Determination of relative permeability curve of unsaturated soils: a pore network modeling approach

Détermination de la courbe de perméabilité relative des sols non saturés: une approche de modélisation des réseaux poreux

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ABSTRACT: Determination of the settlement rate, volume change behavior and two phase flow in unsaturated soils are few examples which need a good knowledge of the relative permeability curve of unsaturated soils. While there are many empirical equations in the literature for the determination of relative permeability curve of unsaturated soils, the lack of physically based models for this purpose is felt. In this study, the required steps to construct a pore network model of an unsaturated soil sample, the utilized algorithm to simulate drainage, and the procedure to determine permeability function based on the pore network model are described. Furthermore, the efficiency and adequacy of the modeling procedure and the proposed model for various soils samples (mainly sand and loamy sand) are examined. The experimental data of relative permeability function are compared against those of pore network simulation. The results reveal that the proposed pore network modeling gives promising results for prediction of relative permeability function of unsaturated soils.

KEYWORDS: Unsaturated soils, pore network, relative permeability

RÉSUMÉ: La détermination du taux de tassement, du comportement de changement de volume et du débit bi-phasique dans les sols non saturés sont quelques exemples qui nécessitent une bonne connaissance de la courbe de perméabilité relative des sols non saturés. Bien qu'il existe de nombreuses équations empiriques dans la littérature pour la détermination de la courbe de perméabilité relative des sols non saturés, l'absence de modèles physiquement basés à cet effet est ressentie. Dans cette étude, nous décrivons les étapes requises pour construire un modèle de réseau poreux d'un échantillon de sol non saturé, l'algorithme utilisé pour simuler le drainage et la procédure pour déterminer la fonction de perméabilité basée sur le modèle de réseau poreux. De plus, on examine l'efficacité de la méthode de modélisation et du modèle proposé pour divers échantillons de sols (principalement sable et sable limoneux). Les données expérimentales de la fonction de perméabilité relative sont comparées à celles de la simulation de réseaux poreux. Les résultats révèlent que la modélisation du réseau poreux proposée donne des résultats propres pour la prédiction de la fonction de perméabilité relative des sols non saturés.

MOTS-CLES: Sols non saturés, réseau poreux, perméabilité relative

1 INTRODUCTION

Permeability is one of the fundamental physical characteristics of soils, determination of which is of high importance when modeling the coupling between hydraulic and mechanical behavior of soil is aimed. In unsaturated soils, the amount of wetting phase permeability changes as the degree of saturation changes. Hence, a permeability curve is defined and expressed as a function of either moisture content or matric suction. The relative permeability curve is then defined as the ratio of wetting phase permeability in unsaturated condition to that of the saturated condition. Knowledge of relative permeability curve is essential for studying phenomena such as consolidation of unsaturated soils, rate of swelling and flow and transport of pollutants in unsaturated soils (Fredlund and Rahardjo 1993).

In unsaturated soil mechanics, several researchers have, therefore, introduced various empirical and semi-empirical methods to determine this curve. The major shortcoming of empirical methods is that they include coefficients which may not have always physical meaning and furthermore, they are restricted to the soil types based on which these equations are obtained. In the physically based methods and semi-empirical relationships presented to date for the relative permeability curve (Brooks and Corey 1964, Van Genuchten M.T. 1980), a conceptual model of the soil porous structure has been employed. This conceptual model is, commonly, a bundle of capillary tube which portrays soil porous structure as a bundle of cylindrical pores.

Contrary to this conceptualization, soil porous structure is made of large voids which are connected to each other by means of narrow inter-particle spaces.

Therefore, soil porous structure can be in fact thought of a network made of nodes and edges (Rojas 2008, Rostami et al. 2013). Where nodes represent large soil voids, hereafter called pores, and edges hereafter called throats connecting two adjacent pores. The throats indeed represent those narrow inter-particle spaces connecting neighboring voids, see figure…

It is noteworthy that the soil porous structure cannot be either portrayed as a set of disconnected spheres or a bundle of disconnected tubes and indeed the flow in soil porous structure from one pore to another is governed not only be the pressure difference between the two pores but also by the hydraulic conductivity of the throat connecting them.

Therefore, the pore network conceptualization of the soil porous structure seems to provide a more realistic and physically plausible platform for determining the relative permeability in unsaturated soils.

In this study, an algorithm for constructing a pore network which resembles the soil porous structure is presented. The most representative porous network is then used to predict the relative permeability curve of two different soils (a sandy soil and loamy sand). Next, the obtained results are compared against those of experiments and finally efficacy and accuracy of the proposed algorithm are discussed and concluding remarks are presented.

