Simulation of Water-Structure Interaction using CPDI

Simulation de l'interaction eau-structure utilisant CPDI

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ABSTRACT: Natural disasters such as floods, coastal erosion, Tsunami involve massive flow of fluid, which cause severe destruction of structures and erosion, as well as the loss of lives. In order to understand the interaction between structures and water for highly dynamic problems, numerical study is performed. However, the analysis is challenging due to the large deformation and the heavy movement of material; which is accommodated by the Material Point Method (MPM) - a particle-based method. It involves tracking particles, which carry the state parameters, moving through a fixed computational mesh. Classical MPM suffers from numerical instability called “grid crossing error”. To overcome this problem, the Convected Particle Domain Interpolation (CPDI) formulation distributes the mass of the particle over a finite domain rather concentrating at particle. Owing to the use of low order element in MPM and CPDI, shows a non-physical increase in the resistance against volume change for material of high bulk modulus. In this paper, a multi-field approach is introduced to mitigate the volumetric locking problem, which is combined with finite deformation formulation. Two examples have been presented to demonstrate the applicability of the proposed scheme: Interaction of a collapsing water column with an elastic body and a solid block sinking in water. The suggested procedure shows fairly smooth stress field and better free surface definition as compared with other numerical methods.

RÉSUMÉ : Les catastrophes naturelles telles que les inondations, l'érosion côtière ou les tsunamis impliquent un flux massif de fluide, qui entraîne une destruction grave des structures et l'érosion, ainsi que la perte de vies. Afin de comprendre l'interaction entre les structures et l'eau pour les problèmes très dynamiques, une étude numérique est réalisée. Cependant, l'analyse est difficile en raison des grandes déformations et du mouvement du matériau. Ceci est pris en compte par la Méthode de Point Matériau (MPM – *Material Point Method*) - une méthode basée sur les particules. Elle implique poursuivre les particules, qui transportent les paramètres d'état, se déplaçant à travers un maillage de calcul fixe. La MPM classique souffre d'instabilité numérique appelée «erreur de croisement de la grille». Pour résoudre ce problème, la méthode d'Interpolation par Domaines de Particules Convectés (CPDI - *Convected Particle Domain Interpolation*) distribue la masse des particules sur un domaine fini au lieu de la concentrer aux particules. En raison de l'utilisation d'éléments avec des fonctions de base d'ordre inférieur dans MPM et CPDI, on peut voir une augmentation non-physique de la résistance contre le changement de volume pour des matériaux à module d’élasticité isostatique élevé. Deux exemples ont été présentés pour démontrer l'applicabilité du plan proposé: Interaction d'une colonne d'eau qui s'effondre avec un corps élastique et un bloc solide qui coule dans l'eau. La procédure proposée montre un champ de tension assez lisse et une meilleure définition de surface libre par rapport à d'autres méthodes numériques.

KEYWORDS: MPM, CPDI, water-structure interaction, anti-locking, large deformation

1 INTRODUCTION.

Geo-mechanical applications are the main interest for developing MPM. The large deformations, involving extreme movement of material and measuring stress/pressure in the domain can be easily obtained through MPM. Practical applications like pile-installation, dropping anchors etc. are simulated using MPM, in which the soil is better modeled with MPM than other numerical methods. So in the present work water is modeled in MPM with some enhancements to address certain aspects related to behavior of water. Finally some cases of water-structure interaction are shown which can be related to applications such as controlling landslides and avalanches, also to study the structural behavior under the interaction of water.

The continuum model using only Lagrangian methods or only Eulerian methods performed significantly well at certain applications but failed to simulate the highly dynamic large deformation problems efficiently. Combining the best features of Eulerian and Lagrangian Material Point Method (MPM) is developed. In this method the material points called *particles* represent the Lagrangian continuum which carries the material information and flows through an Eulerian computational mesh. The computational mesh keeps no permanent information during computation; it only exchanges information with the particles. This method outperformed in large deformation cases (Sulsky et al. 1994, 1995).

