The analysis of the foundations vibrations on wave models in general case of dynamic loading

L'analyse des fluctuations des fondations sur la base des modèles ondulatoires dans le cas général de la charge dynamique

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ABSTRACT: The article describes analytical solutions for the analysis of the dynamic behavior of foundations from industrial, transportation, construction and other sources. The calculated dependence for the amplitude determination is provided for the foundation vibration constituents from force action and kinematic excitation. They are obtained on the basis of elastic half-space model. The dependence includes only general parameter expressions of hardness and damping of the base, actual load and general parameters of the foundation. They are convenient to use for the foundations of different constructions including special types of foundations for machines with dynamic loads, pile foundations and foundations on reinforced base. The usage of the suggested dependence allows simplifying the forecasting of foundation vibrations in design stage, facilitates the choice and optimization of the construction solution and dimensions of foundations used under conditions of dynamic load based on the set vibration level, actual load and dynamic mode of the area.

RÉSUMÉ : Cet article décrit des solutions analytiques pour l'analyse du comportement dynamique des fondations des installations de l'industrie, le transport, la construction et al. En addition, l'article présente les relations de calcul pour déterminer les amplitudes de toutes les composantes des fluctuations de la fondation, incluant l'action de force et l'excitation cinématique. Elles ont été dérivées du modèle d'une demi-espace élastique. Les relations comprennent seulement les valeurs des paramètres de rigidité et d'amortissement, les charges actuelles et les principaux paramètres de la fondation. Elles sont pratiquees à utiliser pour des fondations des différents constructions, y compris des types de fondations pour des machines avec les charges dynamiques, des fondations sur pieux et des fondations sur un basement intensifié. D'utiliser les relations présentées permet de simplifier la prévision des fluctuations des fondations au stade de l'étude du projet, facilite la sélection et l'optimisation de la conception de la structure et des mesures des fondations exploitées dans les conditions de la charge dynamique sur la base du degré des fluctuations déterminé, des charges actuelles et du régime dynamique du site.

KEYWORDS: vibration amplitudes, elastic half-space model, force and kinematic excitation.

1 INTRODUCTION

As we have mentioned this before, one of the main models that describes rather closely the dynamic behavior of foundations from industrial, transport, construction and other sources is elastic semi space model (Nuzhdin 2012). As different from much widely used various modifications of Winkler model (local elastic deformations of basement), its properties pertain wave characteristics. The foundation is regarded as a body, which lies against the semi-space and is subject to wave movements from the same body or to the waves that come to it from other dynamic sources. The waves scatter, as they travel a distance. Mechanical properties of soil media are described by elastic compression E, shift G modules and by Poisson coefficient ν.

2 GENERAL EXAMPLE OF DYNAMIC LOADING

In a general example, foundation dynamic movements may be defined using the principle of superposition, as a sum of oscillations coming from both force and kinematic excitation (Nuzhdin 1995)

$$a = \sum_{i=1}^{n} a_i \sin\left(\omega t + \varphi_i\right) \quad , \qquad (1)$$

where a – is the oscillation amplitude of vertical, horizontal or shift-and-rotation constituents coming from force (i = 1) and kinematic (i > 1) excitation; n – is the number of assumed foundation oscillation sources (or other sources); ω – is the circular oscillation frequency; φ_i – are the accidental phase shifts.

In the first approximation, given the accuracy level of determining initial and predicted task parameters of the foundation oscillation under complex dynamic loading conditions and exposed to a number of the sources, it is obvious that a simple summing up the of oscillation amplitudes may be performed excluding phase shift. Although, it is known that the result received may prove sufficiently higher, especially in case of grouped machines' location producing dynamic loads, with present accidental short-time influences and in other cases. Therefore, in order to ultimately analyse the foundation's dynamic state, it is recommended to take into account effective dynamic loads' character, their combination, performance peculiarities of surrounding oscillation sources, etc. This may be fulfilled in accordance with the initial data present in the technical task, based on materials of special researches or on statistical analysis results and accidental phase shift predictions, for example, using (Tseitlin and Guseva 1979).