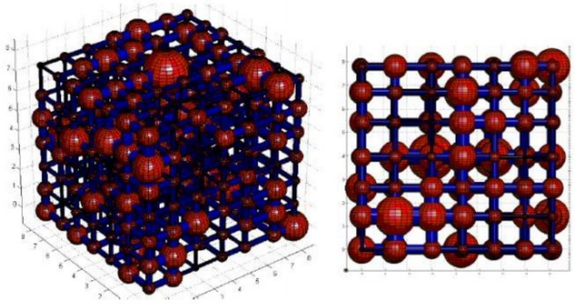

Figure 1. A regular pore network: 3d view (on the left), plan view (on the right) (Daneshian, 2016)

2 COMPUTATIONAL PROCECURE

In this section, the pore network algorithms for predicting soil relative permeability is described; it is composed of the two following stages:
-Construction of the most representative pore network
-Determination of relative permeability by means of the obtained pore network

2.1 *Generation of pore network and its calibration*

In the current study, for modeling the soil porous structure, a regular pore network was used. The pore network was consisted of spherical pore bodies and cylindrical throats. The pores of various sizes were distributed randomly on the nodes of a three-dimensional cubic lattice (20×20×20)). The lateral boundaries of pore network were considered impervious. At the top boundary, the nonweting phase pressure was set equal to that of the nonwetting fluid reservoir and at the bottom boundary the pressure of wetting phase reservoir was exerted. A lognormal distribution function with a specified mean (μ) and standard deviation (σ) was employed to produce the pore radii. In order to find lognormal distribution parameters (μ, σ), pore size-number (r_i, n_i) data were required. In order to find pore size-number datasets, experimentally measured soil water retention data were used. For this purpose, the soil water retention curve was divided into n-intervals. In each interval, the averaged matric suction value, P_{ci}, was considered as a representative value for each interval. The Young-Laplace equation was then used to determine the corresponding pore size for the considered interval, Equation 1

$$r_i = 2T\cos\theta/P_{ci} \qquad (1)$$

Where T is air-water interfacial tension, θ is wetting angle (here it is assumed to be zero) and r_i is the pore size.

Based on the difference between lowest and highest volumetric water content for i-th interval, $\Delta\theta_i$, the number of pores of a certain size, n_i, was obtained:

$$n_i = \Delta\theta_i /[(4/3)\ \pi\ r_i^3] \qquad (2)$$

After that by fitting a lognormal distribution relationship to the obtained pore size-number data, the pore size distribution parameters were found. The EasyFit (Mathwave) software was used for this purpose.

Pore bodies were generated based on the obtained pore size distribution and randomly distributed in a 20×20×20 cubic lattice. Then, for each two pores in the network connected by a throat, the throat radius was computed by using an initial guess for the aspect ratio (A) parameter and the radii of the two neighboring pores. The aspect ratio parameter is defined as the ratio of the throat radius to the maximum radius of the pore bodies connected to it.

In real soil sample, a pore is not connected to all surrounding pores. To have a realistic pore network, therefore, a coordination number parameter (C) is defined. It is defined as the number of throats connecting to a specified pore body. In the current study, it is assumed that all vertical throats are open to flow and connectivity of throats within the XY plane is controlled by the coordination number parameter (C). Each throat in XY plane is labeled by a random number between zero and one. Then, the coordination number parameter (C) is introduced as a probability number. The labels of throats are compared to the coordination number parameter. If throat label is more than probability value, the corresponding throat will close, otherwise it can convey fluids. For instance, when the probability (coordination number) is one, all throats (regardless of their assigned random number) will be open. When the coordination number is zero, the generated network resembles a bundle of capillary tube because all generated numbers are greater than zero and therefore throats in XY plane are closed.

The constructed pore network is used to simulate the experimental data-points of soil water retention curve. A genetic algorithm code was used in order to find the optimum values for structural parameters of pore network (pore-throat aspect ratio, and coordination number parameter) which result in the best matching soil water retention curve.
The procedure to simulate soil water retention curve was as follows:
1. For each experimental suction value, the same suction level is imposed on the pore network using top and bottom boundary conditions.
2. The invasion of non-wetting phase (air, here) into the network was examined using two following criteria:
a. A pore is invaded, if the one of the throats connected to that pore is invaded.
b. A throat is invaded, if the throat entry pressure, P_c (defined as follows), is less than the suction level imposed on the pore network:
A throat can be drained, only if one of the pores connected to that throat is drained (the drainage path should be continuous).

$$Pc= 2T\cos\theta/r_{ij} \qquad (3)$$

When the network calibration with the experimental data points of soil water retention curve is fulfilled, one can start to predict the relative permeability curve.

