by empirical correlations, which implicitly ignoring the effect of fines. This assumption is inappropriate. Recently, significant effects on additions of fines on shear wave velocity or shear modulus compared to clean sand have been found out based on laboratory testing, under the same void ratio and confining pressure (e.g. Wichtmann et al., 2015; Yang & Liu, 2016). A small amount of fines can significantly increase the flow liquefaction potential of clean sand (Yang & Wei, 2012).

This paper presents a new framework which allows a unified evaluation of in-situ state of both clean sand and silty sands through shear wave velocity measurements. Compared with cone penetration resistance, shear wave velocity ( $V_s$ ) measurement is made at small strain and non-destructive.  $V_s$  can be measured in laboratory (e.g. bender element) and in the field (e.g. surface wave method), as well documented by Clayton (2011). It is thus of great interest to explore whether the shear wave velocity can be used to evaluation in-situ state of sand deposits with varying fines content and thereafter to evaluate their potential for liquefaction.

## 2 TESTING PROGRAM

The first step of this study was to create a comprehensive database that allows exploration of the possible relationship between shear wave velocity and liquefaction susceptibility for both clean and silty sands. Two major series of tests were conducted for sand-fines mixtures with different fines contents and at different states (i.e. stress states, density states). One is the direct measurement of shear wave velocity at small strains by bender element test. The two bender elements were mounted in top and bottom caps of the triaxial specimen in a resonant column system. An excitation frequency of 10 kHz and start-to-start method were used to determine the arrival time of the shear wave (Yang & Gu, 2013; Yang & Liu, 2016). The other is monotonic undrained shear tests at large strains using triaxial test apparatus (Yang & Wei, 2012). In this testing program, Toyoura sand was used as the base sand and crushed silica was used as fines. Toyoura sand was uniform fine sand with sub-angular to sub-rounded grains, whereas the crushed silica fines were composed of non-plastic angular grains. The mean particle size ( $d_{50}$ ) of Toyoura sand was 0.216 mm, and the crushed silica fines had a  $d_{50}$  of 0.054 mm. The coefficient of uniformity (Cu) of Toyoura sand was 1.39, and the crushed silica fines had a Cu of 2.18. Fig. 1 shows particle size distribution curves of the sand-fines mixtures, with different percentages of fines content by mass (0%, 5%, 10% and 20%). Using artificially created mixtures not only allows good control of grain characteristics, but also increases the experimental repeatability. The sand-fines mixtures in this study were considered as sand-dominated materials, with the fines content less than about 30%.

All specimens were prepared by the moist tamping method (Ishihara, 1996) in conjunction with the under-compaction technique (Ladd, 1978), in both small-strain and large-strain testing. The purpose to choose this method is because it can produce a wide range of sand densities and has the advantage of preventing grain segregation and enhancing uniformity. After the sample prepared by moist tamping, carbon dioxide and deaired water were firstly applied to flush through the sample. Back pressure was applied to ensure fully saturation. Skempton B-value of the specimen larger than 0.95 was considered as fully saturated. After saturation, the specimen was subjected to an isotropically consideration at different confining pressures, typically stress levels ranging from 50 kPa to 500 kPa.

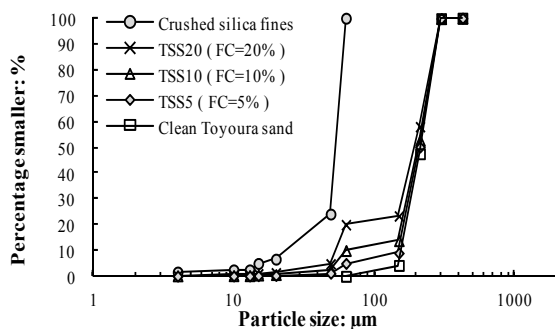


Figure 1. Particle size distribution curves of clean and silty sands

## 3 STATE DEPENDENT SHEAR WAVE VELOCITY

A systematic investigation on state dependent shear wave velocity has been done and some typical results are presented in Fig. 2, where measured  $V_s$  is plotted against void ratio at stress level 50 kPa for different sand-fines mixtures (TS, TSS5, TSS10 and TSS20).

