Thermo-Hydro-Mechanical Behavior of Fractured Unsaturated Porous Media
Comportement Thermohydromécanique des milieux poreux non saturés fracturés

Ehsan Motevali Haghighi[2], Behrouz Gatmiri[1,2,3], Alireza Mokhtari Varnosfaderani[3]
[1]Institut Navier, Université de Paris Est, Ecole Nationale des Ponts et Chaussées, Research Director, Paris, France
[2]Center of Physical Modeling and Soil Dynamic Laboratories, University of Tehran, Research Assistant, Tehran, IRAN
[3]School of Civil Engineering, University of Tehran, Tehran, IRAN
Presenter's Electronic Address, gatmirib@yahoo.com

ABSTRACT: Presence of several discontinuities in the unsaturated porous media can be regarded as an indispensable issue in the geotechnical problems like extracting geothermal energy, oil and gas industries and, so on. The main purpose of the present paper is to evaluate the suction field of the multiphase porous media in the presence of the several cracks. In this model, the multiphase porous media consists of skeleton matrix, moisture, gas, and heat phases. The theoretical model of elastic behavior, generalized Darcy equation, and Philip and de Vries theory are taken into consideration to model deformation, moisture and gas, and heat flow, respectively. Since cohesive material like soil and concrete cannot withstand singular stress field around the crack tip, cohesive crack model is used to obviate the singular field of the crack tip. Extended finite element method and Finite difference method had been implemented for spatial and temporal discretization, respectively. An evaluation of suction field in the presence of several cracks was examine through one numerical problem.

Résumé: L'étude sur des milieux non saturés fractures est indispensable dans les différents domaines comme stockage des déchets nucléaires, géothermie et milieu pétroliers. Ce papier est consacré à l'évaluation de la succion dans un milieu multiphasique fractures. Le milieu non saturé avec les trois phases, squelette, l'eau et l'air sous chargement thermique a été modélisé. Modélisation des fractures cohésives sont considérée afin d'éviter les problèmes de la singularités des champs des contraintes autour de la fin des fractures. Le méthode des éléments finis étendue à été utilisée. The exemple de validations et d'application sont présentés.

KEYWORDS: Porous Media, Cohesive Crack, Unsaturated Soil Mechanics, XFEM

1 INTRODUCTION

The numerical and analytical modeling of porous media had been adopted to the large scale of engineering applications. Since the presence of discontinuity affects the THM behavior of multiphase porous media, in the past new decades, a great deal of attentions have been paid to this area. In this section, a brief glance on the literature of Porous Media, Fracture Mechanic, and the interaction of porous media with discontinuity was provided.

The basic theory of porous media was developed based on the experimental observation (De Boer 2012). Using Darcy law, the concept of potential was introduced to soil-water interaction (Buckingham 1907). The moisture transport due to the thermal gradient was taken into consideration (Philip and De Vries 1957, De Vries 1958).

To independence saturation degree and void ratio from the effective stress, a new degree of saturation and void ration were proposed (Matyas and Radhakrishna 1968). Development of these state surfaces to consider constitutive law was conducted (Gatmiri 1994, Gatmiri and Delage 1995). To couple volumetric moisture content with temperature new degree of saturation was considered (Gatmiri 1997).

To model discontinuity, cohesive crack model is proposed. This model was introduced by pioneer works of(Dugdale 1960, Barenblatt 1962). Later, the concept of fracture energy was appended to the cohesive crack model and several traction-separation was introduced (Hillerborg, Modéer et al. 1976).

The influence of crack on THM media categorized into mechanical force and hydro-thermal leakage fluxes. The former results from cohesive traction, liquid and gas pressure and thermal force and the later roots in heat and moisture leakage fluxes from the crack body. There are large scale of investigation deal with cracked unsaturated porous media. Two-scale modeling of flow in deformable fractured porous media was modeled by (Réthoré, Borst et al. 2007), and he also develop aforementioned model to unsaturated state (Réthoré, De Borst et al. 2008). Using extended finite element method, hydro-Mechanical behavior of multiphase porous media in the presence of cohesive crack was examined (Mohammadnejad and Khoei 2013). To model the propagation of cohesive crack in the presence of heat and moisture flow, new state surface of saturation degree was proposed (Varnosfaderani, Gatmiri et al. 2016). The effects of physical degradation on the propagation of the cracked unsaturated porous media under combined thermal and hydro-mechanical loading was proposed by (Moonen, Sluys et al. 2010).

In the present paper, the thermo-hydro-mechanical behavior of the porous media under the effects of several discontinuities was examined.

