Energy Based Pore Water Pressure Formulation in a Cyclic Plasticity Model for Sand

Formulation de la pression des pores d'eau dans un modèle de plasticité cyclique pour le sable

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ABSTRACT: This paper presents an implementation of a stress-dilatancy relation developed by Ueng and Lee in 1990 to a cyclic plasticity model based on the concept of fuzzy set theory. The constitutive model is a rate independent elasto-plastic model capable of describing repeated load cycles, dilatancy, material memory, critical state soil mechanics features. In particular, the pore water pressure buildup is related to the dissipated energy computed according to the area outside the hysteresis loop between the tanϕ_u and the stress-strain relation. A numerical example on cyclic simple shear test and triaxial test was given to demonstrate the stress-dilatancy relation for further pore water pressure buildup modeling.

RÉSUMÉ : Cet article présente une mise en œuvre d'une relation stress-dilatance développée par Ueng et Lee en 1990 à un modèle de plasticité cyclique basé sur le concept de théorie des ensembles flous. Le modèle constitutif est un modèle élastoplastique indépendant de la vitesse capable de décrire les cycles de charge répétés, la dilatance, la mémoire des matériaux, les caractéristiques critiques de la mécanique des sols. En particulier, l'accumulation de la pression d'eau interstitielle est liée à l'énergie dissipée calculée en fonction de la zone située en dehors de la boucle d'hystérésis entre le tanϕ_u et la relation tension-déformation. Un exemple numérique sur le test de cisaillement cyclique simple et l'essai triaxial a été donné pour démontrer la relation stress-dilatance pour la modélisation future de l'accumulation de pression d'eau

KEYWORDS: Fuzze set plasticity, stress-dilatancy, cyclic loading.

1 INTRODUCTION

Within the framework of classical plasticity theory, isotropic hardening is sufficient to simulate the stress-strain behavior of soil under monotonic loading. Kinematic hardening or mixed hardening (isotropic and kinematic hardening) is typically used to mimic hysteretic phenomena of soil under cyclic loading, such as reverse plastic flow and memory of particular loading events. Koiter (1953) discussed the concept of multiple yield functions. Iwan (1967) extended Koiter's work by adding kinematic hardening to each of the yield surfaces. Independently, Mróz (1967) developed an anisotropic hardening model for metals based on the concept of a field of work-hardening moduli. Prévost (1978, 1985) adopted the work by Mróz in order to study the behavior of clays under monotonic and cyclic loading conditions and the anisotropic effect of initial consolidation. Mróz et al. (1978) proposed an anisotropic hardening model for soils by applying the concept of a field of hardening moduli. Besides the yield surface, a set of nesting surfaces, also called multi-surface plasticity, in stress space specifies the variation of hardening moduli during the deformation process. Both drained and undrained soil behavior under cyclic loading were simulated.

Sture et al. (1982) described details of analysis and application of a multi-surface anisotropic hardening model. The instantaneous configuration of the field of yield surfaces was established by computing the parameters and equations that govern the translation, expansion or contraction of individual surfaces during proportional as well as non-proportional loading and unloading. The surfaces change size with plastic volumetric strain, and the motion of the active yield surface and all interior surfaces is governed by Mróz's kinematic rule. Stress induced anisotropy is captured because the sizes and initial positions of the nesting yield surfaces reflect the past stress-strain history of the soil. However, the model is complicated and requires in excess of 50 input parameters and constants.

Bounding surface plasticity was originally introduced by Dafalias and Popov (1975), and independently by Krieg (1975) in conjunction with an enclosed yield surface for metal plasticity. Since then, bounding surface plasticity has been developed and implemented by numerous researchers mainly to model clay soils. Bardet (1985) proposed a bounding surface plasticity model for sands. The model describes strain-softening, stress-dilatancy, drained and undrained behavior, hysteretic energy dissipation and irreversible strain during cyclic loading test. Along the same line, Poorooshasb and Pietruszczak (1985) developed a generalized two-surface model for granular materials. Pietruszczak and Stolle (1987) modified it and incorporated a non-associated flow rule to perform a finite element analysis of liquefaction potential of a loose sand layer subjected to earthquake loading. Manzari and Dafalias (1997) presented a model within the framework of critical state soil mechanics.