### 3.1 Relationship between $V_s$ and $e$

A relationship was developed among the void ratio ( $e$ ), mean effective stress ( $P'$ ), and  $V_s$  since 1960s, and modified as the following form (Hardin and Richart, 1963; Robertson et al., 1995):

$$V_s = (b_1 - b_2 e)(P'/P_a)^{0.25} \quad (2)$$

Robertson et al. (1992) proposed a normalized shear wave velocity ( $V_{s1}$ ),

$$V_{s1} = V_s(P_a/P')^{0.25} \quad (3)$$

Combining Eqn. (2) and Eqn. (3), the equation of  $V_{s1}$  can be rewritten as:

$$V_{s1} = b_1 - b_2 e \quad (4)$$

This linear relationship has been adopted in many practical projects. The measured  $V_s$  values were normalized with stress levels using Eqn. (3) and plotted against void ratio. Fig. 3 shows a general trend that  $V_{s1}$  decreases with increasing  $e$  for clean Toyoura sand (TS), with relatively large scatter. Besides, sand-fines mixtures data were also plotted in the format of  $V_{s1}$  against  $e$ . It is obvious that large scatters still exist among different sand-fines mixtures. In reality,  $V_s$  is highly influenced by fines content apart from effective confining pressure and void ratio (Fig. 2). At this stage, it seems there exists some problems to find out a general trend to capture the  $V_s$  among different sand-fines mixtures.

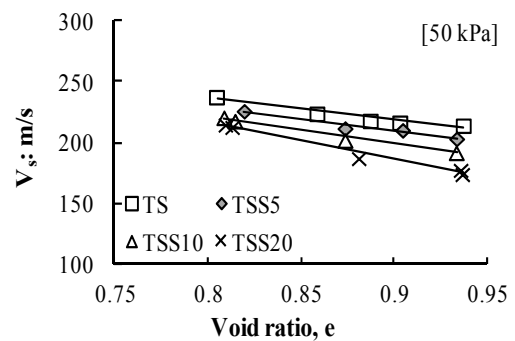


Figure 2. Shear wave velocity versus void ratio at 50 kPa for TSS mixtures

### 3.2 Relationship between $V_s$ and $\Psi$

Compared with void ratio, the state parameter has proved as a more meaningful index for characterizing the stress-strain-strength behavior of sand (e.g. Wood *et al.*, 1994; Yang & Li, 2004). It was defined as the difference between the void ratio ( $e_0$ ) at current state and the void ratio at critical state ( $e_{cs}$ ) under the same mean effective stress (Been & Jefferies, 1985).

$$\Psi = e_0 - e_{cs} \quad (5)$$

It has been found useful in capturing various aspects of the stress-strain-strength behavior of sand (e.g. Yang & Li, 2004). The traditional critical state line (CSL) in  $e$ - $\log p'$  space was defined as follow:

$$e_{cs} = \Gamma - \lambda \log p' \quad (6)$$

where  $\Gamma$  is the intercept of CSL at  $P' = 1$  kPa, and  $\lambda$  is the slope of CSL in  $e$ - $\log p'$  space. However, the critical state line is not a straight line but a curve, once the database is large enough. Several studies on clean sand had found out this phenomena (e.g. Verdugo & Ishihara, 1996). Recently, a power law has been adopted to express the critical state locus for both clean and silty sands, as given by Li & Wang (1998) and Yang & Li (2004):

$$e_{cs} = e_{\Gamma} - \lambda_{cs} (P'/P_a)^a \quad (7)$$

where  $a$  is the stress exponent, taken as 0.6 for quartz sands, typically.  $e_{\Gamma}$  and  $\lambda_{cs}$  are the intercept and slope of CSL, which depend mainly on grain characteristics, such as grain shapes and grading (Yang & Luo, 2015).  $P_a$  is the atmospheric pressure, equals to 101 kPa. The critical state line of Toyoura sand with 20% fines content (TSS20) on the power-law format as example is shown in Fig. 4. Different sand-fines mixtures

have different critical state lines, as fines content increases, the critical state lines tend to shift downward for the sand base sand (Yang & Wei, 2012).

Once critical state lines of the sand-fines mixtures are known, the state parameter ( $\Psi$ ) can be determined. It is possible to find out the relationship between  $V_s$  and  $\Psi$  through the comprehensive database.

