Enhanced interpretation of geotechnical limit analysis solutions using Discontinuity Layout Optimization

Interprétation améliorée des problèmes géotechniques sous analyse limite utilisant Discontinuity Layout Optimization

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ABSTRACT: Discontinuity layout optimization (DLO) is a numerical limit analysis procedure which provides a powerful alternative to other numerical methods such as finite element limit analysis approaches. In its core form it is able to identify the critical collapse mechanism in the form of discrete slip-lines for any problem geometry, enabling the engineer to identify the factor of safety for the system. However the analysis method makes available information which can provide additional valuable insights to the engineer. In this paper it is shown how data from the DLO dual equilibrium solution can supply the following: (a) indication of other possible failure modes of a similar safety factor; (b) generation of faster solutions; (c) indication of low stressed areas allowing earthworks topology optimization.

RÉSUMÉ : *Discontinuity layout optimization* (DLO) est une procédure d'analyse limite qui offre une efficace alternative à d'autres méthodes numériques existantes telles que la méthode des éléments finis. Essentiellement, la méthode est capable d'identifier le mécanisme de rupture critique pour tout type de problème et de la géométrie à moyen de *slip-lines* discrétisées (des lignes glissantes). De cette façon l'ingénieur est en mesure d'estimer le coefficient de sécurité du système. En plus, la méthode est également capable de fournir information supplémentaire et importante pour la conception de l'ouvrage. Cet article montre comment la formulation DLO duale peut apporter : (a) l'indication des autres mécanismes de rupture possibles ; (b) la génération des solutions plus rapides ; (c) la désignation des zones avec des faibles niveaux des contraintes qui éventuellement peuvent permettre une optimisation de la topologie de l'ouvrage géotechnique.

KEYWORDS: limit analysis, DLO, optimization.

1 INTRODUCTION

Discontinuity layout optimization (DLO) is an upper bound numerical limit analysis procedure which provides a powerful alternative to other numerical methods such as finite element limit analysis approaches. In its core form it is able to identify the critical collapse mechanism in the form of discrete slip-lines for any problem geometry, enabling the engineer to identify the factor of safety for the system. Rather than being formulated in terms of elements, the method is based on nodes and the lines connecting these nodes. The method is typically presented in its primal kinematic form (Smith and Gilbert 2007) where a regular square grid of nodes is typically utilized in the solution. The basic principles of the analysis are presented in Figure 1.

However, the dual equilibrium form provides a significant additional amount of information about the nature of the stress field at collapse. The equations may be described in matrix form, for a specific discontinuity i as follows:

$$\max \lambda \tag{1}$$

Subject to:

$$\begin{bmatrix} \alpha_i & \beta_i & \alpha_i & \beta_i \\ \beta_i & \alpha_i & \beta_i & \alpha_i \end{bmatrix} \begin{bmatrix} t_A^x \\ t_A^y \\ t_b^x \\ t_b^y \end{bmatrix} + \lambda \begin{bmatrix} f_{Li}^s \\ f_{Li}^n \end{bmatrix} - \begin{bmatrix} S_i \\ N_i \end{bmatrix} = - \begin{bmatrix} f_{Di}^s \\ f_{Di}^n \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} 1 & \tan\varphi_i \\ -1 & \tan\varphi_i \end{bmatrix} \begin{bmatrix} S_i \\ N_i \end{bmatrix} \le \begin{bmatrix} c_i l_i \\ c_i l_i \end{bmatrix} \tag{3}$$