2 FUZZY SET PLASTICITY MODEL

The concept of the fuzzy set plasticity was first introduced by Klisinski et al. (1988). Unlike conventional elasto-plastic hardening models, the fuzzy set model is physically intuitive and easy to visualize. It provides analytical and simple geometric interpretation to formulate hardening, hysteresis features, material memory, and kinematic mechanisms without using complicated kinematic hardening formulations.

2.1 Membership function

The membership function has been involved in the plastic modulus equations. When $\gamma = 1$, the material behaves purely elastically and the corresponding value of the plastic modulus is infinite, while when $\gamma = 0$, the material reaches a fully plastic state and the plastic modulus is equal to the value on the fuzzy surface, i.e. $H = H^*$.

With the assistance of the membership function γ, we can readily construct reversal plastic loading without resorting to a kinematic hardening rule. The basic rules of kinematic mechanism for the membership functions are: for plastic loading and plastic unloading, $\dot{\gamma} < 0$; for elastic loading and elastic unloading, $\dot{\gamma} \geq 0$. Although the value of the membership function is 1 at a fully elastic state and 0 at the fully plastic state, the assignment of the value in elastoplastic state is deterministic and can be arbitrarily defined as needed. A linear variation with respect to stress state was adopted in this study.

Figure 1 displays an example of the deviatoric stress-strain response and evolution of the membership function for a material subjected to two varied amplitude cyclic loading under a conventional triaxial stress path. The unloading-reloading points take place in two different stress levels q = 156 kPa and q = 231 kPa, respectively. The two graphs on the left highlight cycle 1 with the loading from 0 to 156 kPa and unloading from 156 to 0 kPa (in solid line). The other two graphs highlight the cycle 2 with the loading from 0 to 231 kPa and unloading from 231 to 0 kPa (in solid line).

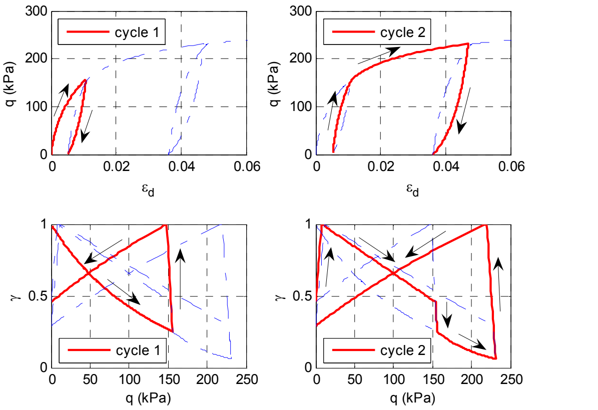

Figure 1. the deviatoric stress-strain response and evolution of the membership function.

2.2 Fuzzy surface

The fuzzy surface is a three-stress-invariant function which can be expressed as:

$$F^c = q\text{-}a_0\text{-}a_1 p \quad \text{in triaxial compression} \qquad (1)$$

$$F^e = q\text{-}b_0\text{-}b_1 p \quad \text{in triaxial extension} \qquad (2)$$

where p is the first stress invariant corresponding to mean stress and q is the second deviatoric stress invariant relating to shear stress. a_0, a_1, b_0, and b_1 are material constants that can be determined from triaxial compression and extension tests.

2.3 Plastic modulus

Since the loading surface is not explicitly defined in the fuzzy set plasticity model, one can think that for the current stress state, there exists a loading surface such that the plastic modulus is defined as follows. For plastic modulus:

$$H = M\gamma^d \qquad (3)$$

d and M are model parameters that can be determined from test data.

2.4 Flow rules

Plastic flow rules are expressed as $\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\boldsymbol{m}$, where $\boldsymbol{m}$ is the direction of plastic deformation increments. In classical plasticity theory, the plastic potential is required to determine the direction of the plastic deformation increment. In the fuzzy set plasticity theory, a forth-order tensor is defined in such a way that $\mathbf{m} = \mathbf{T} : \mathbf{n}$, where T is a forth-order tensor. If $\mathbf{T} = \mathbf{I}$, we have associated plastic flow. Therefore, the effect of non-associated flow rules is achieved by defining a fourth-order tensor **T** so that the plastic potential function need not be known explicitly. This fourth-order tensor **T** is defined as:

$$\mathbf{T} = \mathbf{I}\text{-}\frac{1}{3}(1 - D)\mathbf{1}\otimes\mathbf{1} \qquad (4)$$

where D is called *dilatancy parameter* and will be described in the next section.

