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# Failure probability calculation for machine foundations coupled rocking and sliding vibration on soil with random elastic parameters

Calcul de probabilité de défaillance pour les fondations de machines sur le sol avec des paramètres élastiques aléatoires dans les conditions de vibrations couplé de translation horizontale et de basculement

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**ABSTRACT:** Soil parameters uncertainty is usually accounted for using safety factors. Nonetheless in case of vibration calculation the idea of "safe side" is generally inapplicable. Probabilistic approach is one of the tools to overcome this difficulty. So a 2 degrees of freedom system was analyzed to simulate the most frequent case of coupled rocking and sliding of rigid foundation. The dynamic load was taken sinusoidal having random amplitude with Rayleigh distribution. The expression for the vibration amplitude of the deterministic system with explicit and simple dependence on soil shear elasticity modulus was obtained. This modulus was supposed to have lognormal distribution. The failure probability was expressed as 1-dimensional integral. Thus a simple method was obtained accounting for the effect of random uncertainty, which is inherent to both soil parameters and dynamic load amplitude, and provides an alternative way for design estimation of machine foundations under dynamic loads.

**RÉSUMÉ :** Paramètres de sol incertitude est généralement représenté en utilisant des coefficient de sécurité. Néanmoins, en cas de calcul de vibrations l'idée de "cote de sécurité" est généralement pas applicable. Approche probabiliste est l'un des outils pour surmonter cette difficulté. Un système à 2 degré de liberté pour simuler de vibrations couplé de translation horizontale et de basculement de fondation rigide a été analysée. La charge dynamique est sinusoidale ayant une amplitude aléatoire avec distribution de Rayleigh. L'expression de l'amplitude de vibration d'un système déterministe a été obtenue avec la dépendance explicite et simple sur le cisaillement du sol module d'élasticité. Alors le module était censé avoir distribution log-normale. La probabilité de défaillance a été exprimée en intégrale unidimensionnelle. Ainsi une méthode simple a été obtenue pour le calcul de la probabilité de défaillance de vibration sinusoidale de base rigide. Il représente l'effet de l'incertitude aléatoire, qui est inhérente à la fois les paramètres du sol et de l'amplitude de charge dynamique, et offre une voie alternative pour l'estimation de la conception des fondations de machines soumises à des charges dynamiques.

**KEYWORDS:** random parameters, probabilistic model, failure probability, machine foundations, soil elasticity, rocking and sliding.

## 1 INTRODUCTION

Uncertainty in geotechnical engineering is very important; it is treated with different approaches (Réthati 1988, Nadim 2007, Phoon 2008 and others). Many of them result in limiting values that must not be exceeded to provide safety or serviceability.

The most common way in practical calculation is changing input soil parameters to the conservative side. Unfortunately both overestimation and underestimation of soil stiffness may lead to underestimation of vibration level; the latter may be significant in case of resonance. As a consequence applying of partial reliability factors seems to be impossible.

The most common way to overcome this difficulty in dynamic problems is employing probabilistic approach; some papers using it were reviewed earlier (Kholmyansky 2008). The degree of uncertainty of dynamic soil behaviour is especially great in widespread situations when dynamic parameters are determined not in a direct way, but through correlation dependencies with other geotechnical parameters.

Probabilistic approach to uncertainty is used as the most developed one. It consists in representing soil parameters by random variables having specific distribution laws instead of taking soil variability into account by introducing partial safety factors for soil parameters. This approach describes reliable

operation by the condition of reliability (Bolotin 1984, Sheynin et al. 1990), when the norms are set for probability of failure or of reliable operation. Some results in the field of dynamic soil-structure interaction were obtained (see Tseytlin and Guseva 1979, Kholmyansky 1995, 2008 and 2011; Králik and Králik 2009).

The paper is devoted to failure probability studying in case of soil random stiffness and random load. The most important case of massive block-like (or wall-like) machine foundations is considered.

The simplest problem for single-degree-of-freedom system was solved earlier (Kholmyansky 2011). Now a more complicated 2 degrees of freedom system to simulate coupled rocking and sliding of rigid foundation is analysed.