2. MATHEMATICAL AND PHYSICAL MODEL

This section consists of the three parts. In the first part, the effects of cohesive traction, suction and heat gradient on deformation were demonstrated. In the second section, heat and moisture equation was proposed. The third section looked at the interaction of discontinuities and multi-phase porous media.

2.2 *Deformation of Solid Matrix*

The theory of effective stress was implemented to describe deformation field as,

$$\sigma^{S} = \sigma - \alpha S_w P_w I \qquad (1.1)$$

Where σ^{S}, σ, α are effective stress, total stress, and Biot coefficient, respectively. Also, S_w, P_w and I are defined as degree of saturation, pore water pressure, and identity vector, respectively. To assume the effects of temperature, the thermal strain vector had been considered as,

$$\sigma = D(\varepsilon - \varepsilon_T) \tag{1.2}$$

Where D is selected by constitutive law, ε and ε_T are mechanical and thermal strains, respectively. To consider the effect of water pressure in separation-traction relationship, following equation is considered,

$$t_d = t_d([[u]]_m) + S_w P_w \tag{1.3}$$

The second term of above equation takes account for water pressure. $[[u]]_m$ is an indicative of displacement difference between two sides of crack and it is perpendicular to the crack body. Besides the influence of temperature in deformation field, its effect on traction-separation relationship is proposed as,

$$[[u]]_m = [[u]](1 - \alpha_s[[T]]) \tag{1.4}$$

The relationship between separation, $[[u]]_m$, and traction, t_d, had been introduced in several forms. In the present paper, linear relationship was chosen.

2.3 *Thermo-Hydro mechanical model*

The general governing equation of media is categorized into equilibrium, moisture, and heat equations.

2.3.1 *Equilibrium Equation*

General equilibrium equation for multiphase porous media is obtained as,

$$\nabla\sigma^S + \rho b = 0 \tag{1.5}$$

Where σ^S and b are total stress and body force, respectively. ρ is defined as average volumetric density,

$$\rho = (1-n)\rho_s + \theta\rho_w + (n-\theta)\rho_v \tag{1.6}$$

ρ_s, ρ_w and ρ_v are solid, water and vapor densities. θ is defined as volumetric moisture content.

2.3.2 *Moisture and heat flow*

The moisture flow passes by liquid and vapor phases and it is affected by temperature and moisture content gradients; thus, the moisture movement equation is obtained as,

$$\begin{aligned}\frac{q}{\rho_W} &= \frac{q_{vap}}{\rho_W} + \frac{q_{liq}}{\rho_W} = -(D_{TV} + D_{TW})\nabla T - (D_{\theta v} + D_{\theta W})\nabla\theta - D_w\nabla Z = \\ &-D_T\nabla T - D_\theta\nabla\theta - D_w\nabla Z\end{aligned} \tag{1.7}$$

In the above equation, first, second, and third terms are defined as moisture movement as consequence of temperature, water pressure, and elevation gradients.

To make more accurate simulation, water permeability is introduced as function of viscosity and degree of saturation,

$$D_W = K_w = (\upsilon_r / \upsilon_T)\, K_r\left(S_w\right) K_{\text{int}}(n) \tag{1.8}$$

Where first, second, and third term takes account for viscosity, degree of saturation, and intrinsic permeability. The continuity equation of the moisture movement for multiphase porous media can be given as,

$$\begin{aligned}\frac{\partial\theta_m}{\partial \mathrm{t}} &= \frac{\partial(nS_w\rho_w + n\left(1-S_w\right)\rho_v)}{\partial t} = (\rho_w S_w + \rho_v(1-S_w))\dot{n} + \\ &n(\rho_w - \rho_v)\dot{S}_w + nS_w\dot{\rho}_w + n(1-S_w)\dot{\rho}_v = -div(\rho_w\left(U+V\right))\end{aligned} \tag{1.9}$$

All parameters in above equation should be defined as function of major parameters of media. Solid grains are assumed to be incomprehensible as,

$$\dot{\mathrm{n}} = (1-n)\nabla.\dot{u} \tag{1.10}$$

State surface of saturation degree is proposed as,

$$\mathrm{S_w} = \left[\left(1-\mathrm{S_{w,r}}\right)\left(1+(\alpha_{VG}\left|\mathrm{s}\right|)^{\mathrm{n}_{VG}}\right)^{-(1-\frac{1}{\mathrm{n}_{VG}})} + \mathrm{S_{w,r}}\right]\exp(\mathrm{d_s}\left(\mathrm{T}-\mathrm{T_0}\right)) \tag{1.11}$$

