Direct methods for prediction of shallow foundation settlements in sand and numerical modeling

Les méthodes directes pour la prédiction des tassements des fondations peu profondes sur le sable et la modélisation numérique

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ABSTRACT: A direct method for calculating load – settlement curves for shallow foundations is presented. When using site specific a posteriori correlations between two appropriately chosen applied pressures and in-situ penetration test results, a very good fit is obtained with measurements performed during load tests on five square footings ranging in dimensions from 1m to 3m, at the Texas A&M University. Load – settlement curves for the five footings obtained by using two other direct methods are compared with the first method, as well as results of numerical modeling of the load tests by using an advanced constitutive relationship, previously published for one of the five footings. The two curves from previous direct methods show acceptable deviations from measured data, whereas the chosen type of numerical modeling relies too much on measured settlements.

RÉSUMÉ : Une méthode directe pour calculer les courbes efforts – déformations pour les fondations peu profondes est présentée. Lorsqu'on utilise des corrélations a posteriori, spécifiques pour le chantier, entre deux pressions appliquées, choisies de manière appropriée et les résultats des essais de pénétration in situ, on obtient une très bonne correspondance aux mesures effectuées pendent les essais de charge sur cinq semelles carrées de dimensions allant de 1m à 3m, à l'Université A&M Texas. Les courbes efforts – déformations pour les cinq semelles obtenues en utilisant deux autres méthodes directes sont comparées par rapport à la première méthode, ainsi que les résultats da la modélisation numérique des essais de charge en utilisant une relation constitutive avancée, publiés auparavant pour l'une des cinq semelles. Les deux courbes des méthodes directes antérieures montrent des écarts acceptables par rapport aux données mesurées, alors que le type choisi de modélisation numérique repose trop sur les tassements mesurés.

KEYWORDS: shallow foundations, settlement, bearing capacity, load tests, direct methods, numerical modeling.

1 INTRODUCTION

Settlements of shallow foundations have been the interest of many geotechnical engineers and researchers. There are numerous methods for settlement calculation available, some based on the theory of elasticity, some based on correlations with in-situ penetration test results, some semi-empirical, and numerical modelling with different constitutive relationships is also broadly used.

The Mayne and Poulos (1999) method is presented as the basis for the proposed direct method. Two additional direct methods are presented, Akbas and Kulhawy (2009a) L_1-L_2 method and Mayne et al. (2012) method. The first of the two direct methods relies on the determination of the failure load Q_{L2}, which corresponds well to the Vesić (1975) bearing capacity multiplied by the footing area (Akbas and Kulhawy 2009b), and defining the normalized load Q/Q_{L2} as a hyperbolic function of the ratio of settlement s and the footing width B, expressed in percent. This ratio was used by Fellenius and Altaee (1994) to show that the diagram of applied pressure vs. s/B gives coinciding curves for square footings of different widths on sand.

The Mayne et al. (2012) method uses a single parameter, the CPT tip resistance q_c, to express the applied pressure normalized by q_c as a linear function of the square root of s/B.

The proposed direct method requires the measurements of the shear wave velocity with the depth of the foundation soil, and two known reference points on the load – settlement curve, might it be the loads for $s/B = 0.1$ and $s/B = 0.01$ through correlations with penetration test results, or the loads Q_{L2} and Q_{L1}. Thus, the proposed method can accommodate both mentioned direct methods.

Load tests performed on five square footings ranging in size from 1m to 3m at the Texas A&M University are used herein to compare measured load – settlement data with the two direct methods, the proposed direct method, and numerical modeling with an advanced constitutive relationship. The results of these comparisons are presented in the Conclusion.

2 SOME METHODS TO DETERMINE SETTLEMENTS

2 .1 *The Mayne and Poulos and Mayne methods*

The Mayne and Poulos (1999) expression for settlements on a Gibson type soil profile (Gibson 1967) is

$$s = \frac{p\,d\,I_G I_F I_E \left(1-\nu^2\right)}{E_b} \tag{1}$$

where p is the applied pressure, d is the equivalent diameter of a circular footing having the same area A as the rectangular footing, i.e. $d = 2(A/\pi)^{0.5}$, ν is the constant Poisson ratio and E_b Young's modulus of the foundation soil at the footing base. In the Gibson type soil profile, the Young's modulus E linearly increases with the depth z of foundation soil, so that $E = E_b + k_E z$, where k_E is the increase in foundation soil stiffness per unit depth.

