Optimizing geotechnical site-investigations

Optimisation des études géotechniques

Anders Prästings, Stefan Larsson
KTH Royal Institute of Technology, Sweden, anders.prastings@byv.kth.se

Rasmus Müller
Tyréns AB, Sweden

ABSTRACT: One major question to deal with in the process of updating the current version of Eurocode 7 is how to convert the quality of a performed site-investigation into a measurable context. This should, in turn, affect the safety factor applied to the soil-strength properties in a limit state. This paper presents a study in which the total uncertainty from the evaluation of undrained shear strength in clay is assessed from single and multiple site-investigation methods with regard to both random and systematic uncertainties. A Bayesian procedure is used to convert potential reduction of random and systematic uncertainties into a measurable context in relation to: (1) the type of site-investigation method, (2) the combination of methods, and (3) the number of measurements performed.

RÉSUMÉ: L'une des questions majeures qui se posent lors du processus de mise à jour de la version actuelle de l'Eurocode 7 est la suivante : comment convertir la qualité d'une investigation de site en une quantité mesurable ? Cela devrait alors affecter le niveau de sécurité appliqué aux propriétés de résistance du sol dans un état limite. Cet article présente une étude au cours d e laquelle l'incertitude totale découlant de l'évaluation de la résistance au cisaillement de l'argile non évacuée est évaluée à pa rtir de méthodes d'investigation de site uniques et multiples en relation avec des incertitudes aléatoires et systématiques. Une p rocédure bayésienne est utilisée pour convertir la réduction potentielle des incertitudes aléatoires et systématiques en une incerti tude mesurable en rapport avec : (1) le type de méthode d'investigation de site, (2) la combinaison des méthodes, (3) le nombr e de mesures effectuées.

KEYWORDS: Eurocode 7, Reliability-based design, Geotechnical uncertainties, Partial factor method

1 INTRODUCTION

Site investigations are often proven to be inadequate due to limited time and budget, and geotechnical engineers are regularly forced to make decisions based on too uncertain information. Owing to these practical limitations, an important question is how to perform *cost-effective* site-investigations, rather than *just* reliable. For instance, Jaksa et al. (2003), concludes that there exists little guidance on how to determine the scope of an appropriate site-investigation.

To fully connect the scope of the site-investigation with the reliability of a construction, the uncertainty related to inherent variability of soil (aleatory uncertainty) must be accompanied by the epistemic uncertainties related to the method of soil characterization, which is typically divided into measurement error, statistical uncertainty and transformation uncertainty (Prästings et al. 2016; Ching et al. 2014; Ching et al. 2016). It is possible to reduce the impact of random measurement error and statistical uncertainty on the design value of soil-strength properties by increasing the amount of geotechnical measurements. Furthermore, transformation uncertainty, which to a large extent is systematic, can be reduced by cross-validating different geotechnical site-investigation methods using Bayesian analysis. An approach for the estimation of uncertainties in combination with a Bayesian updating procedure was proposed by Müller et al. (2014), and is referred to as the extended multivariate approach (EMA).

The partial factor, γ_M applied to soil-strength properties in Eurocode 7 (EC7) is fixed. Thus, an important question in the on-going process of updating the current version of EC7 is how to adjust the safety factor applied to the soil-strength properties in a limit state (relative γ_M), depending on the uncertainty related to the inherent variability and to the extent and quality of the geotechnical site-investigation. Today, if the quality of an investigation is increased, the existing procedure of evaluating the *5%-fractile* (CEN 2004) accounts only for the reduction of random uncertainties related to the evaluation of soil-strength properties and, hence, excludes systematic uncertainties, such as the transformation uncertainty. Furthermore, the Swedish national annex to EC7 (IEG 2008) does not use the *5%-fractile* but adjusts the level of safety (relative γ_M) by introducing a conversion factor, η, that is in part evaluated based on a set of *pre-defined values* related to the extent and quality of the site-investigation. The *pre-defined values* that are suggested in the national annex are based on a great amount of subjective judgement, and do not have a robust (objective) connection to the potential reduction in either random or systematic uncertainties.

In this paper, the values of η are calibrated based on input from the EMA and, hence, account for potential reductions of the epistemic uncertainties (both random and systematic). Case data is presented for which the aleatory and epistemic uncertainties related to the evaluation of spatially averaged undrained shear strength, $\bar{s}_u$, in soft clay, are assessed from different site-investigation methods, D_i . The uncertainties obtained in the case data are used as a reference input in the EMA. To investigate the potential reduction of η due to reductions of the epistemic uncertainties, the values of η are calibrated in relation to the number of in-situ and laboratory measurements, and to different combinations of site-investigation methods.