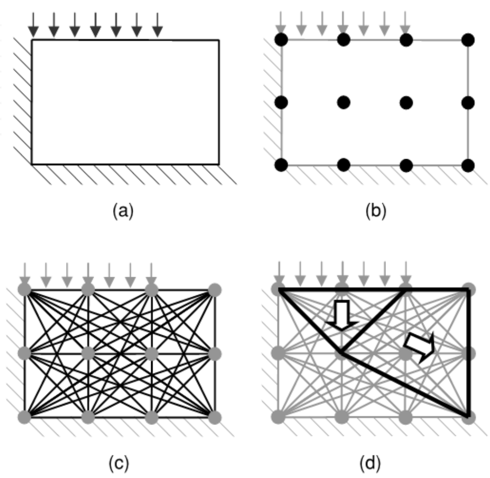


Figure 1. Stages in DLO procedure (after Gilbert et al. (2010)): (a) initial problem definition (surcharge applied to block of soil close to a vertical cut); (b) discretization of soil using nodes; (c) interconnection of nodes with potential discontinuities; (d) identification of critical subset of potential discontinuities using optimization (giving the layout of slip-lines in the critical failure mechanism).

where $\boldsymbol{\alpha_i}, \boldsymbol{\beta_i}$ are the x and y direction cosines of discontinuity i, $\boldsymbol{f^s_{Li}}$, $\boldsymbol{f^n_{Li}}$ and $\boldsymbol{f^s_{Di}}$, $\boldsymbol{f^n_{Di}}$ are the shear and normal external live and dead loads respectively acting on discontinuity i. $\boldsymbol{t^x_A}, \boldsymbol{t^y_A}, \boldsymbol{t^x_B}, \boldsymbol{t^y_B}$ are the horizontal and vertical components of the nodal forces acting at nodes A and B respectively, $\boldsymbol{S_i}, \boldsymbol{N_i}$ are the shear and normal forces acting on discontinuity i, and $\boldsymbol{\lambda}$ is a scalar multiplier. These latter values are the results of the linear optimization.

For a Mohr-Coulomb material, the yield condition is described in equation (3) where $\boldsymbol{c}$ and $\boldsymbol{l}$ are the cohesion and length of discontinuity i, and $\boldsymbol{\varphi}$ is the friction angle.

In general terms, the above process maximizes the live load (via multiplier $\boldsymbol{\lambda}$) while determining equilibrium shear and normal forces acting on every slip-line or potential slip-line connecting any pair of nodes and ensuring they do not violate yield. This remains an upper bound process instead of a lower bound one because yield is only checked on the potential slip lines rather than the whole domain. The yielding lines form the critical mechanism. However those that are below yield can also be utilised. Figure 2 illustrates graphically the shear force magnitudes acting on each line, together with the critical failure mechanism for the same geometry and boundary conditions as Figure 1.

Such data can be used to provide the following: (a) indication of other possible failure modes of a similar safety factor; (b) generation of faster solutions; (c) indication of low stressed areas allowing earthwork topology optimization. Solutions in this paper are generated using both a MATLAB implementation of DLO (Gilbert et al., 2010) and the software LimitState:GEO (LimitState, 2016). For simplicity all problems in this paper are modeled using a weightless, frictionless cohesive material with a unit nodal spacing and unit cohesion within the soil and on boundary interfaces and rigidly loaded. Since the results are largely geometrical in nature, they can be scaled to any dimension and strength.

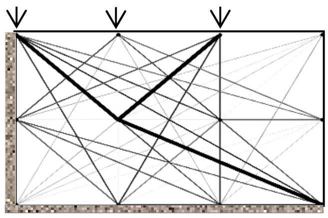
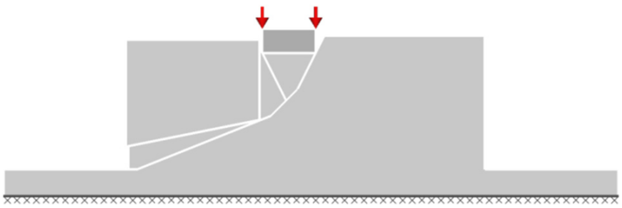


Figure 2. Dual equilibrium output for example problem in Figure 1 (slip-line grey-scale and width both used to represent shear stress magnitude. Black indicates yielding, white indicates zero shear stress).