I_G is the displacement influence factor. Its variation with the normalized Gibson modulus $\beta = E_b/(k_E d)$ is given in Mayne and Poulos (1999) in form of curves for various values of h/d, where h is the distance between the footing base and the incompressible layer. This variation of I_G is approximated as (Lukić Kristić et al. 2017)

$$I_G \approx \frac{1.6(h/d)}{\left(1+\frac{0.6}{\beta^{0.8}}\right)\left[1+1.6(h/d)\right]} \tag{2}$$

The effect of the foundation rigidity and the linearly increasing Young's modulus are taken into account by the rigidity factor which is given by

$$I_F = \frac{\pi}{4} + \frac{1}{\frac{1}{1-\frac{\pi}{4}} + 10\frac{E_f}{E_b + \frac{1}{2}k_E d}\left(\frac{2t}{d}\right)^3} \tag{3}$$

where E_f is Young's modulus of the foundation material, and t is the footing thickness

The depth of embedment factor is given by

$$I_E = 1 - \frac{1}{3.5\exp(1.22\nu - 0.4)\left(\frac{d}{D_f}+1.6\right)} \tag{4}$$

where D_f is the depth of embedment of the footing.

Mayne (2000, 2007) used a special case of the Fahey and Carter (1993) modified hyperbola for the nonlinear stress-strain relationship in equation (1). Based on laboratory shear tests (triaxial and torsional), Fahey and Carter (1993) introduced the following nonlinear relationship

$$\frac{G}{G_0} = 1 - f\left(\frac{\tau}{\tau_f}\right)^g \text{ (a)} \quad \text{or} \quad \frac{E}{E_0} = 1 - f\left(\frac{q}{q_f}\right)^g \text{ (b)} \tag{5}$$

where G is the shear modulus, G_0 the maximum shear modulus at very small strains, τ the shear stress, τ_f the shear strength, and f and g are parameters to be determined from laboratory tests; q is the deviatoric stress, q_f the deviatoric stress at failure, and E_0 the maximum Young modulus at very small strains. For $f = 1$ and $g = 1$, equation (5a) becomes the standard Kondner's hyperbola (Kondner 1963). G and E are related through $G = E/[2(1 + \nu)]$ and G_0 can be calculated from the shear wave velocity v_s by $G_0 = \rho v_s^2$ where ρ is the soil density.

Mayne (2000, 2007) uses equation (5b) with $f = 1$ and $g = 0.3$, and introduces it in equation (1). Furthermore, stating that the ratio of deviatoric stress and deviatoric stress at failure in equation (5b), which is analogous to the ratio $\tau/\tau_f = 1/FS$ (FS is the factor of safety), he replaces q/q_f with p/p_f, where p_f is the foundation soil bearing capacity. The use of this expression showed a good matching with values measured in a load test on a 3 m square footing on sand at the Texas A&M University.

2.2 *Two direct methods*

The concept of plotting the pressure vs. s/B was suggested by Fellenius and Altaee (1994). They showed that such plots coincide for four square footings of dimensions ranging from 0.25m to 1.0m on sand, so that for a given pressure, the settlement is proportional to the footing width. The report on the Texas A&M test site was written by Briaud and Gibbens (1997). It includes five load tests on square footings of dimensions ranging from 1m to 3m. The scale effect was also studied and it was shown that there is no scale effect for the five tested footings, which can be seen "when plotting the load – settlement curves as pressure vs. settlement over width [or equivalent diameter d] curves, [where] all such curves vanish to one curve and the scale effect disappears." Briaud and Gibbens (1999) state that "these data refute the idea of a scale effect for footings on sand up to an s/B of 0.1" where s is the settlement and B is the footing width. Therefore, it is convenient to use the ratio s/B, and several authors (e.g. Amar et al. 1998, Cerato and Lutenegger 2006) suggest that the load required for $s/B = 0.1$ can be taken as the failure load. The load required for $s/B = 0.01$ can roughly be taken as the "allowable" load (Briaud and Gibbens 1997, Akbas and Kulhawy 2009a)