Using above state surface, the dependency of saturation degree on suction and temperature is assumed. As it is noted in the work of (Gatmiri 1997), the density of water is considered as function of temperature and suction,

$$\dot{\rho}_\mathrm{w} = \frac{\partial\rho_w}{\partial P_w}\dot{P}_w + \frac{\partial\rho_w}{\partial T}\dot{\mathrm{T}} = \beta_\mathrm{P}\dot{P}_w + \beta_\mathrm{T}\dot{\mathrm{T}} \tag{1.12}$$

Using local thermo-dynamic law, vapor density can be proposed as function of temperature and moisture content,

$$\rho_\mathrm{v} = \rho_0.\mathrm{h} = \rho_0(T)\exp(\frac{\psi.\mathrm{g}}{\mathrm{R.T}}) \tag{1.13}$$

Where ρ_0, h, R and ψ are defined as saturated water vapor density, relative humidity, gas constant and, thermodynamic potential of water.

Substituting (1-10), (1-11) and, (1-12) into (1.9), final form of moisture movement can be given as,

$$\begin{aligned}&C_{wu}(1-n)\nabla.\dot{u} + C_{ww}\dot{P}_w + C_{wt}\dot{T} + \ldots \\ &\nabla.\left(\rho_w\ D_w\nabla Z + D_T\nabla T + D_P\nabla P_w\right) = 0\end{aligned} \tag{1.14}$$

Paying attention to the theory of Philip and de Vries, the heat flow of unsaturated porous media can be obtained,

$$Q = -\lambda gradT + \rho_W h_{fg} V + [C_{PW}\rho_W U + C_{PV}\rho_W V](T - T_0) \tag{1.15}$$

Where λ, Fourier heat diffusion coefficient, is evaluated as,

$$\lambda = \left(1-n\right)\lambda_s + \theta\lambda_w + (n-\theta)\lambda_v \tag{1.16}$$

The energy conservation equation can be considered as,

$$\frac{\partial\varphi}{\partial t} + divQ = \tag{1.17}$$

Where Q is given by (1.15) and φ, volumetric bulk heat content, is defined as,

$$\varphi = C_T\left(T - T_0\right) + (n-\theta)\rho_v h_{fg} \tag{1.18}$$

Using equation (1.15), (1.16), and (1.18), the final form of energy conservation equation can be obtained as,

$$\begin{aligned}&\chi_\varepsilon(1-n)\nabla.\dot{u} + \chi_w\dot{P}_w + \chi_T\dot{T} - div\left[\lambda\left(\theta\right)\nabla T\right] + \ldots \\ &C_{PW}\rho_w div\left[\left(-D_{Tw}\nabla\mathrm{T} - D_{Pw}\nabla P_w - K_w\nabla\mathrm{Z}\right)\left(T - T_0\right)\right] + \ldots \\ &C_{PV}\rho_w div\left[\left(-D_{TV}\nabla\mathrm{T} - D_{PV}\nabla P_w\right)\left(T - T_0\right)\right] + \ldots \\ &\rho_w h_{fg} div\left[-D_{TV}\nabla T - D_{PV}\nabla P_w\right] = 0\end{aligned} \tag{1.18}$$

2.3.3 *Interaction of unsaturated media with discontinuity*

To attain moisture leakage fluxes from the crack body, moisture mass conservation should be taken into account for local coordination of crack body as following form,

$$\int_{-\mathrm{h}}^{+\mathrm{h}}(C_{wu}(1-n)\nabla.\dot{u} + C_{ww}\dot{P}_w + C_{wt}\dot{T} + \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y})\,\mathrm{dY'} = 0 \tag{1.19}$$

After some simplification, the final form moisture leakage flux can be attained as,

$$\begin{aligned}&\bar{q}_{wd}\left(x,t\right) = \int_{-\mathrm{h}}^{+\mathrm{h}}\frac{\partial q}{\partial y}\mathrm{dY'} = \ldots \\ &-C_{wud}\left(2h < \frac{\partial v_s}{\partial \mathrm{X'}} > + \left[\left[w_s\right]\right]\right) - 2h\left(C_{wwd}\dot{P}_w + C_{wtd}\dot{T}\right) - \ldots \\ &2h\frac{\partial}{\partial \mathrm{X'}}\left(\rho_\mathrm{w} D_{Td}\nabla T.t_{\Gamma d} + \rho_\mathrm{w} D_{Pd}\nabla P_w.t_{\Gamma d} + \rho_\mathrm{w} D_{wd}e_z\nabla\mathrm{u}.t_{\Gamma d}\right)\end{aligned} \tag{1.20}$$