Classical MPM shows instability during the movement of particles between elements in the computational mesh causing sudden jump in gradient of shape functions. In order to avoid this problem, a new variation called CPDI (Convected particle domain interpolation) is introduced by (Sadeghirad et al. 2011). Here the material points are interpolated to a finite spatial domain to have smooth distribution of information between elements. The domain is in parallelogram shape which evolved from circle, square, rectangular shapes of GIMP (Generalized interpolation material Point Method) (Bardenhagen and Kober 2004). The evolution of domains can be seen in Figure 1. Also the parallelogram domain is updated during the computation to eliminate the extensive stretching problem. These enhancements to MPM played an important role in simulating large deformation problems (Sadeghirad et al. 2013, Hamad 2016).

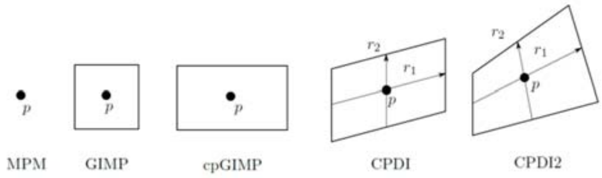

Figure 1. Evolution of Material Point Method and advanced variations

In an example of a hyper-elastic block which is allowed to deform under gravity load, difference between the MPM and CPDI can be seen in Figure 2. In MPM the block is represented by number of particles and as parallelograms in case of CPDI. Due to the concentration of material properties at a single point, there exists a sudden jump in states as particle moves to another grid. This is termed as grid-crossing or cell-crossing error. The discontinuous gradient is avoided by introducing finite domain for the particles which smoothens between the cells.

Another advantage of CPDI is with the post-processing, which can be observed in Figure 2. The domain of the block during deformation is clearly defined, providing the physical description of the body not just the particles (MPM). This is due to the consistent updating parallelogram domain i.e. according to the deformation gradient of the particle, the domain shape is updated. Linking the influence of particles with its deformation gradient causes numerical difficulties when the particle is heavily distorted. For this, (Homel et al. 2014) introduced a geometrical resizing procedure to limit the particle domain size within a fixed circle, whereas the deformation gradients contributes only to the material model.

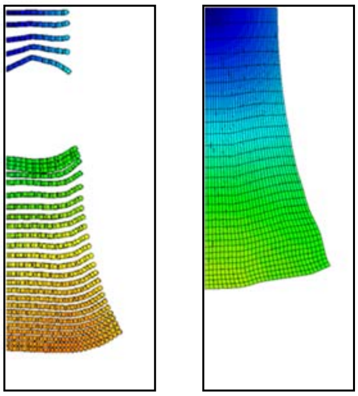

Figure 2. Hyperelastic block in gravity, MPM (left) and CPDI (right)

The paper is organized in the following manner; Section 2 provides the brief description about the enhancements implemented into CPDI formulation to simulate the physical behavior of water, which includes the anti-locking strategy and finite deformation algorithm.

Then in section 3, a validating example to show the effectiveness of the enhancements is presented with a water dam break example followed by examples for water-structure interaction. Interaction of an elastic bar and a rigid bar with water and a solid block sinking in water is presented. Finally in Section 4, the summary and findings are described.

2 ENHANCEMENTS TO CPDI FORMULATION

2.1 *Anti-locking strategy*

CPDI further suffers a non-physical increase in resistance against volume change for nearly or incompressible materials which is referred to as volumetric locking. The linear shape function used in the classical MPM formulation causes this volumetric locking and also discontinuity in gradients (cell-crossing error).

The CPDI formulation helps to avoid gradient discontinuity, multi-field formulation is considered for the elimination of locking. Because of the incomplete representation of isochoric components by linear shape functions, volumetric locking appears. To avoid this, the governing equation is split into deviatoric and volumetric parts to build approximations for stress and strain fields separately in contrast to the classical procedure where only approximations are built for only displacement field. For the construction of approximations, decomposition of fields into volumetric or deviatoric components are made using a filter matrix ($\mathbf{M}$); then a shape matrix $\mathbf{S}(x)$ is selected to interpolate the generalized degrees of freedom ($\boldsymbol{\alpha}, \boldsymbol{\beta}$) which is used to represent the state fields. The fields are then assembled using a reconstruction matrix ($\mathbf{M}^*$). This paper is concentrated mainly in modeling water where volumetric locking is significant due to the incompressibility. To address this, the formulation is made such that only volumetric portions of the fields are approximated without altering the deviatoric components (Mast et al. 2012, Hamad et al. 2016).