2.5 Stress-dilatancy relation

The stress-dilatancy relation implemented in the fuzzy set plasticity was adopted fromn Ueng and Lee (1990), which leads to developing an energy based pore water pressure generation model. This relation was derived assuming that the contact forces between grain must be in equilibrium against the external forces. When the external forces change (e.g. due to earthquake), the contact forces will change both in magnitude and direction, where the deformation of the sand takes place. It was also assumed the deformation of the sand grains is negligible; therefore, the deformation of the sand occurs purely as a result of the movement and rearrangement of the sand grains. The stress-dilatancy relation developed by Ueng and Lee (1990) is given as follows.

$$\frac{\tau}{\sigma_n^{'}} = \pm\tan\phi_\mu \pm b\left(\frac{d\varepsilon_v}{d\gamma}\right) \qquad (5)$$

where ϕ_μ is basic friction angle between sand grains and b is a positive scalar related to shearing plan orientation, friction angle, and sand fabric.

Figure 2 shows the simple shear tests results on Ottawa sand that are presented in the form corresponding to equation 5. It shows that the dilatancy rate and stress ratio give a linear behavior.

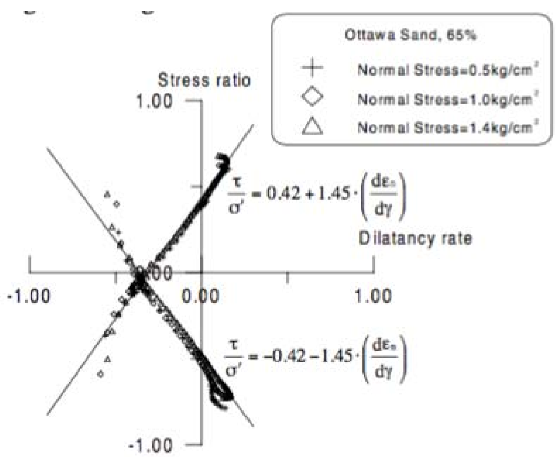

Figure 2. Stress-dilatancy relation for Ottawa sand under simple shear (from Ueng et al. (2000))

2.6 Energy based pore water pressure model

For a saturated sand under undrained cyclic loading, such as earthquake, the total volumetric strain is the sum of the volumetric strains of solid particles and pore water. Assuming the solid particles are incompressible, the volumetric strain of the sand can be presented as:

$$d\varepsilon_v = \text{-}n\frac{du}{K_w} = n\frac{d\sigma_n^{'}}{K_w} \qquad (6)$$

where n is the porosity of the sand, du is the change of pore water pressure, $d\sigma_n$' is the change of effective of normal stress and K_w is the bulk modulus of pore water.

In combining the above mentioned stress-dilatancy relation and introducing the rebound volumetric modulus of sand structure, K_r, the energy form of the pore water pressure and stress-dilatancy can be exressed as follows (Ueng et al. 2000).

$$\left(\frac{n}{K_w}+\frac{1}{K_r}\right)\int \sigma_n' \cdot \mathrm{d}\sigma_n' = \frac{1}{b}\left[\int \tau \cdot \mathrm{d}\gamma - \int \pm\left(\sigma_n' \cdot \tan\phi_\mu\right)\cdot \mathrm{d}\gamma\right] \quad (7)$$

It is worth mentioning that the pore water pressure buildup is not directly related to the energy dissipation within the hysteresis loop shown in the Figure 3. In stead, it is pertaining to the area outside the loop between the tanϕ_μ and the stress-strain relation.

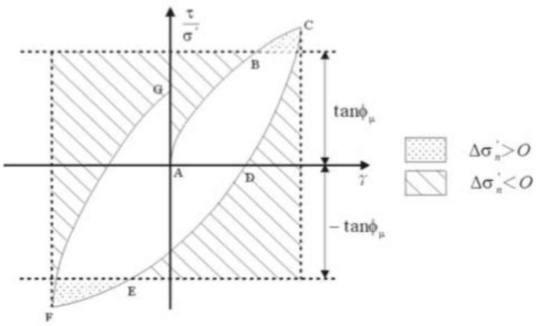

Figure 3. Stress ratio versus strain and effective stress changes under undrained loading (from Ueng et al. (2000))

3 NUMERICAL EXAMPLES

It can be seen from equations 6 and 7 that the volumetric strain obtained under drained condition correlates to the pore water pressure change due to undrained condition. The numerical example given below demonstrates the the stress-strain response under cycle drained loading. The constitutive model parameters are listed in Table 1, where a_1, a_0, b_1 and b_0 are set for the fuzzy surfaces. K and G are the bulk modulus and shear modulus, respectively. d and M are associated to the plastic modulus. Finally, ϕ_u and α are related to stress-dilatancy behavior.