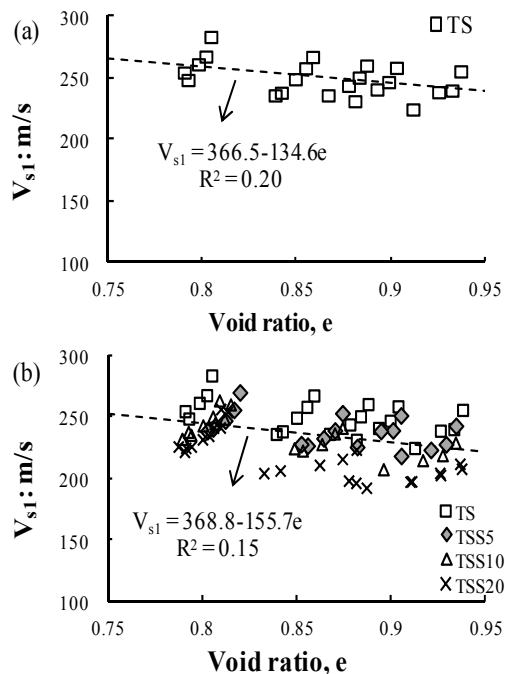


Figure 3. Corrected shear wave velocity versus void ratio: (a) Toyoura sand; (b) TSS mixtures

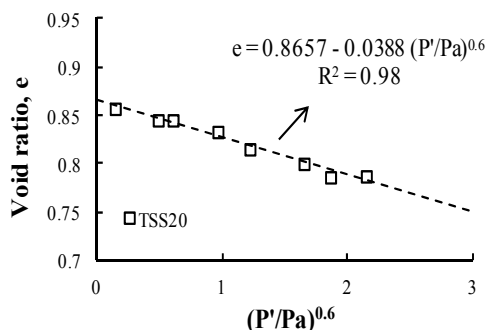


Figure 4. Critical state line of TSS20

Several steps may be applied to the datasets. First, plot the measured  $V_s$  against  $\Psi$  for different sand-fines mixtures for a confining pressure, ranging from 50 kPa to 500 kPa. As shown in Fig. 5,  $V_s$  decreases with  $\Psi$  in an approximately linear manner, which indicates the sound fact that the looser the specimen the lower the shear wave velocity. Further step is stress correction. The shear wave velocity is corrected for stress levels in following equation:

$$V_{s1} = V_s (P_a/P')^n \quad (8)$$

Once the corrected shear wave velocities ( $V_{s1}$ ) are plotted against  $\Psi$  for all sand-fines mixtures data, an approximately linear relationship is achieved between  $V_{s1}$  and  $\Psi$ , as presented in Fig. 6 in the following manner:

$$V_{s1} = A - B\Psi \quad (9)$$

Combining Eqn. (8) and Eqn. (9),  $V_s$  can be predicted by the equation:

$$V_s = (A - B\Psi)(P'/P_a)^n \quad (10)$$

where  $V_s$  is in m/s;  $n$  is stress parameter, best fitted as 0.222 for sand-fines mixture in this study;  $A$  and  $B$  are fitting parameters

which are independent of fines content, as 237.2 m/s and 281.2 m/s, respectively.

#### 4 EVALUATION OF IN SITU STATE

In practice, the horizontal stress is generally unknown and the in-situ mean effective stress  $P'$  (in kPa) can be commonly determined by the following equation:

$$P' = \frac{(1+2K_0)}{3} \sigma'_v \quad (11)$$

where  $K_0$  is the earth pressure coefficient at rest, and  $\sigma'_v$  is the effective vertical stress, in kPa. Substituting Eqn. (11) into Eqn. (10), shear wave velocity can be predicted by

$$V_s = (A - B\Psi) \left( \frac{\sigma'_v}{P_a} \right)^n \left( \frac{1+2K_0}{3} \right)^n \quad (12)$$