Mathematically, the basic equations are

$$\tilde{\boldsymbol{\varepsilon}}(\boldsymbol{x},t) = \mathbf{M}^* \cdot \mathbf{S}(\boldsymbol{x}) \cdot \boldsymbol{\alpha}(t) + \mathbf{P}{:}\, \boldsymbol{\varepsilon} \tag{1}$$

$$\tilde{\boldsymbol{\sigma}}(\boldsymbol{x},t) = \mathbf{M}^* \cdot \mathbf{S}(\boldsymbol{x}) \cdot \boldsymbol{\beta}(t) + \mathbf{P}{:}\, \boldsymbol{\sigma} \tag{2}$$

in which $\tilde{\boldsymbol{\varepsilon}}$ and $\tilde{\boldsymbol{\sigma}}$ are the approximated (smoothed) state fields; $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ are derived from displacement field approximation and $\mathbf{P}$ is the projection tensor. The $\boldsymbol{\alpha}$ can be obtained from the Equation 3, integrating over the domain Ω. Similarly $\boldsymbol{\beta}$ can be obtained.

$$\boldsymbol{\alpha} = \mathbf{H}^{-1} \cdot \int_{\Omega} \mathbf{S}^{\mathrm{T}} \cdot \mathbf{M}{:}\, \boldsymbol{\varepsilon}\, dm \quad \text{and} \quad \boldsymbol{\beta} = \mathbf{H}^{-1} \cdot \int_{\Omega} \mathbf{S}^{\mathrm{T}} \cdot \mathbf{M}{:}\, \boldsymbol{\sigma}\, dm$$
$$\text{with} \quad \mathbf{H} = \int_{\Omega} \mathbf{S}^{\mathrm{T}} \cdot \mathbf{S}\, dm \tag{3}$$

2.2 *Finite deformation algorithm*

As the small deformation theory assumption does not consider the rotation part of the deformation, it is required to adopt finite deformation theory. Finite deformation theory takes care of finite stretches and rotations combined with the stretches. Here, the deformation gradient ($\mathbf{F}$) which carries the complete information is used to obtain the left Cauchy-Green deformation tensor ($\mathbf{b}$). (Malvern 1969, Simo & Hughes 1998)

It is formulated as,

$$\mathbf{b} = \mathbf{F} \cdot \mathbf{F}^{\mathrm{T}} \tag{4}$$

Natural strain $\boldsymbol{\varepsilon}_l$ (logarithmic strain) is the popular and reliable strain measure used in the finite deformation cases; it is obtained from the decomposition of left Cauchy-Green deformation tensor into eigenvalues (b_i) and eigenvector $\mathbf{n}^i$.

$$\boldsymbol{\varepsilon}_l = 1/2\ log(\mathbf{b}) = \sum_{i}^{3} 1/2\ log(b_i)\ \mathbf{m}^i$$
$$where,\ \mathbf{m}^i = \mathbf{n}^i \otimes \mathbf{n}^i \tag{5}$$

3 WATER-STRUCTURE INTERACTION

The modeling of water is tested with a dam break example as shown in Figure 3. Here the water column is considered as to flow under gravity and hit the wall on the other side. In Figure 4, the non-physical behavior of water obtained without enhancements can be compared to the real behavior of water after the implementation of enhancements described in the previous sections.

Now the interaction of water with structure is studied. Usual way of dealing fluid-structure interaction is by modeling the structure as Lagrangian and fluid as an Eulerian body. In general, MPM considers the continuum to behave in Lagrangian way represented by material points or particles. Here, the fluid and structure which interact each other is seen as a single continuum with different material properties. During calculation, material is identified in the computational mesh and corresponding constitutive model is activated. Water is made to flow under gravity after the removal of retaining wall and it is allowed to interact with a structural body placed at a certain distance. A frictional contact algorithm is used in these simulations (Bardenhagen et al. 2000).