Akbas and Kulhawy (2009a) developed a direct method for calculating settlements of shallow foundations on different sands, based on the L_1-L_2 method, according to which the load – settlement curve can be described by the first linear-elastic part ending at the load Q_{L1}, the second curved part, and the third linear part starting at the load Q_{L2}. Using a database with 167 full-scale field load tests at 37 locations (sand, silty sand, sand and gravel) on footings with $B \geq 0.25$m (most of them have $B \leq 1$m), they fitted a hyperbolic function of the form

$$\frac{Q}{Q_{L2}} = \frac{s/B}{0.69(s/B)+1.68} \tag{6}$$

where Q is the load, and s/B is in percent. Q_{L2} is determined as the bearing capacity calculated by the Vesić (1975) method, and presented in Akbas and Kulhawy (2009b). These authors suggest that the failure of the foundation soil occurs at $s/B = 0.0539$, less than 0.1, and that the elastic limit load Q_{L1} is reached at $s/B = 0.0023$.

The ratio Q_{L1}/Q_{L2} is given by Akbas and Kulhawy (2009a) as 0.13 from the normalized curve given by equation (6), and as a mean value of 0.15 from the database. These authors also give tabulated values of Q_{L1} and Q_{L2} for footings from the database, which includes the five footings from the Texas A&M University test site.

Mayne et al. (2012) used a database of 31 footings on 13 different sands, with CPT measurements, and arrived at the following correlation

$$p = 0.585 q_c \sqrt{s/B} \tag{7}$$

where q_c is the CPT tip resistance.

The coefficient of determination for expression (7) $r^2 = 0.933$. Mayne et al. (2012) tabulated the values of q_c for each test site, including the one at Texas A&M University.

3 PROPOSED DIRECT METHOD

The proposed direct method for the calculation of shallow foundation settlements on granular and stiff fine grained soils is derived from Mayne (2000, 2007), with two alterations (Lukić Kristić et al. 2017). First, both Fahey and Carter (1993) parameters f and g are used, and second, the soil bearing capacity p_f is replaced by $p_{0.1}$, the pressure at $s/d = 0.1$. Thus,

$$s = \frac{d\,p}{E_0\left(1 - f\left(\frac{p}{p_{0.1}}\right)^g\right)} I_G I_F I_E \left(1-\nu^2\right) \tag{8}$$

Briaud and Gibbens (1997) offer two types of correlations between penetration test results, Standard Penetration Test (SPT) and cone Penetration Test (CPT), and the two pressures, $p_{0.1}$ at $s/B = 0.1$, and $p_{0.01}$ at $s/B = 0.01$ from results from five load tests at the Texas A&M site. Briaud and Gibbens (1999) offer three types of correlations between penetration test results (slightly different SPT and CPT from 1997, and pressuremeter test) from the same site. It has to be noted that all these correlations are site specific and obtained after the load and

settlement measurements had been taken. Lukić Kristić et al. (2017) calculated settlements according to equation (8) using four types of correlations (two SPT and two CPT) and compared them with measured settlements for the five A&M Texas footings. They showed that all four types of correlations give satisfactory matching with measured settlements, and that the following correlations with SPT results (Briaud and Gibbens 1997) might be distinguished as the best for all five tested footings:

$$p_{0.1} = \frac{N}{12} \text{ with } N \text{ in blows/0.3m and } p_{0.1} \text{ in MPa} \tag{9}$$

$$p_{0.01} = \frac{N}{36} \text{ with } N \text{ in blows/0.3m and } p_{0.01} \text{ in MPa} \tag{10}$$

With two points known on the pressure – settlement curve according to equations (9) and (10), it is possible to calculate parameters f and g explicitly from equation (8). It follows that

$$f = 1 - \frac{p_{0.1}}{0.1E_0} I_G I_F I_E \left(1 - \nu^2\right) \tag{11}$$

$$g = \frac{\log\left[\frac{1}{f}\left(1 - \frac{p_{0.01}}{0.01E_0} I_G I_F I_E \left(1 - \nu^2\right)\right)\right]}{\log \frac{p_{0.01}}{p_{0.1}}} \tag{12}$$