Table 1. Model parameter for the fuzzy set plasticity model

Parameter	Value
$a_1 = b_1$	1.5
$a_0 = b_0$	0.0
K	40000
G	20000
d	1.4
M	18000
ϕ_u	30°
α_0	1.3

Figures 4 and 5 show the stress-strain response under simple shear and conventional triaxial compression conditions. It can be seen in Figure 4 that the hysteresis loops are symmetric to the origin as the effective mean stress remained constant (100 kPa) during simple shear. The deviatoric strain changes back and forth around 3% while the volumetric strain is accumulated up to 6%. As shown in Figure 5, on the other hand, the deviatoric stress is applied between 300 kPa and -100 kpa while the initial effective confining pressure is set as 100 kPa. Both accumulated volumetric strain shows butterfly type behaviors as load cycles continue.

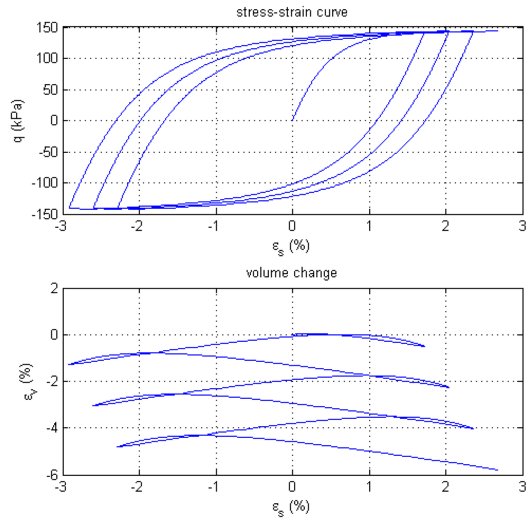

Figure 4. Stress-strain response under simple shear condition

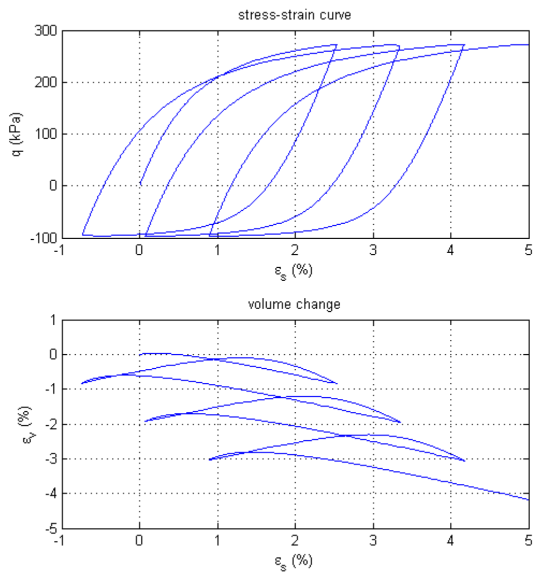

Figure 5. Stress-strain response under conventional triaxial compression condition

4 CONCLUSION

A cyclic constitutive model based on fuzzy set concepts has been developed. The cyclic fuzzy set model is physically intuitive and easy to visualize with the aid of membership functions. The cyclic fuzzy set model provides analytical and simple geometrical interpretation to formulate hardening, hysteresis features, and kinematic mechanisms without invoking complex analytical formulations. In addition, the cyclic fuzzy set model accounts for: realistic stress-strain behavior under repeated load cycles, nonlinear dilatancy behavior, and critical state soil mechanics concepts. The Ueng and Lee type of stress-dilatancy relation was successfully implemented to the described fuzzy set plasticity model. Both numerical examples shows reasonable vomuletric strain behaviors where can be directly correlated to pore water pressure buildup during cyclic loading.

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