Furthermore, this paper exemplifies how to produce general estimates of η that may be used for guidance on how to plan for effective site-investigations performed on clay.

2 PROJECT

This paper investigates measurements performed for a highway project in Sweden. The geotechnical conditions consist of soft clay to a depth of about 100 m . The aim of the site-investigation was to characterize an area of approximately 40 000 m^2 and this paper focuses on the measurements performed in order to evaluate the undrained shear strength, s_u .

Values of s_u was determined via direct simple shear tests (DSS), empirically determined via the pre-consolidation pressure evaluated from oedometer tests (CRS), and determined via the net cone tip resistance evaluated from in-situ cone penetration tests (CPT) (see Fig. 1).

Expert knowledge should preferably be incorporated as á-priori information in Bayesian analysis (Cao et al. 2016). In this paper, expert knowledge on the evaluation of s_u from the effective stresses was incorporated as prior information in the EMA (see Fig 1). All measurements were assumed to be log-normally distributed.

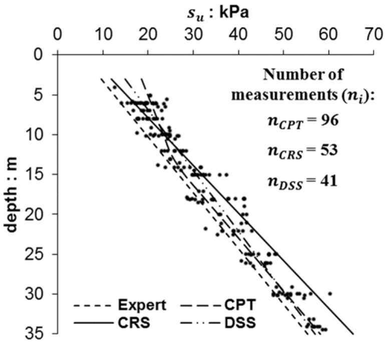

Figure 1. The s_u evaluated from CPT, CRS, DSS, and expert knowledge. CPT is evaluated every fourth meter based on the scale of fluctuation in the vertical direction (in Prästings et al. 2016).

3 THE EXTENDED MULTIVARIATE APPROACH

3.1 *Uncertainties*

The spatially averaged value of n_i measurements from D_i, i.e. $\bar{\delta}_{Di}$, is typically transformed to $\bar{s}_u$ according to:

$$\bar{s}_u \big| D_i = \bar{\delta}_i \times C_i \times e_i , \quad (1)$$

where C_i is the empirical transformation factor; e_i is the error term representing the uncertainties in $\bar{\delta}_i$ (both aleatory and epistemic). The term e_i has a mean value of 1 and variance, σ^2_{ei}. The total uncertainty, $COV_{\bar{s}_u|Di}$ (determined via σ^2_{ei}), can be estimated according to (Müller et al. 2014):

$$COV^2_{\bar{s}_u|D_i} \approx COV^2_{sp|D_i} \times (\Gamma^2 + \psi_{D_i}) + COV^2_{me|D_i} \times \frac{1}{n_i} + COV^2_{tr|D_i} + \xi , \quad (2)$$

where $COV_{sp|Di}$ is the uncertainty related to inherent variability; Γ^2 is a variance reduction factor; ψ_{Di} is a statistical uncertainty factor; $COV_{me|Di}$ is the uncertainty related to measurement error; $COV_{tr|Di}$ is the transformation uncertainty; and ξ is a statistical error term that is ignored in this study.

The $COV_{sp|Di}$ is evaluated from the variability in the performed measurements, i.e. COV_{Di}. This variability includes random measurement error and is typically referred to as *the nugget* (Baecher 1983). In this study systematic measurement error is excluded in $COV_{me|Di}$, and consequently, $COV_{sp|Di}$ is evaluated according to:

$$COV^2_{sp|D_i} = COV^2_{D_i} - COV^2_{me,D_i} . \quad (3)$$

For the CPT-measurements, *the nugget* is estimated from the autocorrelation function (ACF) to 15%, and the random part of $COV_{me|Di}$ is evaluated as $0.15 \times COV_{CPT}$ (in Prästings et al. 2016). On measurements from CRS- and DSS-tests, no ACFs could be constructed because collected soil samples were too sparse. In this case the random part of $COV_{me|Di}$ was estimated to be $0.30 \times COV_{Di}$. In comparison with typical values of $COV_{me|Di}$ found in literature (e.g., Phoon & Kulhawy 1999a), the $COV_{me|Di}$ used in this study is lower. However, the random part of $COV_{me|Di}$ should be put in relation to COV_{Di}, which in this case also is low. Baker (2005) states that random measurement errors account for approximately one-half of the variability evaluated from standard penetration tests (SPT). In relation to the estimated $0.30 \times COV_{Di}$ (for CRS and DSS), SPT is generally assumed to be more crude.