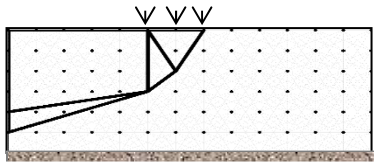
2 SECONDARY FAILURE MODES

Analysis techniques such as the Method of Slices enable the plotting of numerous slip-circles that can be colour coded or contoured according to the factor of safety for that specific slip-circle mechanism (e.g. Leshchinsky 1990). This can be helpful to the engineer in that it can identify different parts of the problem domain where failure may occur even if not identified as the critical mechanism. This is possible with an approach such as the Method of Slices because it considers only a very restricted set of failure mechanisms. In contrast DLO considers a much larger range of mechanisms. For example a model using n nodes will employ of the order of n^2 connections out of which of the order $2^{n^{\wedge}2}$ connection patterns can be composed. Not all of these will provide a viable mechanism, but there will be too many to plot and at present there appears to be no robust simple way of identifying a small subset of distinct mechanism types. An alternative approach is needed. The output of the dual equilibrium method (either directly or indirectly) may provide one such an approach.

Consider an idealized example of the short term undrained stability of a vertically loaded rigid foundation located at the top of a symmetrical vertically sided clay 'embankment', but offset by one half nodal spacing to the left from the centre line as shown in Figure 3(b). The critical mechanism will be found acting towards the nearest edge, but an almost equally likely failure mode would occur in the other direction. It is possible to identify this by suppressing the critical mechanism in the analysis by strengthening the soil on the left hand side, but this is difficult to do automatically and generally.

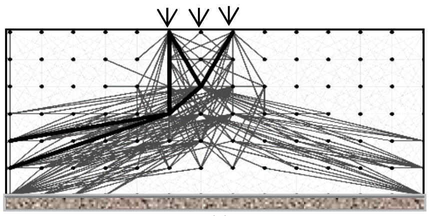


(a)

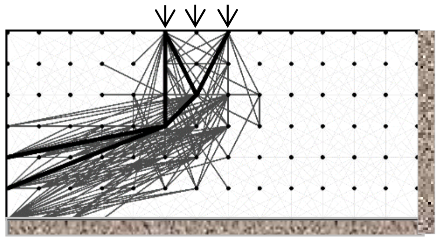


(b)

Figure 3. Example problem of foundation on vertically sided 'embankment', (a) failure geometry and mechanism generated using Limit State: GEO. (b) schematic nodal layout.



(a)



(b)

Figure 4. Dual equilibrium output for example problem (a) potential sliplines >80% yield in grey and critical mechanism in black (b) Same problem as (a) with right side constrained.

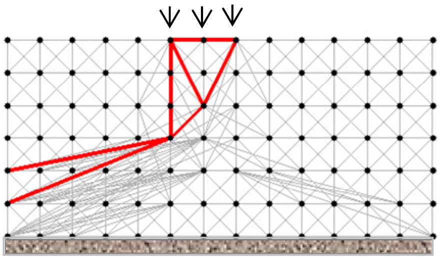
However the likelihood of such a failure can also be seen by plotting either (i) all potential sliplines >X% yield (where a suitable value for X has been found to be ~80%), Figure 4a, or (ii) by plotting the additional lines identified during an adaptive nodal connection procedure, used for solving large problems, Figure 5a. The adaptive nodal connection procedure (Smith and Gilbert 2007) initially solves the problem with a reduced connectivity between nodes and then iteratively adds connections that improve the solution. This provides a robust high speed solution procedure for large problems. It can be clearly seen that the possibility of failure to the right is indicated in both figures.

For comparison, the results for the same model where the right hand boundary is fixed are shown in Figure 4b and 5b. As it can be observed from the figures, the right side constraint helps to prevent failure on this side, thus not triggering any associated lines.