2.1 *Prediction of relative permeability curve*

The determination of unsaturated hydraulic conductivity using pore network was based upon the following assumption. It was basically assumed that in unsaturated condition and at each saturation, it is mainly the part of pore network that is filled with the wetting fluid and has access to both bottom and top boundaries which provides conductive paths for the wetting fluid to flow.

Therefore, at each suction level, the wetting-fluid-filled part of the pore network was determined. Then, in order to find the hydraulic conductivity of this network, a small pressure gradient was imposed at top and bottom boundaries of this sub-network. The wetting fluid flow rate was set to zero in throats that are not connected to these two boundaries and the throats that end in trapped region of wetting fluid (where wetting fluid flow could not be indeed established). Poiseuille flow is assumed in all other throats, with the flow rate given by the equation for steady Poiseuille flow in a pipe. For a throat of radius r and a pressure drop of Δp across the throat length l, the flow rate passing through the throat is given by

$$q=\frac{\pi r_{ij}^4}{8\mu l_{ij}}\Delta p_{ij} \qquad (4)$$

$$\Delta p_{ij}=p_i-p_j$$

where q_{ij} is the flow rate in the throat connecting pore bodies i and j, and pi, and pj denote the pressure of wetting fluid at pores i and j, respectively. l_{ij} , and r_{ij} are the throat length and radius for the throat connecting pores i and j, respectively. The continuity equation at i-th pore body filled with wetting fluid can be written as

$$\sum q_{ij}=0 \qquad (5)$$

Combining Equations 4 and 5 results in a system of linear equations in terms of pore pressures. When the pressures of all pores are determined, the flow rate in each throat, and consequently the flow across the network can be obtained. Total flow rate can be calculated by sectioning through the network and computing the flow rate passing within the intersected throats. Once the total flow rate is calculated, the absolute permeability can be computed by means of Darcy law, Equation (6).

$$k=\frac{\mu Q l}{A\Delta p} \qquad (6)$$

Where k is the absolute permeability, Q is the total flow rate, L is a length of network in the direction of flow, A is the cross section of the network perpendicular to the flow direction and Δp is the imposed pressure gradient.

The same procedure was pursuit to obtain absolute permeability at al suction levels including zero suction (saturated condition), where it was assumed all pores of the pore network are filled with the wetting fluid.

Finally, the relative permeability is obtained which is the ratio of absolute permeability at each suction level to that of saturated condition.

3 RESULTS AND DISCUSSION

In this section results of pore network modeling are compared to the experimental data for two soil types, namely, a sandy soil and a loamy sand. The basic soil properties of these two soils which are adopted from soil vision 2016 database are presented in Table 1.

Table 1. Basic soil properties of sandy soil and loamy sand sample used in the current study

Soil type	Void ratio (e)	Porosity (n)	G_s	Saturated conductivity K_s (m/s)
Sand(11167)	0.70	0.41	2.65	4.14×10^{-6}
Loamy sand(11184)	0.75	0.43	2.67	1.01×10^{-5}

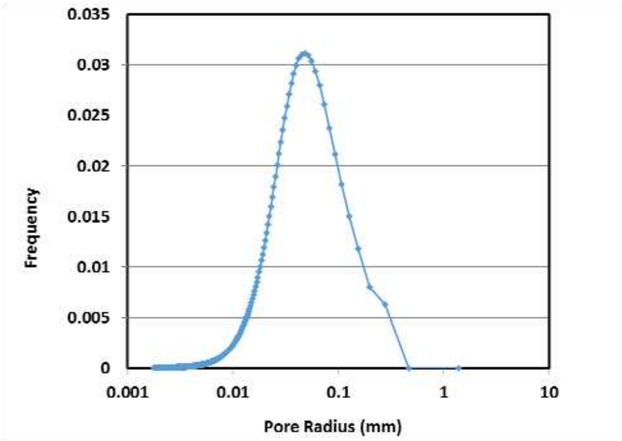

Figure 2. Pore size distribution of sand sample

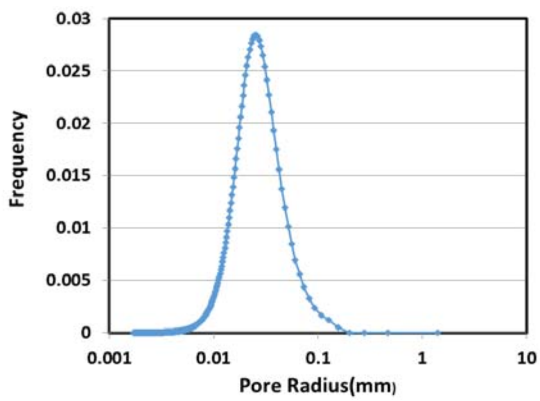

Figure 3. Pore size distribution of loamy sand

Pore size distribution curves obtained from experimental data which were used to construct pore network models are presented in Figures 2, and 3. The optimal values of the pore network structural parameters for these two soil types are presented in Tables 2.