The foundation oscillation constituents' amplitudes are determined by considering separately each kind of movements. Analysis of equations, which correspond to each kind of oscillations as applied to various kinds of foundations (in a natural basement, pier-type, pile-type, considering their embedment depths, etc.) was repeatedly conducted by different researchers. While the most of the analytical formulas acquired are specific, they are based on different theoretical suppositions and include initial parameters, which are typical for a certain computation model, thus requiring specific determinations. A combined application of these formulas for a complex computation of the dynamic behavior is practically impossible.

Application of more "universal" computation formulas is desirable for comparing the results acquired from foundation computation, when changing its design, optimizing design solution or choosing a rational variant of reinforcement. It may be reasonable, therefore, to apply computational dependencies that include only general equations of hardness and damping parameters in "foundation-to-basement" system, oscillation process characteristics and foundation design.

When considering the foundation oscillations, we assume as known the resultant dynamic loads applied to it: P_z, P_x and M – vertical, horizontal forces and moment, correspondingly, as well, as vertical z_s, horizontal x_s and rotational φ_s components of the soil surface oscillations from kinematic excitation at the point of the foundation location. The foundation has m, $Q = mg$, θ – the mass, weight and mass moment of inertia relative to its gravity centre axis, which is perpendicular to the plane of the oscillations considered. Basement of the foundation has the following stiffness K_{zz}, K_{xx}, $K_{x\varphi}$, $K_{\varphi x}$, $K_{\varphi\varphi}$ and damping C_{zz}, C_{xx}, $C_{x\varphi}$, $C_{\varphi x}$, $C_{\varphi\varphi}$ parameters, correspondingly characterized by vertical components' parameters at vertical movement, horizontal ones – at shift and rotation and rotational – at shift and rotation.

3 OSCILLATIONS FROM FORCE EFFECT

3.1 *Vertical foundation oscillations*

Amplitudes of vertical translational foundation oscillations resulting from force effect may be determined with the following equation:

$$m\ddot{z} + C_{zz}\dot{z} + K_{zz}z = P_z \quad .$$

The amplitudes of vertical translational foundation oscillation resulting from the force effect ($i = 1$) may be defined a known expression that includes only common expressions of stiffness and damping parameters in vertical system oscillations

$$a_{z.1} = \frac{P_z}{K_{zz}} \cdot \frac{1}{\sqrt{(1 - \frac{m\omega^2}{K_{zz}})^2 + (\frac{C_{zz}\omega}{K_{zz}})^2}} \quad . \tag{2}$$

3.2 *Horizontal foundations oscillations*

On analogy to the discussed equation of the horizontal component in the foundation oscillations

$$m\ddot{x} + C_{xx}\dot{x} + K_{xx}x = P_x \quad .$$

There may be acquired an equation for determining amplitudes of horizontal translational foundation oscillations from the force effect ($i = 1$)

$$a_{x.1} = \frac{P_x}{K_{xx}} \cdot \frac{1}{\sqrt{(1 - \frac{m\omega^2}{K_{xx}})^2 + (\frac{C_{xx}\omega}{K_{xx}})^2}} \quad . \tag{3}$$

3.3 *Shift-and-rotation foundation oscillations*

To acquire amplitudes of shift-and-rotation foundation oscillation resulting from force effect, it is necessary to consider generalized horizontal rotation oscillations, which set of equations is as follows:

$$\begin{cases} \theta\ddot{\varphi} + C_{\varphi\varphi}\dot{\varphi} - C_{x\varphi}(\dot{x} - h\dot{\varphi}) + \bar{K}_{\varphi\varphi}\varphi - K_{x\varphi}(x - h\varphi) = M \\ m\ddot{x} + C_{xx}(\dot{x} - h\dot{\varphi}) + K_{xx}(x - h\varphi) = P_x \end{cases}$$

where the rotation component of basement hardness including rotational moment resulting from the foundation's own weight is

$$\bar{K}_{\varphi\varphi} = K_{\varphi\varphi} - mgh \quad .$$

Linear system determinant

$$\Delta = (-m\omega^2 + K_{xx} + i\omega C_{xx})(-\theta\omega^2 + hK_{x\varphi} + \bar{K}_{\varphi\varphi} + i\omega hC_{x\varphi}) - (hK_{xx} + i\omega hC_{xx})(K_{x\varphi} + i\omega C_{x\varphi}) = D_1 + iD_2$$

$$D_1 = m\theta\omega^4 - (\theta K_{xx} + mhK_{x\varphi} + m\bar{K}_{\varphi\varphi} + C_{xx}C_{\varphi\varphi})\omega^2 + K_{xx}\bar{K}_{\varphi\varphi} \quad ,$$

$$D_2 = -(\theta C_{xx} + mhC_{x\varphi} + mC_{\varphi\varphi})\omega^3 + (K_{xx}C_{\varphi\varphi} + \bar{K}_{\varphi\varphi}C_{xx})\omega \quad .$$

