A series of fully undrained cyclic loading simulation on unsaturated soils using an elastoplastic model for unsaturated soils

Une série de simulations de chargements cycliques complètement non-drainés sur des sols non saturés en utilisant un modèle élastoplastique pour les sols non saturés

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ABSTRACT: This paper describes a series of simulations of fully undrained cyclic loading on unsaturated soils, conducted using an elastoplastic constitutive model. This model is a critical state soil model formulated for unsaturated soils using Bishop's effective stress tensor, which incorporates the following concepts: (a) the volumetric movement of the state boundary surface containing the critical state line owing to the variation in the degree of saturation; (b) the soil water characteristic curve considering the effects of specific volume and hydraulic hysteresis; and (c) the subloading surface concept for considering the effect of density. Void air is assumed to be an ideal gas obeying Boyle's law. The proposed model is validated through comparisons with past results. The simulation results show that the proposed model properly describes the fully undrained cyclic behavior of unsaturated soils. Finally, the effects of the degree of saturation and void ratio on the cyclic strength of unsaturated soils are described by the simulation results. The liquefaction resistance of unsaturated soils increases as the degree of saturation and the void ratio decrease. Furthermore, the degree of saturation has a greater effect on the liquefaction resistance than void ratio.

RÉSUMÉ : Cet article décrit une série de simulations de chargements cycliques complètement non-drainés sur des sols non saturés, effectuées à l'aide d'un modèle constitutif élastoplastique pour de tels sols. C'est un modèle constitué avec de la terre à l'état critique, utilisant le tenseur des contraintes effectives de Bishop, qui incorpore les concepts suivants: (a) Le mouvement volumétrique de la surface de l'état frontière contenant la ligne de l'état critique en raison de la variation du degré de saturation; (b) La courbe caractéristique sol-eau qui prend en compte les effets du volume spécifique et de l'hystérèse hydraulique; et (c) Le concept de surface subloading pour prendre en compte l'effet de densité. On suppose que l'air vide est un gaz parfait qui obéit à la loi de Boyle. Le modèle proposé est validé grâce aux comparaisons des résultats passés. Les résultats de la simulation montrent que le modèle proposé décrit correctement le comportement cyclique totalement non-drainé des sols non-saturés. Finalement les effets du degré de saturation et de l'indice de vide sur la force cyclique des sols non-saturés sont décrits dans les résultats de la simulation. La résistance à la liquéfaction des sols non-saturés augmente parallèlement à la diminution du degré de saturation et de l'indice de vide. De plus, le degré de saturation a un effet plus grand sur la résistance à la liquéfaction que l'indice de vide.

KEYWORDS: unsaturated soils; cyclic loading; elastoplastic model; simulation; degree of saturation; void ratio.

1 INTRODUCTION

Soils are often subjected to cyclic loading under unsaturated conditions in practical situations. In Japan, the Sanriku-Minami earthquake triggered a landslide in the town of Tsukidate on May 26, 2003. An artificial fill in this disaster area lost its effective stress under cyclic loading although the degree of saturation was around 70% (Unno et al. 2006).

Several researchers investigated the cyclic behavior of unsaturated soils through laboratory tests under fully undrained cyclic loading conditions. Ishihara et al. (2004) studied the effects of relative density and the degree of saturation on the undrained behavior of nearly saturated sand. Unno et al. (2006, 2008, and 2013) conducted a series of strain-controlled cyclic triaxial tests on the unsaturated sand under fully undrained conditions to study the liquefaction of such soils. Importantly, the existing experimental studies have revealed that the mean effective stress of an unsaturated soil having a relatively high degree of saturation gradually decreases and such soil can be finally liquefied in a similar manner to the saturated soils.

The main purpose of this paper is to present a series of simulations of fully undrained cyclic loading on unsaturated soils using an elastoplastic constitutive model for unsaturated soils (Kikumoto et al. 2010). The validity of the proposed model is checked via comparisons with the experiment results (Unno et al. 2013). Finally, the effects of the degree of saturation and void ratio on the cyclic strength of unsaturated soils under low confining pressure condition are studied through the simulations.