As stated previously, the pore network with such characteristics results in the best matching soil water retention curve and has therefore been considered as the representative soil pore network. Figure 4 and 5 illustrate the best matching soil water retention curve as well as experimental data points for these two soils types.

Table 2. Optimal values of the pore network structural parameters

Soil Type	C	A	μ(mm)	σ	r_{min}	r_{max}
Loamy Sand	0.8985	0.8119	-3.87	0.5308	0.0017094	0.155556
Sand	0.4439	0.8936	-3.51	0.7966	0.00178334	0.28

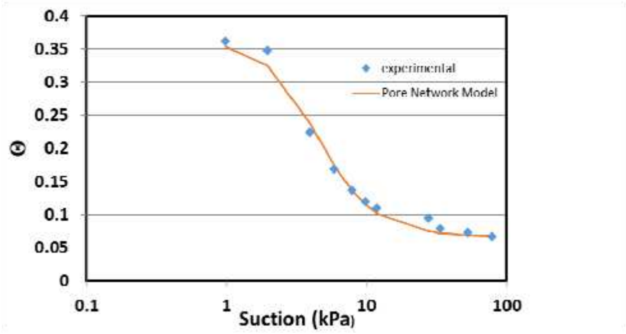

Figure 4. The simulated and experimental soil water retention curves for sand

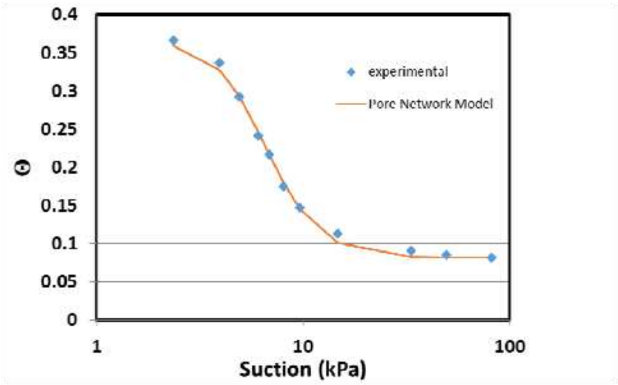

Figure 5. The simulated and experimental soil water retention curves for loamy sand.

In order to minimize the effect of random distribution of the pores inside the pore network, 50 random placements of pores inside a pore network with optimal structural parameters has been used to obtain a possible range of pore network results. Based on this bound, the mean permeability curve has been obtained and reported as the prediction result.

The predicted relative permeability curves are presented in Figures 6 and 7. Mean square error of prediction for sand sample is 0.002, and for loamy sand is 0.01. As it can be seen a fairly well agreement between simulated curves and experimental data has been achieved.

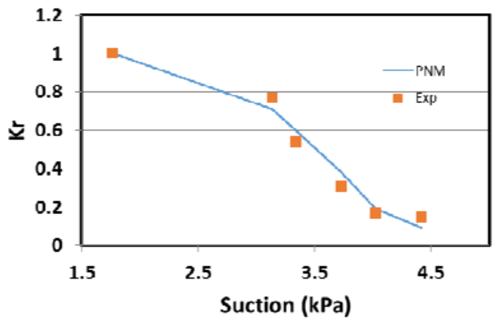

Figure 6. The simulated and experimental relative permeability curves for sand.

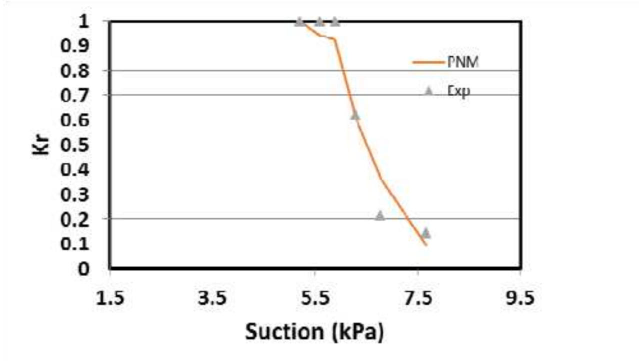

Figure 7. The simulated and experimental relative permeability curves for loamy sand.

4 CONCLUDING REMARKS

In this study, an algorithm for constructing pore network model of a soil based on its water retention curve was presented. The presented algorithm was utilized in order to predict the soil relative permeability curve. The predicted relative permeability had a good agreement with the experimental data. Further research is recommended to examine:

1. The effect of soil type on the network parameters by employing a database containing a wide variety of soil types.
2. The influence of larger pore networks and other pore and throat geometries on the accuracy of the method.

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