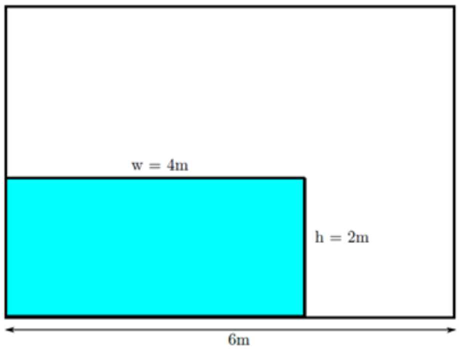

Figure 3. Initial configuration of water dam break example

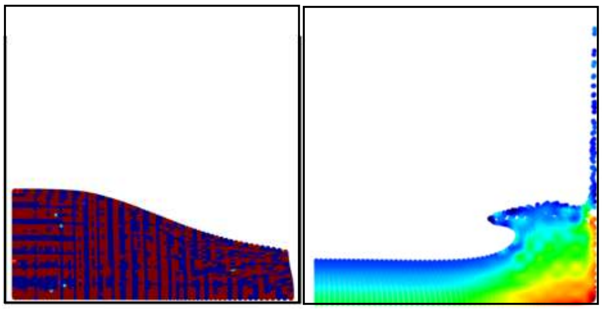

Figure 4. Water dam break example shown without (left) and with enhancements for CPDI formulation (right).

Two examples are considered to show the simulation of water interacting with a structure; a bar under the effect of water and a solid block sinking in water. The results are compared with a numerical method called Particle FEM (PFEM) and also compared with analysis using ABAQUS, where the coupled Eulerian-Lagrange (CEL) method is used to simulate the problem. In this method, the fluid domain is created as Eulerian part and the water material is assigned to it. The volume fraction tool is used to assign the material into Eulerian part by intersecting a reference body. The structural body is modeled as Lagrangian body. (Qiu et al. 2011)

3.1 Interaction with an elastic and rigid bar

The Initial configuration of the problem is shown in the Figure 5. Water column of height and width of 2L and L respectively, is considered. This is assumed to be in a box of width, 4L and held with the support of a retaining wall on the right face. The elastic bar is considered with the dimensions H and W, as shown in Figure 5. This body has the following material properties, density (ρ) = 2500 kg/m^3, Young's modulus (E) = 10 GPa and Poisson's ratio (ν) = 0. Water properties are taken as, density (ρ) = 1000 kg/m^3, Bulk Modulus (K) = 2.13GPa and Dynamic viscosity (μ) = 0.001 Pa s.

In the early stage of the simulation, water column collapses due to gravity attaining the hydrostatic pressure distribution, can be seen in the first picture of Figure 7.

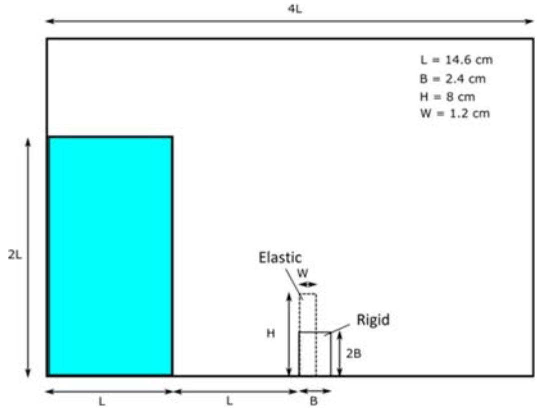

Figure 5. Initial configuration of the water-structure interaction problem

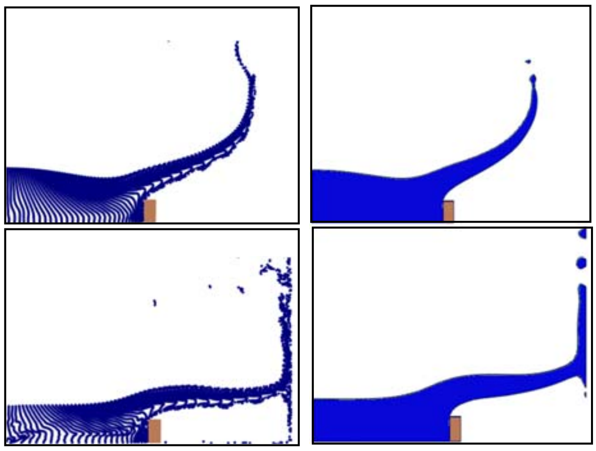

Figure 6. Water interaction with the rigid body, CPDI (left) and ABAQUS (right)