2 BASIC CONCEPTS

The basic concepts applied to formulate a model for unsaturated soils (Kikumoto et al. 2010) are described herein.

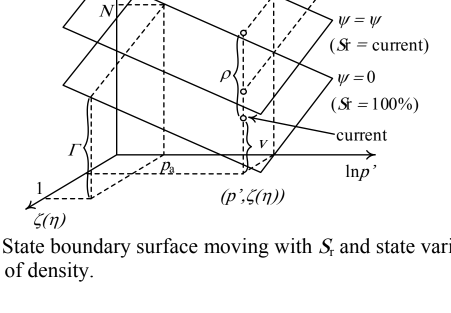


Figure 1. State boundary surface moving with S_r and state variable ρ for the effect of density.

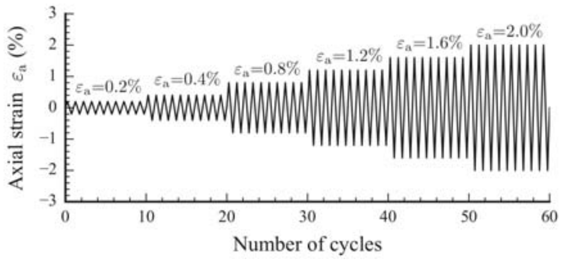


Figure 2. Time history of axial strain during cyclic shearing.

2.1 *The Bishop's effective stress*

The critical state stress ratio is uniquely defined by Bishop's mean effective stress vs. deviator stress plane regardless of the degree of saturation (Sivakumar, 1993). We thus formulate a model for unsaturated soils using Bishop's effective stress:

$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma} - u_a\mathbf{I} + \chi(u_a - u_w)\mathbf{I} = \boldsymbol{\sigma}^{net} + S_r s\mathbf{I} \quad (1)$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}^{net}$, u_a, u_w, S_r, and s represent Cauchy's total stress tensor, Cauchy's net stress tensor, pore air pressure, pore water pressure, degree of saturation, and suction, respectively. χ is a variable given as a monotonic increasing function of S_r and it is assumed to be equal to S_r in this study for simplicity. $\mathbf{I}$ is the second order identity tensor.

2.2 *Soil water characteristic curve (SWCC) considering the effects of density and hydraulic hysteresis*

It is well-known that the SWCCs trace hysteretic paths according to drying and wetting histories. It is also indicated through the past experimental studies (e.g. Tarantino and Tombolato, 2005) that volumetric behavior influences the SWCC and that denser soils tend to retain a higher degree of saturation S_r. Thus we extended a classical SWCC model that assumes a unique relationship between suction s and S_r to incorporate the effects of
hysteresis and volumetric behavior (Kikumoto et al. 2010).

$$S_r = I_h S_r^d + (1 - I_h) S_r^w \quad (2)$$

Here, S_r^d and S_r^w are S_r on the main drying and wetting curves given by classical model proposed by van Genuchten (1980) as

$$\frac{S_r^A - S_{min}}{S_{max} - S_{min}} = S_e^A(s^*) = \left\{1 + \left(\alpha^A s^*\right)^n\right\}^{-m} \ , \mathrm{A} = \mathrm{d}, \mathrm{w} \quad (3)$$

and I_h is a variable reflecting the wetting and drying histories whose variation is given as

$$\frac{dI_h}{dS_r} = \begin{cases} -\xi_h (1 - I_h)^3 & \text{when } \dot{S}_r \le 0 \\ -\xi_h {I_h}^3 & \text{when } \dot{S}_r > 0 \end{cases} \quad (4)$$

where ξ_h is a material constant. s^* is modified suction in which effect of density on retention characteristic is considered.

$$s^* = s\left(\frac{e}{e_{ref}}\right)^{\xi_e} \quad (5)$$

Here, e_{ref} and ξ_e are material parameters.