The expression of shift-and-rotation oscillation amplitudes at the foundation upper edge, which result from force effect ($i = 1$) may be determined as:

$$a_{x\varphi.1} = \sqrt{\frac{B_1^2 + B_2^2}{D_1^2 + D_2^2}} \quad . \tag{4}$$

$$B_1 = [\bar{K}_{\varphi\varphi}(h + h_1) - m\omega^2 h_1]M + [\bar{K}_{\varphi\varphi} + K_{x\varphi}(h + h_1) - \theta\omega^2]P_x$$

$$B_2 = C_{xx}(h + h_1)\omega M + [C_{\varphi\varphi} + C_{x\varphi}(h + h_1)]\omega P_x$$

h_1 – is the distance from the foundation upper edge point to the foundation gravity centre; h – is the distance from the foundation gravity centre to the foundation foot.

3.4 *Vertical foundation oscillations with account of its rotation in vertical plane*

Strictly speaking, when determining the amplitudes of vertical foundation oscillations, it is reasonable to take into account its additional movements from rotational component of the oscillations. Consequently, in order to determine the amplitudes of the foundation vertical oscillations from force effect ($i = 1$), taking into account its rotation in the vertical plane, we arrive to:

$$a_{z\varphi.1} = \sqrt{\frac{B_3^2 + B_4^2}{D_1^2 + D_2^2}} \quad , \tag{5}$$

$$B_3 = [K_{xx}\bar{K}_{\varphi\varphi} - K_{xx}\theta\omega^2 - K_{x\varphi}m\omega^2 h - \bar{K}_{\varphi\varphi}m\omega^2 + m\theta\omega^4 - C_{xx}C_{\varphi\varphi}\omega^2]a_{z.1} + [K_{xx} - m\omega^2]\frac{b}{2}M + K_{x\varphi}\frac{b}{2}P_x$$

$$B_4 = [K_{xx}C_{\varphi\varphi} + \bar{K}_{\varphi\varphi}C_{xx} - C_{xx}\theta\omega^2 - C_{x\varphi}m\omega^2 h - $$
$$-C_{\varphi\varphi}m\omega^2]\omega a_{z.1} + C_{xx}\omega\frac{b}{2}M + C_{x\varphi}\omega\frac{b}{2}P_x$$

where b – is the foundation's side length, along which the oscillations are considered.

4 OSCILLATIONS FROM KINEMATIC EXCITATION

When analyzing oscillations from kinematic excitation (from waves coming though the soil, for example, from dynamically loaded foundations) with recipient foundation dimensions sufficiently smaller than length of waves distributing through the soil, a quite accurate allowance may be assumed in that it will commit only vertical and horizontal translational movements (Nuzhdin and Zabylin 1991).

4.1 *Foundation vertical oscillations*

Let us consider equation of vertical component in the foundation oscillations from kinematic excitation

$$m\ddot{z} + C_{zz}(\dot{z} - \dot{z}_s) + K_{zz}(z - z_s) = 0 \quad .$$

The expression for determining the amplitudes of the vertical component in the foundation oscillations at kinematic excitation from one ($i = k$) source is

$$a_{z.k} = a_{zs}(k)\sqrt{\frac{1 + \left(\frac{C_{zz}\omega}{K_{zz}}\right)}{\left(1 - \frac{m\omega^2}{K_{zz}}\right)^2 + \left(\frac{C_{zz}\omega}{K_{zz}}\right)^2}} \quad . \quad (6)$$