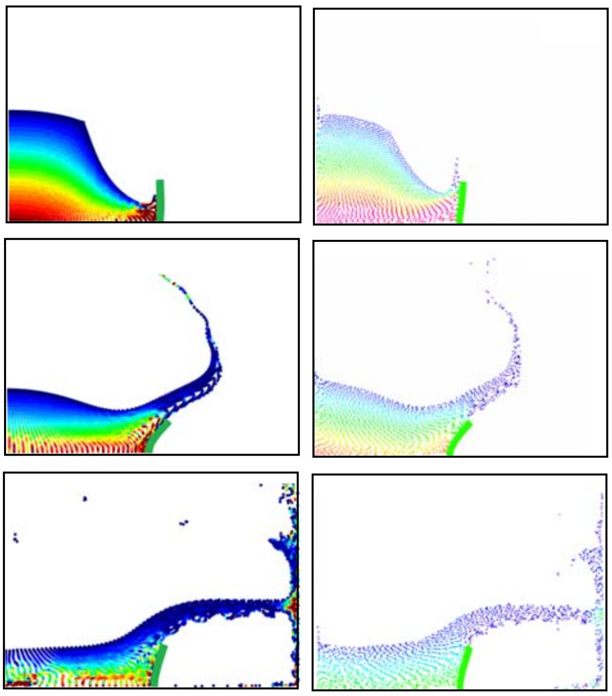

Figure 7. Water interaction with the elastic body, CPDI (left) and PFEM (right).

Then due to the exertion of pressure by water, the elastic body deflects and allows water to flow over it as seen in the second picture of Figure 7. Finally the wave hits the other side of the wall. This can be compared to the work of (Idelsohn et al. 2008), where the simulation is performed using PFEM method. Whereas in case of rigid bar no deformation or deflection is observed as the water flows over it can be seen in Figure 6.

3.2 Solid body sinking in water

Another example is considered to demonstrate the interaction of structural body with water. Here, a box of length = 0.3m is dropped from the height of 0.1m into a water tank of height = 2m and width = 2.5m. The density of the box is taken to be 10% more than water i.e., 1100 kg/m³ and high Young's modulus.

The box enters the water by splitting the surface creating a depression on its way. This can be observed in the first picture of Figure 8. Then as box continues to move inside, water covers it from the sides as shown in the second picture in Figure 8. Finally, due to the difference in density water covers the box from all the sides by sinking the box.

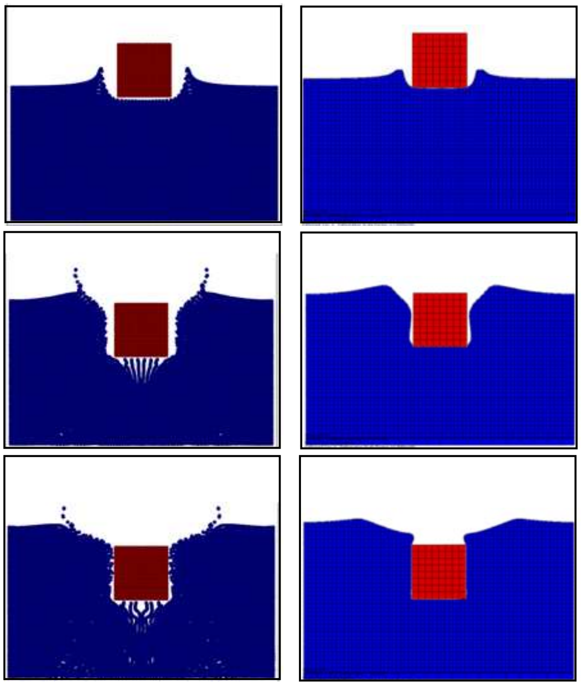

Figure 8. Solid block sinking in water simulated with CPDI (left) compared to ABAQUS (right)

4 SUMMARY

In this work, enhancements to CPDI procedure for the simulation of fluid-structure interaction problems is implemented, which provides the physical behavior of fluids. The enhancements provided to mitigate numerical errors is found to be effective than adopting higher order shape functions. The simplicity of linear shape functions served the purpose by providing economical results.

Also the consideration of finite deformation formulation and unified treatment of the fluid and solid makes the procedure accurate and simple. It can be observed that the results obtained in the present work provides a close match with the experimental and numerical results from different methods like PFEM, ABAQUS (FEM). This provides insight to the application of CPDI procedure into various water-structure interaction problems.

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