2.3 *Elastoplastic model for unsaturated soil*

An elastoplastic model for the unsaturated soil (Kikumoto et al. 2010) is formulated based on the Modified Cam clay (Roscoe and Burland 1968), incorporating the subloading surface concept (Hashiguchi 1977) and the volumetric movement of the state boundary surface (Figure 1) due to the variation in S_r. Yield function of the proposed model is expressed as follow.

$$f = \frac{\lambda - \kappa}{v_0}\left\{\ln\frac{\mathrm{tr}\boldsymbol{\sigma}''}{\mathrm{tr}\boldsymbol{\sigma}''_0} + \ln\left[1 + \left(\frac{\eta}{M}\right)^2\right]\right\} - \frac{\Psi - \Psi_0}{v_0} + \frac{\Omega - \Omega_0}{v_0} - \mathrm{tr}\boldsymbol{\varepsilon}^p \quad (6)$$

Here, Ψ represents the effect of degree of saturation as:

$$\Psi = \psi(1 - S_r) \quad (7)$$

Table 1. Parameters for stress-strain characteristics.

λ	0.123	Compression index
κ	0.022	Swelling index
M	1.5	Stress ratio in critical state
ν	0.3	Poisson's ratio
N	1.90	Reference specific volume
ω	90.0	Effect of density
ψ	0.90	Effect of S_r on the position of the state boundary surface

Table 2. Parameters for water retention curve.

$S_{r_{max}}$	1.0	Parameters for van Genuchten's SWCC equation
$S_{r_{min}}$	0.20	
α^d (1/kPa)	0.04	
α^w (1/kPa)	2.00	
n	1.724	
m	0.42	
ξ_h	10.0	Influence of suction histories
ξ_e	2.5	Influence of void ratio
e_{ref}	0.90	Reference void ratio

Table 3. Initial state of cyclic shearing simulation (Unno et al. 2013).

Case No.	c-1	c-2	c-3
Air pressure (kPa)	0.0	6.0	14.8
Water pressure (kPa)	0.0	0.0	0.0
Suction (kPa)	0.0	6.0	14.8
Net stress (kPa)	20.8	18.5	19.9
Mean effective stress (kPa)	20.8	23.2	30.8

Note The mean effective stress is calculated on the basis of Bishop's eff ective stress equation, and the pressure is gauge pressure, which exclud es atmospheric pressure (98 kPa).

with a parameter ψ. Ω represents the effect of the density and is defined by the volumetric distance from the current state to the state boundary surface. The evolution for Ω is given as:

$$\frac{\dot{\Omega}}{v_0} = -\omega\Omega|\Omega|\left\|\dot{\boldsymbol{\varepsilon}}^p\right\| \quad (8)$$

with a parameter ω. An associated flow is assumed and the function f is also used as the plastic potential function.

3 SIMULATION RESULTS AND DISCUSSION

All the analyses were performed using the parameters for Tsukidate volcanic sand (non-plastic sand), which has a specific gravity of 2.478 (Unno et al. 2013), as shown in Tables 1 and 2. In the simulation of the cyclic triaxial tests for the calibration of the model parameters and validation, the initial states of cyclic shearing simulation were considered (see Table 3). Cyclic axial strain as shown in Figure 2 was applied to the specimens under the fully undrained conditions at a constant confining pressure.

3.1 *Fully undrained simulation*

In order to simulate the unexhausted air condition, we assume that air is an ideal gas and the temperature is constant. Therefore, Boyle's law, which states that the absolute pressure of a given mass of an ideal gas is inversely proportional to its volume at a constant temperature, can be used as

$$u_a V_a = \mathrm{constant} \quad (9)$$

where V_a is the volume of air.

