Bearing capacity of shallow impervious footing in soil under sub-vertical seepage

Capacité portante d'une fondation superficielle et imperméable sur sol soumis à percolation sous-vertical

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ABSTRACT: Seepage in soils changes the effective stresses in the soil masses due to seepage forces. Bearing capacity of shallow foundations in soils with strictly vertical seepage can be determined applying the classic bearing capacity equation using a unit weight taking into account the seepage forces. This strictly vertical seepage can only be possible if the footing is pervious. Impervious foundations change the flow net and therefore the seepage forces. The present paper evaluates the influence of this change on the bearing capacity of shallow foundations. A finite element code with the capability of performing seepage and upper-bound plastic calculations was used. A parametric analysis of geometric (soil height to footing width ratio), hydraulic (average gradient) and mechanical (soil friction angle) parameters is performed and their influence on the bearing capacity and on the failure mechanisms is presented and analyzed. Results show that: (1) the effect on the bearing capacity is greater for soils with lower friction angles; (2) this effect is nonlinear and therefore cannot be described by a change in the soil unit weight only; and (3) for the same average hydraulic gradient, the geometric parameter has little influence on bearing capacity.

RÉSUMÉ: La percolation d'eau à travers le sol modifie la répartition des contraintes effectives à cause des forces de percolation. La capacité portante des fondations superficielles sur sols soumis à un écoulement strictement vertical peut être déterminée avec l'expression classique de la capacité portante en utilisant un poids volumique prenant en compte les forces de percolation. Un écoulement strictement vertical n'est possible qu'en présence d'une fondation perméable. Si la fondation est imperméable le réseau de flux est altéré et, donc, les forces de percolation aussi. Le travail présenté étudie l'influence de ce changement dans la capacité portante des fondations superficielles. Un code d'éléments finis ayant la capacité de résoudre des problèmes de l'écoulement et de calcul plastique utilisant le théorème cinématique de l'analyse limite a été utilisé. Une analyse paramétrique concernant la géométrie (rapport entre la hauteur du sol et la largeur de la fondation), les conditions hydrauliques (gradient moyen) et un paramètre mécanique (angle de frottement du sol) a été réalisée. Les résultats montrent que : (1) l'effet sur la capacité portante est plus élevé dans les cas des sols avec un angle de frottement plus bas ; (2) il s'agit d'un effet non-linéaire qui ne peut pas être décrit par le simple changement du poids volumique ; et (3) que l'influence du paramètre géométrique est petite pour la même valeur du gradient hydraulique.

KEYWORDS: bearing capacity, shallow foundations, seepage forces, finite element method, upper bound theorem.

1 INTRODUCTION. FIRST LEVEL HEADING

Bearing capacity of shallow foundations in soils under drained conditions is a classical soil mechanics problem. It can be given by:

$$q_r = 0.5\gamma B N_\gamma + q' N_q \quad (1)$$

for vertical centered loading and for a long footing, where γ is the soil unit weight, B is the footing width, q' is the vertical effective stress at the level of the base of the footing and N_γ and N_q are bearing capacity factors, depending on the soil friction angle.

If a foundation was placed at the soil surface, without any embedment, the second parcel of eq. (1) would be null and in the first one, γ would be the total unit weight of the soil or the submerged one, γ', depending on the water level. Values of N_γ can be determined using different proposals, but the results given by Martin (2005) should be mentioned.

In the present work, the effect of sub-vertical seepage on the bearing capacity of shallow foundations is studied. Two scenarios are considered (Figure 1). In both cases a footing with width B is considered (in Figure 1 the with b is equal to $B/2$), and a rigid stratum is underlying the soil at depth H. The water level is assumed at the soil surface. The hydraulic head, h, is defined by:

$$h = z^* + \frac{u}{\gamma_w} \quad (2)$$

where z* is the elevation relative to a reference plane, u is the pore pressure and γ_w is the unit weight of the water. The difference between the heads at the rigid stratum and at the soil surface is Δh.

For the first scenario, the footing is assumed pervious and therefore seepage is vertical in every point of the soil. In the second scenario the footing is impervious, which will affect seepage in its vicinity and causing non-null water pressures at its base. For the first scenario, the solution is well known and it can be obtained by replacing γ in eq. (1) by:

$$\gamma^* = \gamma' - i\gamma_w = \gamma'\left(1 - i / i_{cr}\right) \quad (3)$$

where i is the hydraulic gradient given by $\Delta h/H$ and i_{cr} is the critical gradient, given by γ'/γ_w. The second scenario is the aim of the present work, and the first scenario is included in this paper as a reference and a verification case.