4.2 *Foundation horizontal oscillations*

Disregard of the foundation rotation in vertical plane, when considering horizontal translational oscillations there appears:

$$m\ddot{x} + C_{xx}(\dot{x} - \dot{x}_s) + K_{xx}(x - x_s) = 0 \quad .$$

A similar dependency may be acquired for the amplitudes of the horizontal component in the foundation oscillations at the kinematic excitation from one ($i = k$) source:

$$a_{x.k} = a_{xs}(k)\sqrt{\frac{1 + \left(\frac{C_{xx}\omega}{K_{xx}}\right)}{\left(1 - \frac{m\omega^2}{K_{xx}}\right)^2 + \left(\frac{C_{xx}\omega}{K_{xx}}\right)^2}} \quad . \quad (7)$$

4.3 *Foundation shift-and-rotation oscillations*

In a general example, the foundation oscillations resulting from elastic soil waves that come from other dynamic sources (as in the kinematic excitation example) have a complicated and complex character. Besides the vertical and horizontal components, almost in any case, there may be distinguished a rotational component (Nuzhdin 2011). Equations of generalized shift-and-rotation oscillations resulting in the foundation from the kinematic excitation have the following form:

$$\begin{cases} m\ddot{x} + C_{xx}[\dot{x} - \dot{\bar{x}}_0 - h(\dot{\varphi} - \dot{\varphi}_s)] + \\ +K_{xx}[x - \bar{x}_0 - h(\varphi - \varphi_s)] = 0 \\ \Theta\ddot{\varphi} + C_{\varphi\varphi}(\dot{\varphi} - \dot{\varphi}_s) - C_{x\varphi}[\dot{x} - \dot{\bar{x}}_0 - h(\dot{\varphi} - \dot{\varphi}_s)] + \\ +\bar{K}_{\varphi\varphi}(\varphi - \varphi_s) - K_{x\varphi}[x - \bar{x}_0 - h(\varphi - \varphi_s)] = 0 \end{cases} ,$$

where the horizontal transferred shift of the foundation gravity centre is

$$\bar{x}_0 = x_s + h\varphi_s \quad .$$

Two cases of phase shift between x_s and φ_s consider equal to 0 and π. Determinant of the system

$$\Delta = \left(-m\omega^2 + K_{xx} + i\omega C_{xx}\right)\cdot\begin{pmatrix} -\Theta\omega^2 + hK_{x\varphi} + \\ \bar{K}_{\varphi\varphi} + i\omega C_{\varphi\varphi} + i\omega h C_{x\varphi} \end{pmatrix} -$$
$$-\left(hK_{xx} + i\omega h C_{xx}\right)\left(K_{x\varphi} + i\omega C_{x\varphi}\right) = D_3 + iD_4$$

$$D_3 = m\Theta\omega^4 - \left(\Theta K_{xx} + mhK_{x\varphi} + m\bar{K}_{\varphi\varphi} + C_{xx}C_{\varphi\varphi}\right)\times$$
$$\times\omega^2 + K_{xx}\bar{K}_{\varphi\varphi}$$

$$D_4 = -\left(\Theta C_{xx} + mhC_{x\varphi} + mC_{\varphi\varphi}\right)\omega^3 +$$
$$+\left(K_{xx}C_{\varphi\varphi} + \bar{K}_{\varphi\varphi}C_{xx}\right)\omega \quad .$$

The expression for determining the amplitudes of the foundation shift-and-rotation oscillations resulting from one ($i = k$) source excitations has the following form:

$$a_{x\varphi.k} = \sqrt{\frac{B_5^2 + B_6^2}{D_3^2 + D_4^2}} \quad . \quad (8)$$

$$B_5 = (K_{xx}\bar{K}_{\varphi\varphi} - \Theta\omega^2 K_{xx} - \omega^2 C_{xx}C_{\varphi\varphi})a_{xs} +$$
$$+(m\omega^2 K_{x\varphi} + \omega^2 C_{xx}C_{x\varphi})h_1 a_{xs} \pm (K_{xx}\bar{K}_{\varphi\varphi} -$$
$$-\omega^2 C_{xx}C_{\varphi\varphi})\cdot(h + h_1)a_{\varphi s} \mp m\omega^2\bar{K}_{\varphi\varphi}h_1 A_{\varphi s}$$

$$B_6 = (\omega K_{xx}C_{\varphi\varphi} + \omega\bar{K}_{\varphi\varphi}C_{xx} - \Theta\omega^3 C_{xx})a_{xs} +$$
$$+m\omega^3 C_{x\varphi}h_1 a_{xs} \pm (\omega K_{xx}C_{\varphi\varphi} + \omega\bar{K}_{\varphi\varphi}C_{xx})\times$$
$$\times(h + h_1)a_{\varphi s} \mp m\omega^3 C_{\varphi\varphi}h_1 a_{\varphi s}$$