In this paper we consider the amplitude of sinusoidal load corresponding to the action of theoretically balanced machines with uniformly rotating parts. There are some reasons to suppose that this amplitude is a random variable with well-known Rayleigh distribution. In particular, it may be derived from supposition that the dynamic load is caused by random variations of inertial and geometrical properties of the machine rotating parts. It is reasonable to suppose that the random load is statistically independent of soil stiffness.

First task to be solved is obtaining deterministic response in the most convenient form. The second task is finding the failure probability itself.

## 2 PROBABILISTIC PROBLEM STATEMENT

Mainstream approach (Bhatia 2008, SP 26.13330.2012) to reliability of operation of machine foundations under dynamic loads requires limiting the vibration amplitude with the inequality

$$a \leq a_u \quad (1)$$

where  $a_u$  is the admissible value. For the probabilistic approach Eq. (1) is replaced by the condition of sufficient reliability:

$$\text{Prob}[a > a_u] = P_f < P_u = 1 - R_u \quad (2)$$

where  $P_f$ — failure probability,  $R_u$ — limiting reliability and  $P_u$ — limiting failure probability.

The most important input data for the calculation is soil elastic shear modulus  $G$ , especially for rigid foundations (block-like and wall-like). The other parameters of the soil-structure system — foundation dimensions, densities of structural materials and soil, machine mass and moments of inertia — have significantly less uncertainty and are supposed deterministic.

The load is harmonic with deterministic angular frequency  $\omega$  and with amplitude  $F$  being a random variable with well-known Rayleigh distribution as stated earlier.

## 3 MACHINE-FOUNDATION-SOIL SYSTEM DYNAMIC RESPONSE

The matrix equation of small vibrations of the rigid foundation on soil reads as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \quad (3)$$

where  $\mathbf{q}$  and  $\mathbf{Q}$  are column vectors of generalized displacements (translations and rotations) and generalized forces (corresponding forces and moments);  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the matrices of inertia, damping, and stiffness of soil correspondingly. Generally the system has 6 degrees of freedom; in the case under consideration (symmetrical coupled rocking and sliding of rigid foundation) this value is reduced to 2.

The final expression for the vibration amplitude reads (Kholmyansky 2011)

$$a = F / |D(G, \omega)| \quad (4)$$

where  $D(G, \omega)$  is complex scalar dynamic stiffness,  $|\dots|$  — absolute value of a complex number,  $\omega = 2\pi f$  — angular frequency.

The expression for  $D(G, \omega)$  is obtained from Eq. (3):

$$D(G, \omega) = \frac{1}{\mathbf{1}^T (-\omega^2 \mathbf{M} + i\omega \sqrt{G} \mathbf{C}_0 + G \mathbf{K}_0)^{-1} \mathbf{g}} \quad (5)$$

where  $\mathbf{1}$  and  $\mathbf{g}$  are constant vectors describing observation point and moment loading correspondingly. Matrices  $\mathbf{M}$ ,  $\mathbf{C}_0$  and  $\mathbf{K}_0$  do not depend on  $G$  in a broad range of situations, the simplest being elastic half-space (Richart et al. 1970) or soil elasticity model of Russian codes (SP 26.13330.2012).

## 4 COUPLED ROCKING AND SLIDING OF RIGID FOUNDATION ON ELASTIC HALF-SPACE

Equations for vibrations of symmetrical rigid foundation may be written using the displacement  $x$  of a preselected point and rotation angle  $\psi$ . The point is chosen to be either the centre of mass of the machine-foundation system (see Richart et al. 1970) or the centroid of the foundation footing (Bhatia 2008); the latter case is selected, giving

$$\mathbf{M} = \begin{bmatrix} m & mh_0 \\ mh_0 & I \end{bmatrix} \quad (6)$$

$$\mathbf{K} = G \mathbf{K}_0 = \begin{bmatrix} k_x & 0 \\ 0 & k_\psi \end{bmatrix} \quad (7)$$