2 NUMERICAL MODEL

For determining approximations of the collapse load q_r a numerical program developed at the Department of Civil Engineering of Faculty of Sciences and Technology of UNL is used. The program numerically implements the upper bound theorem of limit analysis and uses the finite element method. It is described by Vicente da Silva and Antão (2008) and by Antão et al. (2012). Upper bound collapse load approximations can be obtained by its application to mechanical problems and it has been applied to different types of geotechnical structures (Antão et al., 2011, 2012; Simões et al., 2014).

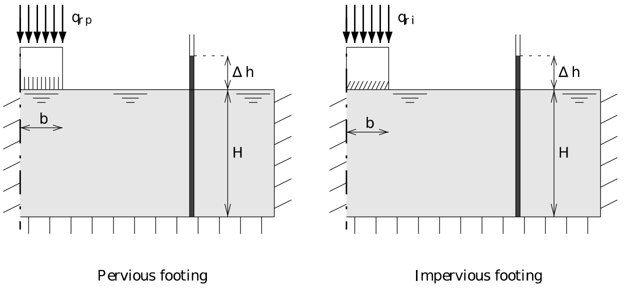

Figure 1: Schematic representation of the scenarios analyzed.

In a saturated soil the effective stresses depend on the pore pressures. In case of seepage, seepage forces are added. These forces are defined by:

$$\vec{F}_P = -\gamma_w \vec{\nabla} h \tag{4}$$

Therefore, the determination of the collapse loads of geotechnical structures involving soil submitted to seepage needs the previous determination of the hydraulic head and of the corresponding seepage forces, which are then introduced as loading for the determination of the collapse loads.

Only permanent flow is considered in the present work. The solution involves solving Laplace's equation, considering border and flow conditions. The solution of this problem using the finite element method is described in several text books, such as Zienkiewicz and Taylor (2000).

The finite element program which determined the collapse loads of mechanical problems under 2D conditions uses 3 and 6-noded triangular elements, with linear and quadratic approximations, respectively, for the velocity field. They are used in the program because they assure a strict implementation of the upper bound theorem. In the present work, only the 6-noded elements are used.

Solution of Laplace's equation allows determining the head, h, at each element, that can be written as:

$$h(x, z^*) = [N] \cdot \{\underline{h}\} \tag{5}$$

where $[N]$ contains the shape functions for the 6-noded elements and $\{\underline{h}\}$ the heads at the element nodes.

Seepage forces in eq. (3) can be approximated by:

$$\{F_P\} = -\gamma_w \{B\} [N] \cdot \{\underline{h}\} \tag{6}$$

where $\{B\}$ is the gradient differential operator.

Displacements of the element are approximated by:

$$\begin{Bmatrix} u_x \\ u_{z^*} \end{Bmatrix} = [N^*] \cdot \{u^n\} \tag{7}$$

where $[N^*]$ is a matrix containing the shape functions and $\{u^n\}$ contains the displacements in the x and z* directions of the element nodes. Nodal forces caused by seepage forces can be written as:

$$-\gamma_w \iint [N^*]^T \{B\} [N] \cdot \{\underline{h}\}\, dx dz^* \tag{8}$$

These forces are added to the forces caused by the submerged unit weight, γ. Both types of forces are assumed fixed in the finite element module that determines the collapse load q_r, and the mechanism that minimizes q_r is searched for.

3 CALCULATIONS

For the two scenarios presented in Figure 1, three values of the soil friction angle were considered and a submerged unit weight of *10 kN/m³* was assumed. The footing was considered rigid and perfectly rough. Calculations were performed for ratios *H/b* of *2* to *6*. For several values of the hydraulic gradient the bearing capacity q_r was determined using different finite element meshes, with different finite element sizes. As observed in other situations (Simões et al., 2014), a linear relation between the values of q_r and the element size was obtained. Therefore, the values reported in this work were derived by extrapolating the results that would be obtained by an ideal mesh, composed of an infinite number of infinitesimal elements, based on the actual computational results.