4.4 *Foundation vertical oscillations with account of its rotation in vertical plane*

When determining the amplitudes of the foundation vertical oscillations from kinematic excitation, taking account of its additional movement from rotation in the vertical plane is recommended as well. the expression for determining the foundation vertical oscillations amplitudes with account of its rotation at the kinematic excitation from one ($i = k$) source

$$a_{z\varphi.k} = \sqrt{\frac{B_7^2 + B_8^2}{D_3^2 + D_4^2}} \quad . \tag{9}$$

$$B_7 = (m\Theta\omega^4 - \Theta\omega^2 K_{xx})a_{z.k} + (K_{xx}\bar{K}_{\varphi\varphi} -$$
$$-m\omega^2\bar{K}_{\varphi\varphi} - \omega^2 C_{xx}C_{\varphi\varphi})\left(a_{z.k} \pm \frac{b}{2}a_{\varphi s}\right) -$$
$$-m\omega^2 K_{x\varphi}\left(hA_z - \frac{b}{2}a_{xs}\right) + \omega^2 C_{xx}C_{x\varphi}\frac{b}{2}a_{xs}$$
$$B_8 = -\Theta\omega^3 C_{xx}a_{z.k} + (\omega K_{xx}C_{\varphi\varphi} + \omega\bar{K}_{\varphi\varphi}C_{xx} -$$
$$-m\omega^3 C_{\varphi\varphi}) \times \left(a_{z.k} \pm \frac{b}{2}A_{\varphi s}\right) -$$
$$-m\omega^3 C_{x\varphi}\left(h\,a_{z.k} - \frac{b}{2}a_{xs}\right) \quad .$$

Here $a_{z.k}$ – are the amplitudes of the foundation vertical oscillations resulting from the kinematic excitation regardless of its rotation in (6).

5 RESULTS

Taking the use of the above computation dependencies (2), (3), (4), (5), (6), (7), (8) and (9), a sufficiently easier determination of all the foundation oscillations components, which result from both the force effect, as well, as from the kinematic excitation may be carried out. Thus acquired analytical dependencies contain only general expressions of stiffness and damping in "foundation-to-basement" system, oscillation process characteristics and the foundation design parameters. They may be conveniently applied for various foundation designs, including specific types of foundations for machines with dynamic loads, pile foundations and reinforced basement foundations.

This facilitates forecasting the foundation oscillations at the designing stage, conducting comparative analysis of application efficiency in various design solutions and optimizing dimensions and foundation design based on set oscillation level, effective loads and dynamic state of site.

It is necessary to mention that the dependencies given in this paper differ only in their forms from similar ones acquired by other researchers. They possess a universal character and, after certain hardness and damping parameters of the basement are inserted, they may be transformed into corresponding expressions used for mold foundations (foundations in a natural basement), embedded pier foundations, pile foundations, etc.

6 COMPUTATION SOFTWARE

A software developed on the basis of the formulas acquired is used for various design oscillation forecasts and for finding their optimum design parameters.

The main criterion used for selection of the optimum design of a pile foundation is to avoid situations when oscillation amplitudes exceed the preset limit.

Since the optimization problem is primarily of practical importance, in addition to minimization of the function of oscillation amplitude there is an additional target function of economic expenses with its main optimized parameter – material consumption for manufacturing of a foundation.

The software package includes modules for statistical processing of data received from geological engineering research, soil dynamic load computing, as well, as for determining hardness and damping parameters of a single pier or of a pier basement.

One of the advantages of this software system is a possibility to refine calculation formulas and to introduce new influencing parameters during modeling the behavior of a dynamic system. For example, to use additional modules with calculation of dynamic soil properties using different techniques, taking into account dynamic interaction of the piles in the group, taking into account lateral backfilling of the foundation frame, etc.

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