$$\mathbf{C} = \sqrt{G} \mathbf{C}_0 = \begin{bmatrix} c_x & 0 \\ 0 & c_\psi \end{bmatrix} \quad (8)$$

$$k_x = \frac{32(1-\nu)}{7-8\nu} G r_0^3 \quad (9)$$

$$k_\psi = \frac{8}{3(1-\nu)} G r_0^3 \quad (10)$$

$$c_x = \frac{18.4(1-\nu)}{7-8\nu} r_0^2 \sqrt{\rho G} \quad (11)$$

$$c_\psi = \frac{0.8}{(1-\nu)(1+B_\psi)} r_0^4 \sqrt{\rho G} \quad (12)$$

$$B_\psi = \frac{3(1-\nu)I}{8\rho r_0^5} \quad (13)$$

$$\mathbf{1} = [1 \quad h]^T; \quad \mathbf{g} = [1 \quad h_F]^T \quad (14)$$

where  $m$  and  $I$  are the foundation and machine mass and moment of inertia,  $h_0$  — the height of the machine-foundation system centre of mass above the footing centroid.

Elastic homogeneous half-space possesses elastic shear modulus, Poisson ratio and density  $G$ ,  $\nu$ , and  $\rho$  correspondingly. The values  $h$  and  $h_F$  are heights of observation point and dynamic load line above the base correspondingly.

## 5 PROBABILISTIC PROPERTIES OF INPUT DATA

Experimental data (Barkan et al. 1974) are compatible with the hypothesis of lognormal distribution of elasticity modulus (Kholmyansky 2008); coefficient of variation for elastic shear modulus is taken equal to 0.3. This distribution takes place when the elasticity modulus is taken to be proportional to the deformation modulus; this relation is used in Russian code (SP 26.13330.2012). All the other parameters of soil and foundation are supposed deterministic

The dynamic load is sinusoidal time-dependent with random amplitude. This assumption corresponds to the action of theoretically balanced machines with uniformly rotating parts. There are some reasons to suppose that this amplitude is a random variable with well-known Rayleigh distribution:

$$\text{Prob}[F < F_0] = 1 - \exp\left(-\frac{\pi F_0^2}{4\bar{F}^2}\right) \quad (15)$$

In this equation  $F_0 > 0$ ;  $\bar{F}$  is the mathematical expectation of the random load amplitude  $F$ ; coefficient of variation of  $F$  equals 0.523. The random load is supposed stochastically independent of soil stiffness.

## 6 FAILURE PROBABILITY CALCULATION

The general formula for failure probability (Kholmyansky 2008) is

$$P_f = \int_0^{\infty} p_G(G) \text{Prob}[F > F_u(G)] dG \quad (16)$$

In case of Rayleigh load from Eq. (15) the integral in Eq. (16) becomes:

$$P_f = \int_0^{\infty} p_G(G) \exp\left(-\frac{\pi F_u^2(G)}{4\bar{F}^2}\right) dG \quad (17)$$

The desired failure probability in Eq. (17) may be computed numerically without difficulty due to rapid decrease of integrand for large  $G$ .

## 7 WORKED EXAMPLE

Consider machine supported by cylindrical foundation with the following parameters:

radius  $r_0 = 1$  m

height  $h = 2$  m

density (average, with cavities)  $= 2$  t/m<sup>3</sup>

soil shear elastic modulus  $\bar{G}$  (expected value)  $= 50\,000$  kN/m<sup>2</sup>

soil Poisson ratio  $\nu = 0.35$

soil density  $\rho = 1.8$  t/m<sup>3</sup>

machine force amplitude  $\bar{F}$  (expected value)  $= 1$  kN

admissible amplitude  $a_u = 5 \times 10^{-5}$  m

machine lumped mass  $= 0.4 \times$  foundation mass  $= 5.0$  t

horizontal action line height above foundation  $= 2.5$  m

Consider the deterministic problem first — random soil elastic modulus and machine load are substituted by their expected values. The result is shown on Figure 1 (a). This resonant curve shows no second resonance due to high damping. The design is supposed to be reliable if  $a/a_u < 1$ .