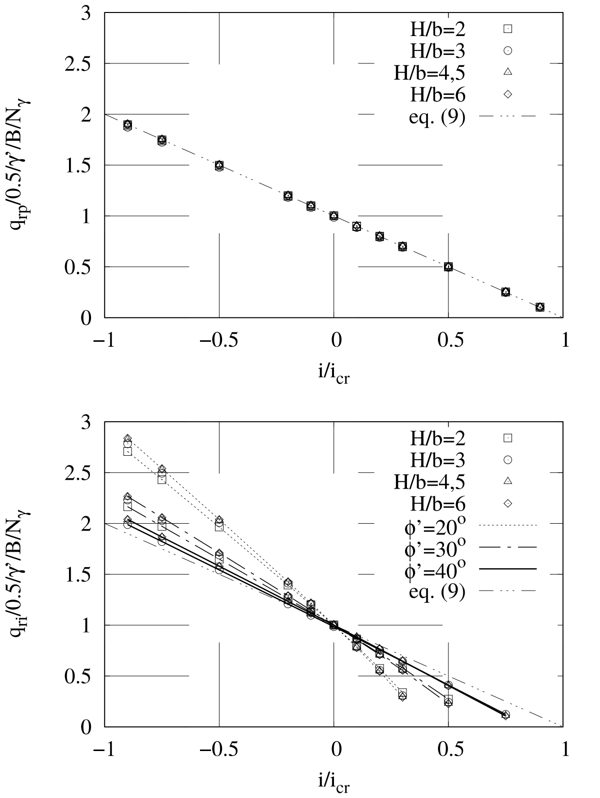

Figure 2; Dimensionless bearing capacities determined from the calculations, using the values of Ng obtained for pervious footing

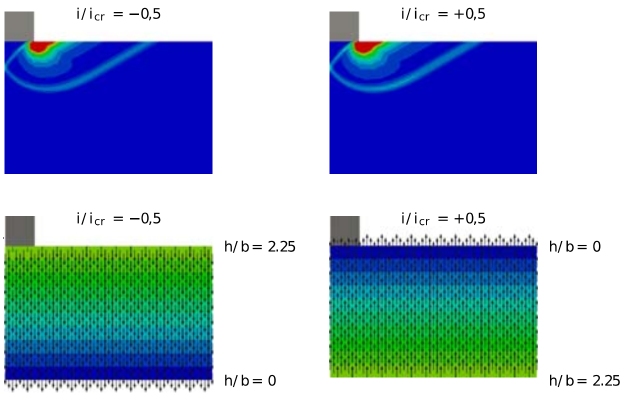

Figure 3: Pervious footing: plastic strains, hydraulic heads and seepage forces for $\phi'=30^o$, $H/b=4.5$ and $i/i_{cr}=\pm0.5$.

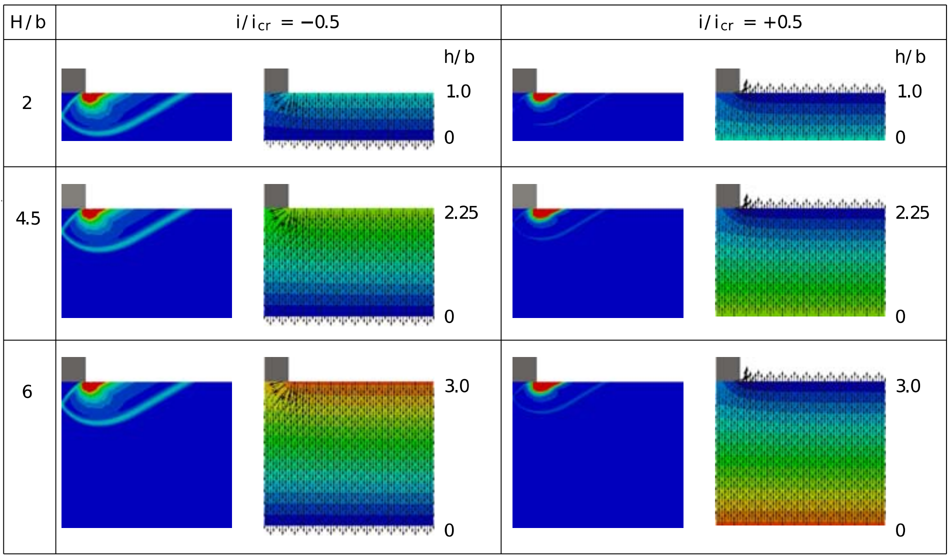

Figure 4: Impervious footing: plastic strains, hydraulic heads and seepage forces for $\phi'=30^o$, $H/b=2$, 4.5 and 6, and $i/i_{cr}=\pm 0.5$.