The random part of $COV_{me|Di}$ may be influenced by the degree to which data is de-trended in order to achieve stationarity (Jaksa et al. 1997). This study assumes the opposite, that the random part of $COV_{me|Di}$ is consistent for a particular investigation (see ch. 3).

The first step in the evaluation of COV_{Di} is to perform a Bayesian linear regression on δ_{Di} (eq. 4), and secondly, to evaluate COV_{Di} from the variance in δ_{Di} (eq. 5).

$$\bar{\delta}_{Di}(z) = \hat{a}_i + \hat{b}_i \times z \quad (4)$$

$$\sigma^2_{\delta} = \frac{\sum_{j=1}^{n_i} \left(\delta_{i,j} - \bar{\delta}_i\right) - \hat{b}^2 \sum_{j=1}^{n_i} \left(z_j - \bar{z}_i\right)^2}{n_i - 2} \quad (5)$$

In the equations above, $\hat{a}_i$ and $\hat{b}_i$ are the intercept and slope of the regression line, respectively, in the direction of depth, z.

The statistical uncertainty factor, ψ_{Di} is evaluated according to Tang (1980):

$$\psi_{Di} = \frac{(n_i - 1)}{(n_i - 3)} \left[\frac{1}{n_i} \left(1 + \frac{n_i}{n_i - 1} \times \frac{(z - \bar{z}_i)^2}{\sigma^2_{zi}} \right) \right], \quad (6)$$

where $\bar{z}_i$ and σ^2_{zi} are the mean and variance of the measurements, respectively, when each measurement point, n_i (Fig. 1), is defined by its value on z.

The transformation uncertainty was evaluated from measurements of CPT and CRS, with DSS as the benchmark test (guidance by Phoon & Kulhawy 1999b).

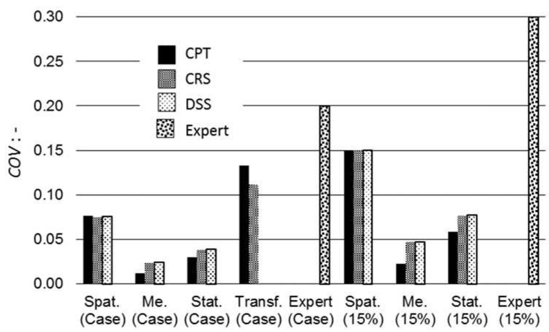

Figure 2. "*(Case)*" represents uncertainty values obtained in the case project, "*(15%)*" represents uncertainty values evaluated based on an assumed inherent variability ($COV_{sp|Di}$) of 15% (see ch. 4).

The s_u evaluated from CPT, CRS, DSS, and expert knowledge is presented in Fig. 1. The corresponding uncertainties are evaluated and presented in Fig. 2, in terms of *COV*.

3.2 *Variance reduction*

Variance reduction by the factor, Γ^2 (eq. 2), can potentially reduce the uncertainty related to inherent variability on account of spatial averaging by comparing the size of the failure domain with the horizontal and vertical scale of fluctuation in the soil. This paper aims at producing general estimates on η. Therefore, the effect of Γ^2 is not included directly in the main analysis, and is set to unity in eq. 1. A simple way of incorporating Γ^2 in the results is discussed in ch. 5.

3.4 *The multivariate analysis*

The Bayesian updating procedure that is used in the EMA was originally proposed by Ching et al. (2010) and is referred to as the multivariate analysis (MVA). The MVA calculates the Bayesian posteriors, hereafter denoted with double prime (´´), of the spatially averaged value, $\bar{s}_u$, and the associated $COV_{\bar{s}_u}$ based on the uncertainty in C_i (i.e. $COV_{tr|D_i}$). The essential difference between the EMA and the MVA is the procedure for estimating the total uncertainty in $\bar{s}_u | D_i$ (eq. 2).

The updating procedure in the MVA specifically reduces $COV_{tr|D_i}$ due to cross-validation between different site-investigation methods, D_i. As the EMA evaluates the total uncertainty, a framework is provide in which the epistemic uncertainties in δ_{D_i} are possible to reduce due to the number of measurements, n_i, taken before calculating the posterior $COV_{\bar{s}_u}''$.