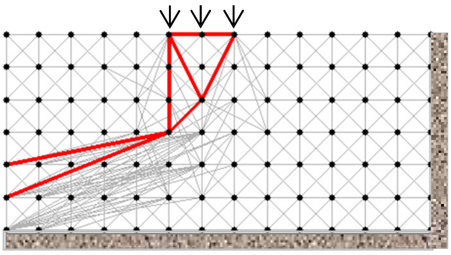
From assessment of many problems, the display of the member adding lines appears to give a marginally clearer indication of those areas where failure may occur that allows the engineer to determine potential remedial measures.

3 SOLUTION ACCELERATION

The accuracy of a DLO solution depends on the number and distribution of nodes employed. Strategic addition or movement of nodes have the potential to accelerate the solution process, e.g. He and Gilbert (2016) who moved nodes to locations that would enhance the solution, using non-linear optimization. An alternative is to add nodes in the vicinity of highly stressed zones rather than in areas that are lightly stressed.



(a)



(b)

Figure 5. Member adding lines for example problem (a) member adding lines in grey and critical mechanism in black (b) Same problem as (a) with right side constrained

In a similar way to the approach outlined in Section 2, the shear forces on the sliplines can be used to identify areas where yield will not occur and can be used to inform an algorithm which will add nodes where they are required. For example if a square grid of nodes is utilised as a starting point, the forces along each square boundary and diagnoals can be determined. If all fall below a certain %yield violation, then there is no need to add additional nodes within this square. It is beyond the scope of this paper to describe the additional details in implementing such an approach, but the above gives the inherent flavour of the method. The results of one variant of this approach is shown in Figure 6. This is a solution of the Prandtl punch problem and is able to get to within 0.1% of the known solution with only 167 nodes starting from a coarse square grid. To acheive a similar accuracy with regular square grid of nodes, requires an inital nodal spacing ~4 times smaller as shown in Figure 7 and takes around 10 times longer to solve.

4 EARTHWORKS TOPOLOGY OPTIMIZATION

A third variant of the approach is to identify those parts of the problem that are lightly stressed and use as candidates to remove material from the problem geometry. This has applications in determining the optimum earthworks topology to achieve a specified Ultimate Limit State design goal utilizing the minimal amount of material, for example when an artificial embankment is to be built using only the minimum amount of material, thus saving costs and energy.

Topology optimization has been widely used in mechanical engineering and manufacturing, however its usage in civil and more precisely in geotechnical engineering is, to date, very limited. Some authors have made use of it for the assessment of settlements (Pucker at al. 2010) and geotechnical structures (Seitz 2015) although always linked to a Finite Element Method implementation and advanced constitutive models for Serviceability Limit State.

An example based on the DLO procedure can be seen in Figure 8. For simplicity the problem is based on a frictionless and weightless soil with unit cohesion (or problem where undrained strength dominates over weight) loaded on top by a rigid footing and constrained on the lower side. In Figure 8b the slip-line shear forces distribution indicates that the soil in the top right and left hand corners and at the base in the centre is lightly loaded and could be removed. Figure 8c shows the modified geometry and collapse mechanism, which fails at the same failure load as Figure 8a. If soil is removed from higher stressed areas, the stability of the problem is changed (Figure 8d)

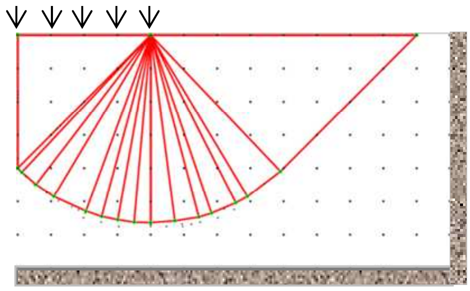


Figure 6. Solution of Prandtl punch problem in a unit cohesion soil (symmetric half space) using adaptive nodal addition (solution N_c = 5.15, 167 nodes)