Proceed to the probabilistic problem. It seems reasonable to prescribe relatively high admissible failure probability:  $P_u = 0.1$  (Tseytlin and Guseva 1979). The result is shown on Figure 1 (b). The design is supposed to be reliable if  $P_f/P_u < 1$ . It may be seen from Figures 1 (a) and 1 (b) that for the supposed conditions the frequency range of reliable operation significantly broadens with the shift to the probabilistic problem statement.

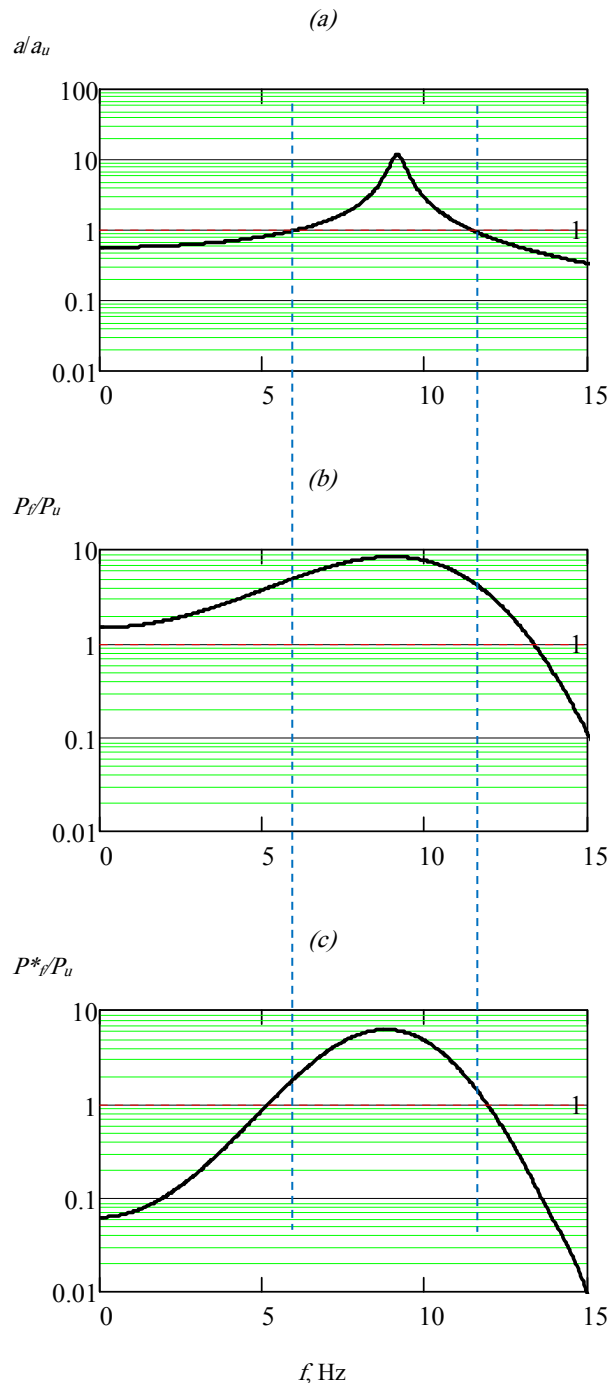


Figure 1. Failure frequency zones for deterministic problem statement (a), probabilistic problem statement (b) and probabilistic problem statement with twice increased admissible amplitude (c)

Attempting to bring together the two results suppose that the admissible amplitude may be increased. Such an attempt is supported by some case studies where measured amplitudes were greater than limiting value while no damage was observed. The results for increased admissible amplitude  $2a_u$  are shown on Figure 1 (c). Increasing admissible amplitude leads to closer conformity of reliable operation frequency ranges for deterministic and probabilistic problem statements.

## 8. CONCLUSIONS

Sinusoidal vibration of rigid foundation with coupled sliding and rocking may be easily analysed with account for probabilistic indeterminacy of both soil parameters and dynamic load amplitude giving failure probability and providing an alternative way of machine foundation project assessment.

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