Similarly, the heat leakage flux obtains,

$$\begin{aligned}&\overline{q}_{Td}\left(x,t\right) = -\chi_{\varepsilon d}\left(2h < \frac{\partial v_s}{\partial \mathrm{X'}} > + \left[\left[w_s\right]\right]\right) - \ldots \\ &2h(\chi_{wd}\dot{P}_w + \chi_{Td}\dot{T}) - 2h\frac{\partial}{\partial X'}(\mathrm{Q})\end{aligned} \tag{1.21}$$

3. DISCRETIZATION

The discretization is performed in spatial and temporal domains. Displacement distribution is selected as,

$$u(x,t) = \sum_{i\in\Omega} N_i^u(x)u_i(t) + \sum_{i\in\Omega_{enr}} N_i^u(x)(H(x) - H(x_i))\,\tilde{u}_i(t) \tag{1.22}$$

Where H regarded as Heaviside function and it makes displacement field as strong discontinuity. For water pressure and temperature,

$$
\begin{aligned}
P_w(x,t) &= \sum_{i\in\Omega} N_i^{P_W}(x)P_{W^i}(t) + \sum_{i\in\Omega_{enr}} N_i^{P_W}(x)(D(x) - D(x_i))R(x)\tilde{P}_{W^i}(t) \\
T(x,t) &= \sum_{i\in\Omega} N_i^{T}(x)T_i(t) + \sum_{i\in\Omega_{enr}} N_i^{T}(x)(D(x) - D(x_i))R(x)\tilde{T}_i(t)
\end{aligned}
\qquad (1.23)
$$

D is defined as distance function and takes account for weak discontinuity.
Temporal discretization is executed by finite difference method.

4. RESULTS

This problem is designed to figure out the suction behavior of THM media in the presence of several discontinuities. The simple plate with the dimension of 0.25×0.25m is considered and different arrangements of nine cracks as it is demonstrated in Fig.1 is proposed.

In the Fig 1, all cracks are located in the horizontal location. In the Fig 2, central crack is positioned at 15, 45 and, 75 degree above the horizontal line. Material properties are given in the Tab.1. Plate is stretched from two horizontal edges with constant fixed vertical velocity of 2.36E-06 m/s.

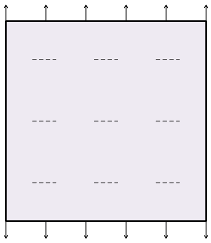

Figure 1a. All cracks are horizontal

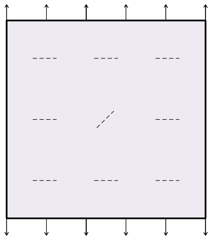

Figure 1b. Central crack is above the horizontal line

4 nodes quadrilateral element size of 0.25/85 is considered. All cracks have constant length equal to (10×element size) and the center to center distance of cracks is set to constant value. Since there is a severe computational procedure when crack propagates, the iterative solution is stopped when the non-local stress field, at least, around one crack tip, reach to the material yield criteria.

Table 1. Material properties.

Youngs modulus (GPa)	25
Poisson's ratio	0.2
Tensile strength (MPa)	2.7
Fracture energy (N/m)	95
Initial saturation degree	1
Residual saturation degree	0
α_{vG}	$1E-6$ 1/Pa
n_{vG}	2.3
Porosity	0.2

As a consequence of this assumption, there is a few changes in the suction field; however, the results can be considered for any case with same quality.

To make more accurate examination, three permeability are selected, 1E-09 m/s, 1E-10 m/s and 1E-11m/s. In the first examination, the effects of several cracks on suction field are investigated (figure 2-3).

In the second examination, to survey the effect of crack opening on suction field in the presence of several cracks, the central crack is located in the different angles above the horizontal line and the suction field will be noticed (figure 4-9).