A classic equation for solving problems involving three-phrase relationships (solid, water, and air) can be used to satisfy the undrained water condition as

$$w = eS_r/G_s = \text{constant} \tag{10}$$

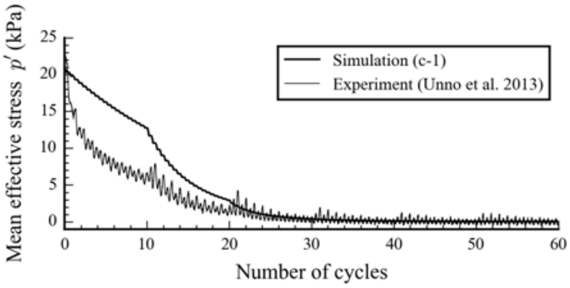


Figure 3. Comparison between the simulation result and the experiment result of Case c-1 (S_r = 100%, e_0 = 1.09 kPa).

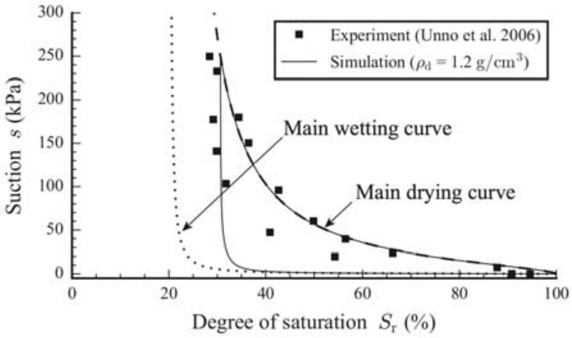


Figure 4. Comparison between the simulation results and the experiment results of the water retention test under drying and wetting paths (ρ_d = 1.2 g/cm3).

where w, e, and G_s re the water content, void ratio, and specific gravity of the soil, respectively.

Finally, a model for unsaturated soils can be formulated by applying these basic concepts in order to predict the cyclic behavior of unsaturated soils under fully undrained conditions.

3 .2 *Model validation*

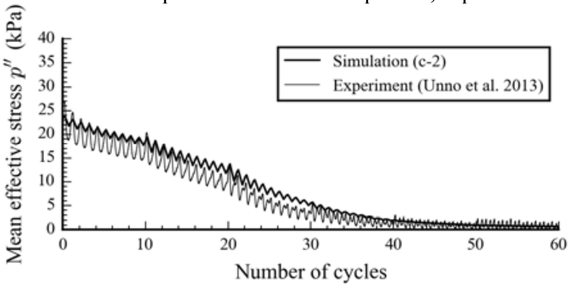
Parameters for stress-strain relationship in Table 1 were firstly calibrated using the simulation of undrained cyclic triaxial tests on saturated soil (Case c-1) as shown in Figure 3. Parameters for SWCC in Table 2 were calibrated using the simulation of water retention test (Unno et al. 2006) as shown in Figure 4.

The validity of the proposed model is verified through a series of simulations of cyclic triaxial tests on unsaturated soils under fully undrained conditions.

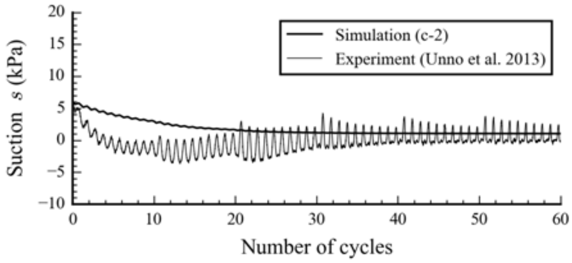
Cyclic triaxial tests, conducted by Unno et al. (2013), have been performed on two types of unsaturated samples (initial degree of saturation = 78.9% and 73.5% with the same initial void ratio of 0.93). In the simulation, the initial states of cyclic shearing simulation were first set as Case c-2 and c-3 for S_r of 78.9% and 73.5%, respectively. Cyclic axial strain as shown in Figure 2 was then applied to the specimens under the unexhausted air and undrained water conditions at a constant confining pressure. Finally, the comparison between the experiment results of Case c-2 and their corresponding simulation results, i.e., the time histories of suction, mean effective stress, air pressure, water pressure, and void ratio, have been obtained as shown in Figures 5 (The simulation results of Case c-3 cannot be shown because of space limitations).