4 NUMERICAL AND DIRECT VERIFICATIONS

Load – settlement curves from measurements at the Texas A&M University test site (Briaud and Gibbens 1997) were used for comparison between measured values and (a) numerically calculated ones, and (b) values from three direct methods given by equation (6), equation (7) and equations (8) through (12) respectively. Five square footings, with dimensions of 1m, 1.5m and 2.5m, and two footings of 3m (North and South) were loaded to the settlement of 15cm each. The footings were embedded 0.76m into the 11m thick layer of medium dense, fairly uniform, silty fine silica sand (Briaud and Gibbens 1997, 1999), underlain by stiff clay. The water table is 4.9m below the soil surface. Load increments were imposed in 30min time intervals, and some loads were kept constant for 24h. The sand exhibited creep under constant loads. Measured load settlement curves for the five footings are shown in Figures 1a through 1d along with calculations from numerical modeling and those from the three direct methods. Vertical lines connecting measured data in Figure 1 denote creep.

Extensive field and laboratory testing took place for the test site foundation soil. It was, thus, possible to use numerical modeling with sophisticated constitutive models, such as Hardening-Soil-Small (Benz 2007) to simulate load tests on the five footings.

Benz (2007) assumed that the foundation soil was overconsolidated in order to match calculated results with measured data, so he used preloading of the foundation soil when simulating the load – settlement curve on the 3m North footing. He calibrated the model in order to determine the required input parameters by using results of drained triaxial tests on reconstructed samples under three different confining pressures, of the resonant column test, and also of in-situ cross-hole seismic test (Benz et al. 2003). Benz (2007) published all parameters used for the HSSmall model in simulating the behavior of the 3m North footing under load.

The Authors repeated the calculation of settlements with the HSSmall model and given parameters, with Plaxis 2D software (Brinkgreve 2002), to make sure the proper preloading of the foundation soil was used. When they matched the Benz load settlement curve, including unloading and reloading (with the preloading of 125 kPa), they repeated the calculations for the other four A&M Texas footings. These results are presented in Figures 1a through 1d as Plaxis 2D curves.

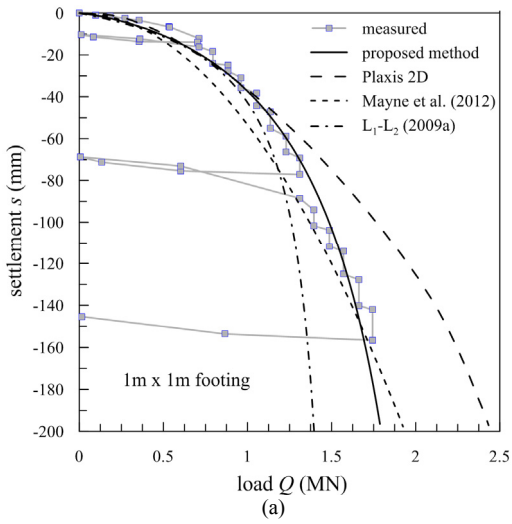
The "proposed method" curves in Figure 1 are those calculated by using equations (8) through (12). An average value of $N = 18.8$ blows/0.3m was used, $E_0 = 230.4$ MPa, $\nu = 0.2$, and $1/\beta = 0$.

Mayne et al. (2012) curves in Figure 1 are obtained from expression (7) with $q_c = 7.5$ MPa.

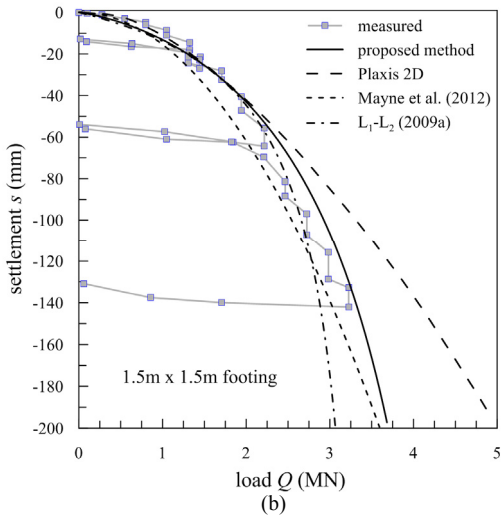
For the L_1-L_2 method in Figure 1, values of Q_{L2} were taken from Akbas and Kulhawy (2009a) and are presented in Table 1. The column denoted by (e) in this Table refers to the footing 3m South, whereas only curves for the footing 3m North are presented in Figure 1d. The reason for this is that the corresponding two curves for the footings 3m North and South differ negligibly.