4 FROM SITE SPECIFIC UNCERTAINTIES TO GENERAL ESTIMATES

In the following analysis, values of the conversion factor, η, is calibrated based on the posterior results from the EMA (i.e. $COV_{\bar{s}_u}''$). This paper aims at investigating the potential reduction of η and at exemplifying how to produce general estimates of η that can be used for a variety of geotechnical conditions. For this purpose, eq. 2 is applied using the following assumptions: (1) $COV_{me|D_i}$ represents a fraction (*percentage*) of $COV_{sp|D_i}$; the *percentage* is partly related to the investigation method and may be used for a variety of geotechnical conditions, (2) $COV_{tr|D_i}$ is related to the investigation method and to the accuracy in C_i, thus, it is also possible to use for a variety of geotechnical conditions, (3) $COV_{me|D_i}$ and ψ_{D_i} is dependent on the amount and location of the collected soil samples and/or in-situ measurements (n_i), and (4) $COV_{sp|D_i}$ is dependent on the inherent variability at the specific site. The evaluation of the different uncertainties in $COV_{\bar{s}_u|D_i}$ (used as input in the calculation of $COV_{\bar{s}_u}''$) was in the succeeding analysis based on the case value of $COV_{sp|D_i} = 7.5\%$ and an assumed value of $COV_{sp|D_i} = 15\%$, and following the assumptions (1) to (4) (see Fig 2). By varying $COV_{sp|D_i}$ (which is exemplified in this study), it is possible to produce a number of η-charts that can be used for a variety of geotechnical conditions (ch. 5.2 and 5.1). Furthermore, the number of in-situ and laboratory measurements (n_i), and the different combinations of site-investigation methods, were varied in order to investigate the potential reduction of η.

5 CALIBRATION OF η

5.1 *The Swedish national annex*

Design in accordance with the Swedish national annex to Eurocode 7 allows practicing engineers to adjust the otherwise fixed partial factor, γ_M, according to:

$$X_d = \frac{\gamma_M}{\eta} \times X_k \, , \tag{12}$$

where X_k is the characteristic value of soil strength properties and γ_M is a fixed value of 1.5 for s_u. In this paper, values of η were calibrated according to:

$$\frac{\gamma_M}{\eta} = e^{\alpha \times \beta_T \times COV} \, , \tag{13}$$

where α is a sensitivity factor; β_T is the target reliability index; and COV is selected as the posterior uncertainty ($COV_{\bar{s}_u}''$). The analysis in this study was performed with $\alpha = 0.8$ and $\beta_T = 4.7$ (in accordance with consequence class 2 in Eurocode 7).

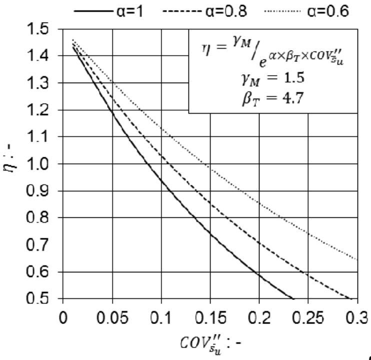

Figure 3. Relation between η and $COV_{\bar{s}_u}''$.

5.2 *The η in relation to amount of site-investigations – Case*

To investigate how statistical uncertainty and measurement error are reduced by increasing n_i, a simulated borehole with assumed measurements on depths 5, 10, 15, 20, 25 and 30 m ($n_i = 6$) was encoded for CRS and DSS, and on depths 2, 9, 13, 17, 21, 25, 29, 33 m ($n_i = 8$) for CPT. The $COV_{\bar{s}_u|D_i}$ was subsequently calculated for the scenario of 1 to 48 boreholes for each site-investigation method based on the case value of $COV_{sp|D_i} = 7.5\%$ and by following assumptions (1) to (4) as applied to Eq. 1. The results of $COV_{\bar{s}_u|D_i}$ evaluated for 1 to 48 boreholes was then used as input in the Bayesian updating procedure to also investigate the reduction of transformation uncertainty due to cross-validation between different combinations of site-investigation methods. As a final step, η was calibrated based on the posterior $COV_{\bar{s}_u}''$ (eq. 13) in relation to the number of boreholes, and to different combinations of site-investigation methods (Fig. 4). The updated information in Fig. 4, i.e. expert (prior) knowledge in combination with CRS, CPT and/or DSS is interpreted as one borehole of each site-investigation method and start from 1, 2 and 3 boreholes respectively.