According to Figure 5, the proposed model can precisely describe the cyclic behavior of unsaturated soils under fully undrained cyclic loading conditions. The unsaturated soil lost its effective stress because of the development of pore-air and pore-water pressures (Unno et al. 2008) and a decrease in suction because the development of air pressure is less than that of water pressure during cyclic shear.

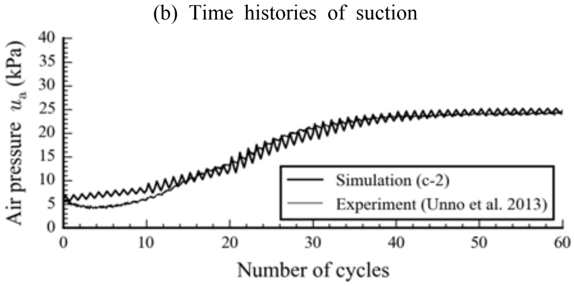
The proposed model can illustrate the fact that liquefaction does not occur when suction becomes zero (Unno et al. 2008). Based on the Bishop's effective stress equation, liquefaction of



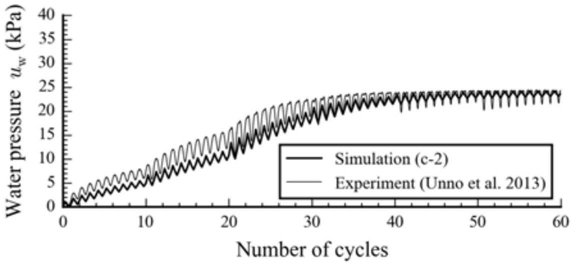
(a) Time histories of mean effective stress



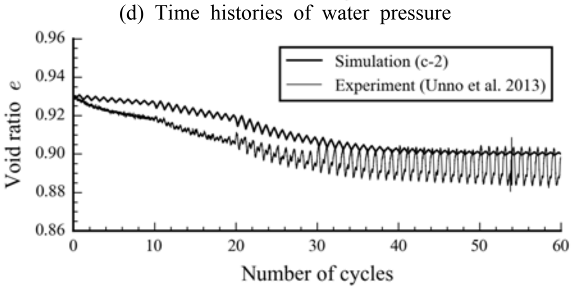
(b) Time histories of suction



(c) Time histories of air pressure



(d) Time histories of water pressure



(e) Time histories of void ratio

Figure 5. Comparison between the simulation results and the experiment results of Case c-2 (Sr = 78.9%, s = 6.0 kPa).

the unsaturated soil will occur when the suction and the net stress become zero. This situation means that air pressure, water pressure, and total confining pressure must be equal.

Moreover, the proposed model incorporating Boyle's law can capture the compression behavior of unsaturated soils under fully undrained conditions. As air pressure increases during cyclic loading, air volume will automatically decrease following Boyle's law. The magnitude of the decrease in the void ratio was also predicted accurately.

The proposed model for water retention curve can predict the increase in the degree of saturation due to volumetric contraction (Unno et al. 2006) as shown in Figure 6.

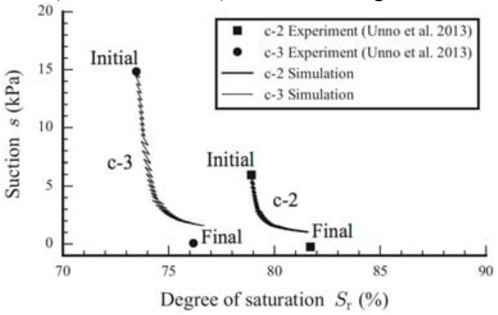


Figure 6. Increase in the degree of saturation during fully undrained cyclic triaxial tests of Case c-2 and c-3.