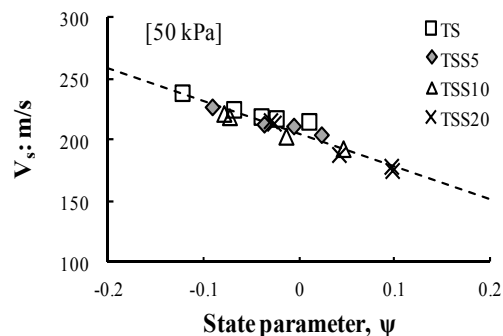


Figure 5. Shear wave velocity versus state parameter at 50 kPa for TSS mixtures

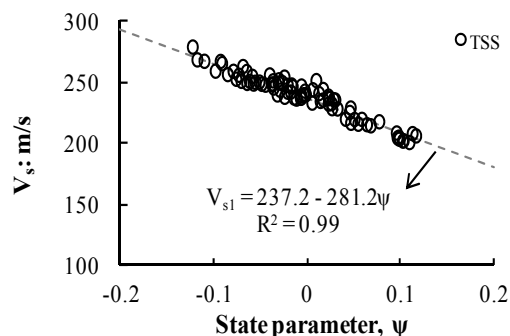


Figure 6. Corrected shear wave velocity versus state parameter for TSS mixtures

For normally consolidated, loose and medium dense sands that are of interest in liquefaction evaluation, typical values of  $K_0$  vary between 0.4 and 0.6 (Been et al., 1986; Wride et al., 2000). Based on Eqn. (12), a set of  $V_s$ -profile can be plotted against  $\sigma'_v$  at different  $K_0$  (e.g. 0.5 and 1) and  $\Psi$  (-0.2 to 0.2) values in Fig. 7. Typical state parameter  $\Psi$  values ranging from -0.2 (very loose and contractive) to 0.2 (very dense and dilative). Contractive and dilative behavior of soils can be distinguished by the  $V_s$ -profile when  $\Psi = 0$ . Once the values of  $V_s$ ,  $K_0$  and  $\sigma'_v$  are known, which can be measured or achieved from site investigation, the in-situ state of soils can be predicted by these charts. If the initial evaluation gives a conclusion of possible liquefaction, a more detailed evaluation may be carried out using the framework but with the material constants determined from project-specific laboratory tests.

#### 5 SUMMARIES AND CONCLUSIONS

This paper proposes a unified framework for evaluation of the in-situ state of soils with varying fines content through shear wave velocity. The framework is built upon a comprehensive experimental program involving shear wave measurements for both clean sand and sand-fines mixtures in conjunction with

interpretation and analysis in the theory of critical state soil mechanics. Several key results are summarized below:

- The shear wave velocity ( $V_s$ ) is not only influenced by void ratio ( $e$ ) and effective confining pressure ( $P'$ ), but also fines content. The traditional  $V_s$ - $e$  relationship fails to unify the sand with fines.
- Usage of traditional critical state line in  $e$ - $\log P'$  space should be in caution when data are limited, leading a large overestimation on critical state void ratio value at low effective confining pressures. A power law relationship of critical state locus is strongly suggested to evaluate the state parameter and further evaluate liquefaction potential.
- Under the framework of critical state soil mechanics, a unique linear relationship between corrected shear wave velocity ( $V_{s1}$ ) and state parameter ( $\Psi$ ) was found and it is independent of fines content. The correlation indicates that  $V_{s1}$  decreases with increasing  $\Psi$  for both clean sand and a certain amount of fines.
- An alternative relationship is proposed for in-situ anisotropic stress conditions by considering the earth pressure coefficient at rest ( $K_0$ ). A set of velocity profiles is provided for evaluation of in-situ state of soils with different  $K_0$  and  $\Psi$  values. One of the specific velocity profile for  $\Psi=0$  serves as the boundary approximately distinguishing between dilative and contractive response.
- Further studies on different materials with different grain characteristics are suggested to accomplish, in order to generalize the material constants.

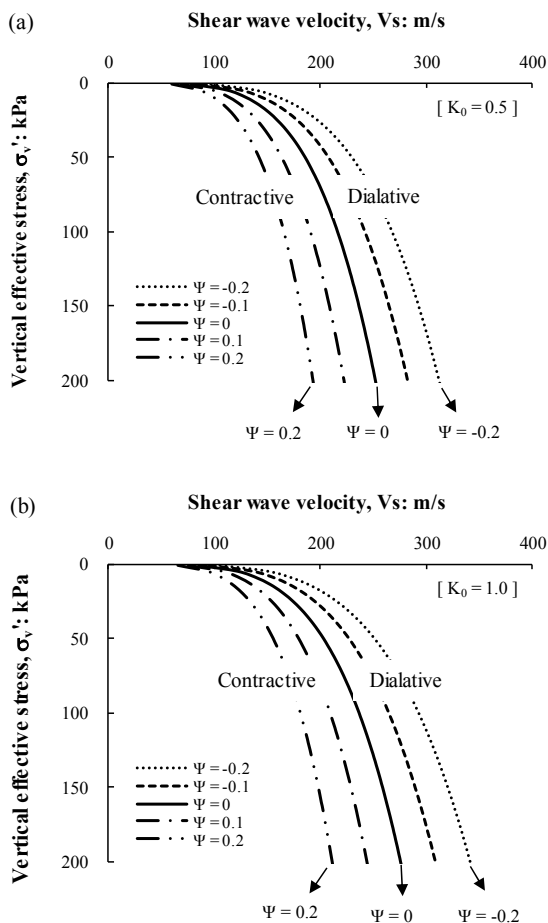


Figure 7. Profile of state parameter in  $V_s$ - $\sigma'_v$  plane: (a)  $K_0=0.5$ ; (b)  $K_0=1.0$