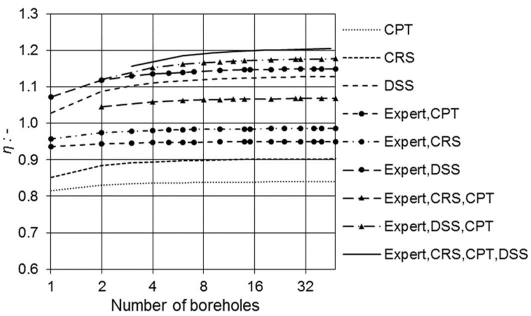

Figure 4. The η in relation to number of site-investigations and to different combinations of site-investigation methods performed in the case project.

The increase in η for single site-investigation methods, i.e. only increasing the amount of n_i for CPT, CRS or DSS, is due to the reduction of random measurement error and statistical uncertainty. The measurement error is directly proportional to n_i, while the statistical uncertainty accounts for the location of

n_i towards the depth (eq. 6). To conclude, the measurement error and statistical uncertainty approaches zero between 4 and 8 boreholes.

In studying the increase in η for different combinations of site-investigation methods, the potential gain in conducting parallel methods becomes obvious. In this case project, the most effective combination would be to combine expert knowledge with the measurements of DSS, mainly due to the fact that DSS is lacking transformation uncertainty.

The site-investigation performed in the case project included: $n_{i,CPT}=96$ (~12 boreholes), $n_{i,CRS}=53$ (~8 boreholes) and $n_{i,DSS}=41$ (~6 boreholes). Regarding the effectiveness of the site-investigation, performing additional CPTs would not further increase η. However, a greater value for site-investigation expenses would have been achieved by shifting some of the CPTs to 1 or 2 additional boreholes where samples for DSS tests were collected.

5.3 *The η in relation to D_i to amount of investigations – 15% inherent variability*

In order to exemplify how to produce general estimates of η the evaluation of the different uncertainties in $COV_{\bar{s}_u|D_i}$ (used as input in the calculation of $COV_{\bar{s}_u}''$) were calculated for $COV_{sp|D_i}=15\%$, and by following assumptions (1) to (4). Studying eq. 2, it is evident that the inherent variability governs the outcome of the calibrated values on η (ch. 3). Thus, different geological situations, i.e. different inherent variability, entail a nearly systematic displacement of η. By varying $COV_{sp|D_i}$ it is possible to produce a number of η-charts that can be used for a variety of geotechnical conditions. As an example, a supplementary analysis is presented in Fig. 5 (based on $COV_{sp|D_i}=15\%$). The results indicate smaller values of η than for the case (see ch. 5.2) due to the higher uncertainty included in the analysis, but also due to the fact that a slightly higher number of boreholes is needed for the measurement error and statistical uncertainty to approach unity (i.e. where η is approaching a constant value).

This study excludes variance reduction (Γ^2). For the case project the calibrated values of η in Fig. 4 should, therefore, be considered as conservative. However, if Figs. 4 and 5 are used for guidance on selecting η-values for another site, the $COV_{sp|D_i}$ for which an η-chart is produced should correspond to the expected or evaluated $COV_{sp|D_i}^2 \times \Gamma^2$ to also include variance reduction (i.e. $\Gamma^2<1$).

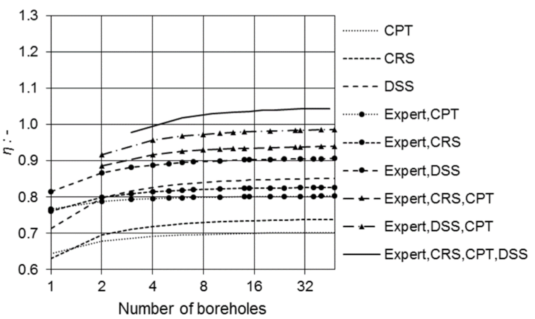

Figure 5. The η in relation to the number of boreholes and to different combinations of site-investigation methods performed in the case project, with $COV_{sp|D_i}=15\%$ (Fig. 3).

6 CONCLUDING REMARKS

The EMA can be of assistance in evaluating and planning for geotechnical investigations (Müller et al. 2014). The results presented in this paper indicate the importance of cross-validating information from several site-investigation methods. In the upcoming version of Eurocode 7 it is necessary to incorporate a methodology to adjust the safety factor applied to the soil-strength properties in a limit state (relative the fixed γ_M), due to the extent and quality of geotechnical site-investigations. In a simplified reliability-based design (like the partial factor method), the procedure presented in this paper can be used for providing guidance on selecting such differentiating values (η-values) that have a connection to the actual uncertainty in the evaluation of soil-strength properties and to a failure probability designated by the target reliability index.

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