3 .3 *The effects of the void ratio and the degree of saturation on the cyclic strength of unsaturated soils*

A series of simulations of cyclic triaxial tests on unsaturated soils
under fully undrained conditions has been performed here in order to study the effects of the void ratio and the degree of saturation on the cyclic strength of unsaturated soils.

A series of simulations of cyclic triaxial tests on unsaturated soils was performed on the prescribed void ratio (initial void ratio of 0.60 to 1.09). In the simulation, the suction was increased from zero to the prescribed values for the desired degree of saturation (initial degree of saturation of 100% to 40%) under constant confining pressure of 20 kPa, and cyclic axial strain as shown in Figure 2 was then applied to the specimens under the unexhausted air and undrained water conditions at a constant confining pressure.

The result of a series of simulations is shown in Figure 7 in 3D space. Mean effective stress reduction ratio (Unno et al. 2008) is the decreasing rate of the mean effective stress of unsaturated soils after subjected to cyclic shear loading, which can be used to describe the liquefaction resistance of unsaturated soils

$$\text{Mean effective stress reduction ratio} = 1 - \frac{p''}{p_0''} \qquad (11)$$

where equals 1.0 at the complete liquefaction state.

It is seen from Figure 7 that the liquefaction resistance of unsaturated soils increases as the degree of saturation and the void ratio decrease. Moreover, the effect of the degree of saturation on the liquefaction resistance of unsaturated soils is higher than the effect of the void ratio. The unsaturated soils easily liquefied when the degree of saturation is higher than 70% (Mean effective stress reduction ratio is higher than 0.9). This result can be explained by the following reasons. Based on the volumetric movement of the state boundary surface due to the variation in the degree of saturation, unsaturated soils behave more similar to a dense soil, which dense soils gain the cyclic strength due to their dilatancy characteristics (Kazama et al. 2000).

4 CONCLUSIONS

An elastoplastic model for the unsaturated soil (Kikumoto et al. 2010), which is able to predict the cyclic behavior under fully undrained conditions, has been introduced. This model was formulated using the Bishop's effective stress tensor incorporating the following concepts: the volumetric movement of the state boundary surface containing the critical state line due to the variation in the degree of saturation, the soil water characteristic curve model considering the effects of specific volume and hydraulic hysteresis, the subloading surface model, and Boyle's laws.

The validity of the proposed model was verified through a series of cyclic triaxial tests on unsaturated soils under fully undrained conditions. The results showed that the proposed

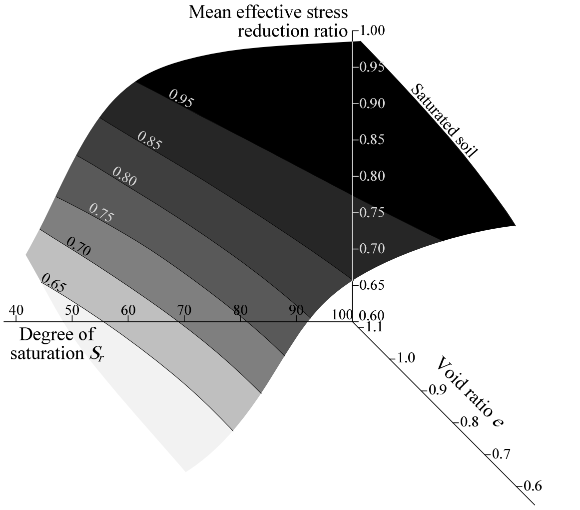


Figure 7. Three-dimensional surface of cyclic strength of unsaturated soils: effect of degree of saturation and void ratio.

model properly describes the fully undrained cyclic behavior of unsaturated soils such as liquefaction, compression behavior, and
an increase in the degree of saturation.

According to the simulation results in 3D space, the cyclic strength of unsaturated soils increases as the degree of saturation and the void ratio decrease. Moreover, the effect of the degree of saturation on the cyclic strength of unsaturated soils is higher than the effect of the void ratio. The unsaturated soils easily liquefied when the degree of saturation is higher than 70%.

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