4. RESULTS OBTAINED FOR THE PERVIOUS FOOTING

Results obtained for the pervious footing scenario show a linear decrease with the increase of the gradient, as follows:

$$\frac{q_{rp}}{0.5\gamma' BN_\gamma} = 1 - i / i_{cr} \quad (9)$$

where the values of N_γ, obtained from the calculations are *2.91*, *14.95* and *88.02* for ϕ' equal to *20*, *30* and *40º*, respectively. These values of N_γ are close to the values obtained by Martin (2005), which are *2.84*, *14.75* and *85.57*, for the same friction angles. Results of q_{rp} adimensionalized by $0.5\gamma' BN$ can be seen in Figure 2 (top). Plastic strains, hydraulic heads and seepage forces for an example of $\phi'=30^o$ and two values of i/i_{cr}, namely *-0.5* and *+0.5*, are represented in Figure 3. The mechanisms than can be inferred from this figure for both situations (and for all other values of i/i_{cr}) are the same, corresponding to a constant value of N_γ. The hydraulic head is not affected by the presence of the pervious footing and therefore the lines of equal head are horizontal and the seepage forces are constant in the soil. Results for *H/b* equal to *2* and $\phi'=40^o$ are not represented, as for this situation the presence of the rigid stratum affects the mechanism and the study of this problem is not intended in the present work.

5 RESULTS OBTAINED FOR THE IMPERVIOUS FOOTING

Results obtained for the impervious footing scenario adimensionalized by the same quantities are shown in Figure 2 (bottom). For comparison purposes, eq. (9) is also represented in this figure. The effect of the impervious footing can clearly be seen: for negative gradients (downwards seepage) the bearing capacity is less that the corresponding one for the pervious footing and for positive gradients the opposite occurs.

The bearing capacity is slightly affected by the *H/b* ratio, with greater bearing capacities obtained for greater *H/b* ratios. The similarity of the bearing capacity is accompanied by the similarity of the mechanisms. Figure 4 shows the mechanisms obtained for two values of i/i_{cr} (*-0.5* and *+0.5*) and for $\phi'=30^o$ for three different *H/b* ratios. For the same value of i/i_{cr} the mechanisms are very similar for the different *H/b* ratios. The effect of the impervious footing on the hydraulic head distribution is visible in Figure 4, and seepage forces near the footing are no longer vertical. Their values are greater in areas where the mechanism is developed and this explains the influence of seepage on the mechanism and on the bearing capacity of the footing.

Results for the greatest values of *H/b* are not included in Figure 2 (bottom), given that for these (upwards) gradients collapse occurs for virtually null loading of the footing, because of the seepage forces and the problem is no longer a bearing capacity one. Some of the mechanisms corresponding to these cases are included in Figure 5, where the mechanisms for different friction angles and different gradient ratios i/i_{cr} are represented for the case *H/b* equal to *4.5*. From the case of i/i_{cr} equal to *-0.75* to a certain positive value of this ratio that depends on the friction angle and is around *+0.2* to *+0.5*, the mechanisms are of the same type; as i/i_{cr} increases the volume involved in the mechanisms decreases and the mechanism changes significantly up to the previous mentioned case of collapse with null loading.

6 COMPARISON BETWEEN THE RESULTS OBTAINED FOR THE PERVIOUS AND IMPERVIOUS FOOTING

Figure 6 represents the ratio between the bearing capacity q_{ri} and q_{rp}, obtained for the impervious and pervious footing, respectively, for different *H/b* ratios and different friction angles. For negative gradient ratios (downwards seepage) the impervious footing can reach increases of *50%* relatively to the pervious one for $\phi'=20^o$ and $i/i_{cr}=-0.9$; this increase is smaller for greater values of the friction angle; it is about *20%* for $\phi'=30^o$ and *5%* for $\phi'=40^o$. For upwards seepage decreases are very significant and can reach about *50* to *60%*. This occurs for relatively small i/i_{cr} ratios for the friction angle of *20º*, for greater friction angles it occurs for larger i/i_{cr} ratios. This means that considering the pervious footing solution, as a simplified approach, for an impervious footing case would lead to an unsafe scenario.

7 CONCLUSIONS

For upwards seepage the effect of the impervious footing is quite significant and leads to an important decrease of the bearing capacity. Results obtained for the impervious footing highlighted the changes that its influence on the head distribution and on the seepage forces has on the bearing capacity, changing the mechanisms involved in collapse.