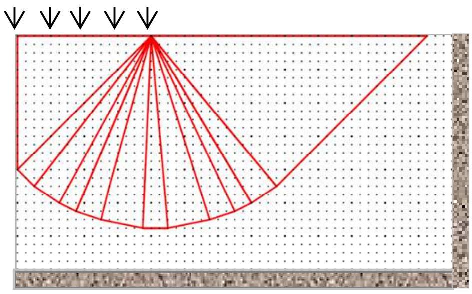


Figure 7. Required nodal density to achieve a similar accuracy to the result in Figure 6 using a uniform starting nodal grid with standard member adding (solution N_c = 5.15, 1537 nodes)

It is noted that, for clarity, only slip-lines below 60% yield have been plotted. Thin light grey lines indicate a value between 0% and 35% yield whereas thick deep grey lines indica te values from 35% up to 60% yield. Also for the sake of clarity only lines shorter than 3 units have been displayed, however all lines have been taken into account in the analysis.

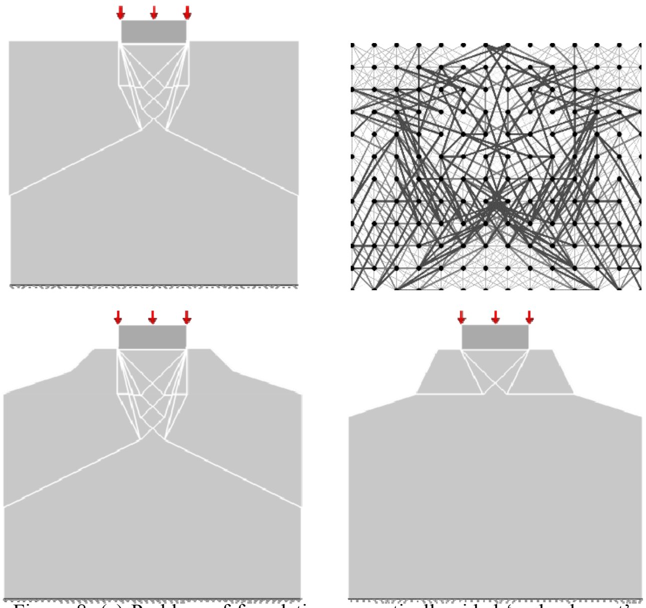


Figure 8. (a) Problem of foundation on vertically sided 'embankment' using Limit State GEO (b) low stressed squares identified in DLO solution. (c) modified geometry based on (a) removing low stressed areas. (d) Different failure mechanism due to removing material from higher stressed zones.

When material is removed from low stressed zones, the failure mechanism hence the factor of safety does not change (Figure 8c). However, when the same procedure is performed on higher stressed areas the failure mechanism changes together with the factor of safety (Figures 8d).

The method is promising and research is ongoing to refine the process so that it is fully automatic and can achieve specified goals rigorously, and to extend it to optimal placement of soil reinforcement.

5 DISCUSSION

While the dual equilibrium solution can identify forces acting in all parts of the domain, these forces are only unique in zones that are failing. In any other zone, the DLO method only has to find a set of forces that are in equilibrium and do not violate yield. Hence this distribution is in part determined by the nature of the linear programming solver used to generate the solutions. The examples presented here use the MOSEK interior point solver (Mosek, 2013) and in general have been found to generate smooth variations of forces. Other solvers may generate different dsitributions. One area of ongoing research is to determine a procedure that guarantees repeatable consistent results for non-yielding zones. However zones that are close to yield will not be subject to significant variation.

6 CONCLUSIONS

Three promising approaches utilising the dual equilibriumm form of the DLO formulation have been illustrated with a range of examples, covering (a) indicative stress fields, (b) accelerated solution and (c) toplogy optimization. Work is ongoing in all three areas and several avenues of approach are being examined in each case.

7 ACKNOWLEDGEMENTS

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