## 6 ACKNOWLEDGMENTS

Financial support provided by the National Natural Science Foundation of China (No. 51428901) and by the Research Grants Council of Hong Kong (No. 719105) is gratefully acknowledged.

## 7 REFERENCE

- Been, K. & Jefferies, M. G. (1985). A state parameter for sands. *Géotechnique* **35**, No. 2, 99–102.
- Been, K., Crooks, J.H. A., Becker, D.E., Jefferies, M.G. (1986). The cone penetration test in sands. Part 1: State parameter interpretation. *Géotechnique* **36**, No. 2, 239–249.
- Casagrande, A. (1975). Liquefaction and cyclic deformation of sands, a critical review. *Proceedings, 5th Pan - American Conference on Soil Mechanics and Foundation Engineering*, Buenos Aires, 5, 79–133.
- Clayton, C.R.I. (2011). Stiffness at small-strain: research and practice. *Géotechnique* **61**, No. 1, 5–38.
- Hardin, B. O. & Richart, F. E. (1963). Elastic wave velocities in granular soils. *J. Soil Mech. Found. Div., ASCE* **89**, No. SM1, 39–56.
- Ishihara, K. (1993). Liquefaction and flow failure during earthquakes. *Géotechnique* **43**, No. 3, 351–451.
- Li, X.S. & Wang, Y. (1998). Linear representation of steady-state line for sand. *J. Geotech. Geoenviron. Eng. ASCE* **124**, No. 12, 1215–1217.
- Robertson, P.K., Woeller, D.J., Kokan, M., Hunter, J. and Luternauer, J. (1992). Seismic techniques to evaluate liquefaction potential *Proceedings, 45th Canadian Geotechnical Conference*, Toronto, 5:1-5:9.
- Robertson, P.K., Sasitharan, S., Cunning, J.C. & Sego, D.C. (1995). Shear wave velocity to evaluate flow liquefaction. *J. Geotech. Eng., ASCE* **121**, No. 3, 262–273.
- Verdugo, R. & Ishihara, K. (1996). The steady state of sandy soils. *Soils Found.* **36**, No. 2, 81–91.
- Wichtmann, T., Hernandez, M. & Triantafyllidis, T. (2015). On the influence of a non-cohesive fines content on small strain stiffness, modulus degradation and damping of quartz sand. *Soil Dynamics Earthquake Eng.* **69**, 103–114.
- Wood, D.M., Belkheir, K., and Liu, D.F. (1994). Strain softening and state parameter for sand modeling. *Géotechnique* **44**, No. 2, 335–339.
- Wride (Fear), C.E., Robertson, P.K., Biggar, K.W., Campanella, R.G., Hofmann, B.A., Hughes, J.M.O., Küpper, A. & Woeller, D.J. (2000). Interpretation of in situ test results from the CANLEX sites. *Can. Geotech. J.* **37**, No. 3, 505–529.
- Yang, J. (2002). Non-uniqueness of flow liquefaction line for loose sand. *Géotechnique* **52**, No. 10, 757–760.
- Yang, J. & GU, X.Q. (2013). Shear stiffness of granular material at small strains: does it depend on grain size? *Géotechnique* **63**, No. 2, 165–179.
- Yang, J. & Li, X.S. (2004). State-dependent strength of sands from the perspective of unified modeling. *J. Geotech. Geoenviron. Eng., ASCE* **130**, No. 2, 186–198.
- Yang, J. & Liu, X. (2016). Shear wave velocity and stiffness of sand: the role of non-plastic fines. *Géotechnique* **66**, No. 6, 500–514.
- Yang, J. & Luo, X.D. (2015). Exploring the relationship between critical state and particle shape for granular materials. *J. Mech. Phys. Solids* **84**, 196–213.
- Yang, J. & Wei, L.M. (2012). Collapse of loose sand with the addition of fines: the role of particle shape. *Géotechnique* **62**, No. 12, 1111–1125.