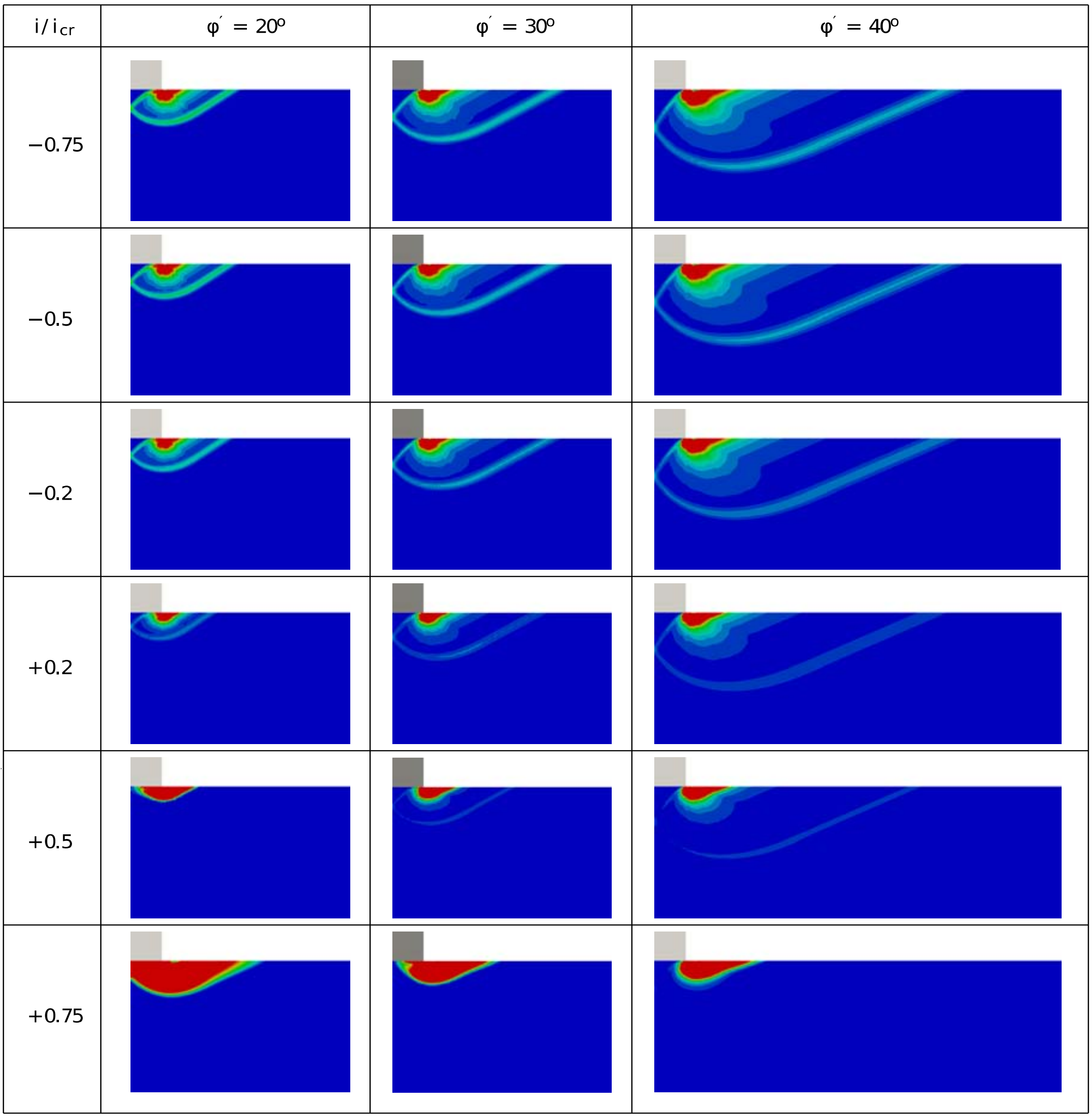

Figure 5: Impervious footing: plastic strains, hydraulic heads and seepage forces for different values of *ϕ'* and i/i_{cr}, for *H/b=4.5*.

Results for the pervious footing confirm the expected behaviour, which allows the use of bearing capacity equations corrected with the appropriate value of the soil unit weight.

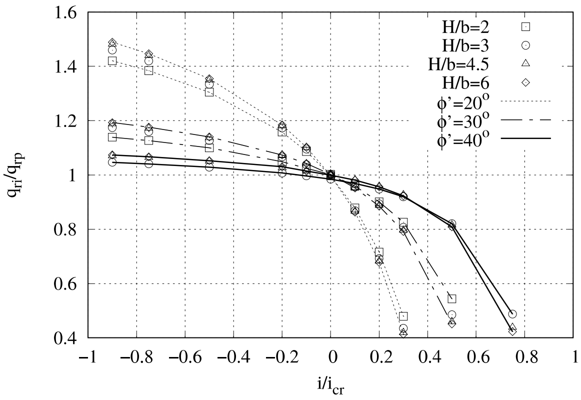

Figure 6: Ratio between the bearing capacity obtained for the impervious and pervious footing for all cases.

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