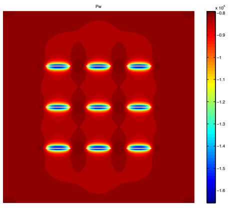

Figure 2. Suction Distribution

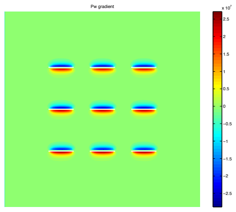

Figure 3. Gradients of Suction

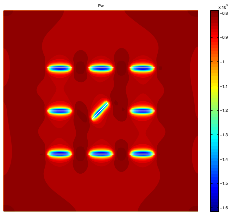

Figure 4. Suction Distribution(45)

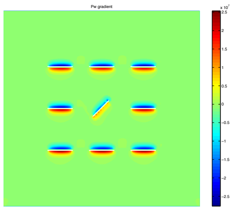

Figure 5. Gradients of Suction(45)

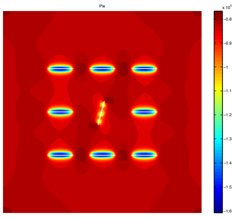

Figure 6. Suction Distribution(75)

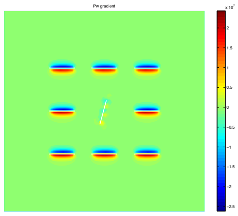

Figure 7. Gradients of Suction(75)

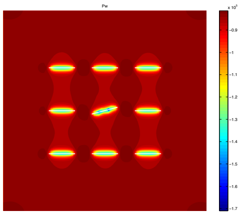

Figure 8. Suction Distribution(15)

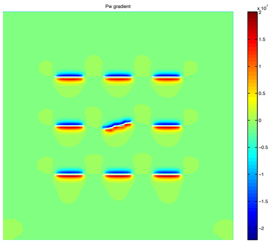

Figure 9. Gradients of Suction(15)

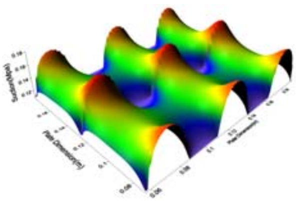

Figure 4. K=1E-11m/s

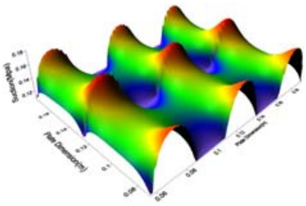

Figure 5. K=1E-10m/s

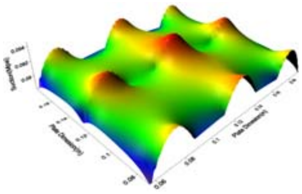

Figure 6. K=1E-09m/s

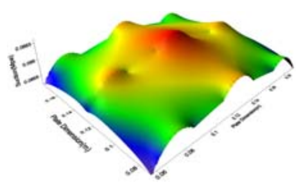

Figure 7. K=1E-08m/s

One indispensable fact about the relationship between movement of moisture and permeability is that moisture moves more conveniently when the permeability increases. As a consequence of the fact, in the case of high permeability, central crack is highly affected by surrounding cracks (figure 4-7). Moreover, in low permeability case, since moisture flows with more difficulty than high permeability, the suction distribution is concentrated at the each crack body. Consequently, when the permeability decreases, moisture has more tendency to pass through the crack and high suction field appears around the crack body (figure 4-7).

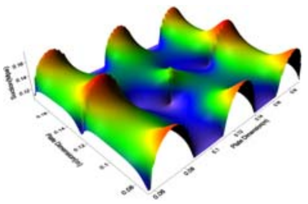

Figure 8. Central crack 45 above the horizontal line

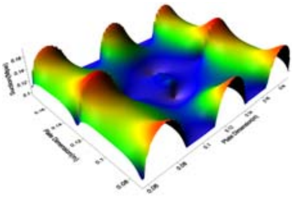

Figure 9. Central crack 75 above the horizontal line

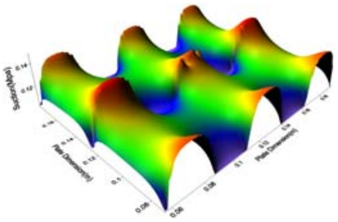

Figure 10. Central crack 15 above the horizontal line

As the central crack is located higher above the horizontal line, less tensile force is exerted on the central crack body; thus, less opening is observed for central crack. Considering figures 8-10, it can be concluded that in the case of less tensile force, less suction appears around the central crack (figure 9); however, in the figure 10, suction field is increased due to higher tensile stress. This observation reveals the clear role of suction to minimize the crack opening.

5. CONCLUSION

Suction field plays an important role in the THM behavior of cracked unsaturated porous media. The effects of such critical component become more attractive in the presence of several cracks.

Based on the proposed model, the suction filed is largely affected by presence of several cracks when moisture moves more conveniently. In other words, if the porous media has higher permeability, moisture moves more readily and the suction field concentrated on the central crack.

Also, exerting less opening to the central crack results in less suction value around the central crack body. This fact can demonstrate the impact of suction field on the crack body to decrease its opening.

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