Table 1. Values used in equation (6) for L_1-L_2 curves in Figure 1 (Akbas and Kulhawy 2009a).

	(a)	(b)	(c)	(d)	(e)
B (m)	0.99	1.49	2.49	3.0	3.02
Q_{L2} (MN)	1.079	2.5	7.271	10.0	10.295



(a)



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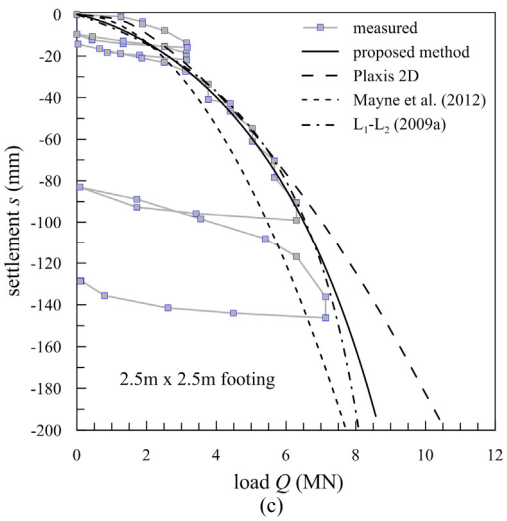
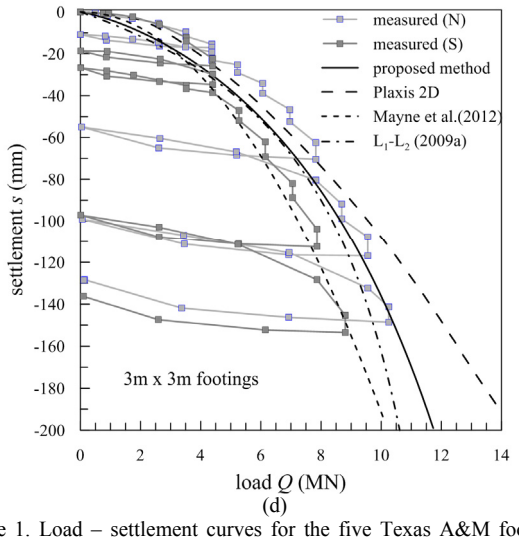


(c)



(d)

Figure 1. Load – settlement curves for the five Texas A&M footings (measured data from Briaud and Gibbens 1997; L_1-L_2 method from Akbas and Kulhawy 2009a).

5 CONCLUSION

Five load – settlement curves from measurements at the Texas A&M University test site (Briaud and Gibbens 1997) were used for comparison between measured values and four types of calculations: (a) numerical modeling, (b) the Mayne et al. (2012) correlation given by equation (7), (c) the Akbas and Kulhawy (2009a) L_1-L_2 method given by equation (6), and (d) the proposed direct method given by equations (8) through (12).

The proposed direct method gives the best matching, because the two required points on the load – settlement curve, that is the pressure $p_{0.1}$ for $s/B = 0.1$ and $p_{0.01}$ for $s/B = 0.01$ are site specific, and determined from correlations obtained after measured data was known. However, this method has the advantage that it can accommodate any two points on the load – settlement curve (such as Q_{L1} and Q_{L2}), which was done for both other direct methods, and very similar results to those calculated by the original methods given by equations (6) and (7) respectively, were obtained. This method also requires the profile of shear wave velocities in the foundation soil.

The Mayne et al. (2012) method is the simplest, and it requires only one parameter, the CPT tip resistance. It is certainly the best start for the prediction of shallow foundation settlements.

The Akbas and Kulhawy (2009a) L_1-L_2 method requires the bearing capacity of the soil to be determined and the authors suggest the Vesić (1975) method (Akbas and Kulhawy 2009b). They also suggest that soil failure occurs at loads required to produce $s/B = 0.0539$, which is less than $s/B = 0.1$, as suggested by other authors.

The previous two direct methods give acceptable deviations from measured data. This is also the case with numerical modeling, but the path to the results is strenuous. First, a set of numerous input parameters has to be determined, preferably by triaxial and resonant column tests, as well as the profile of shear wave velocities in the foundation soil. Second, and more problematic, is the issue of overconsolidation of the foundation soil, which can hardly be known prior to load tests.

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