Modelling the influence of rock variability on geotechnical structures

Modélisation numérique de l'influence de la variabilité du roche sur les structures géotechniques

A.M.G. Pedro, D. Ferreira, P.A.L.F. Coelho, J. Almeida e Sousa
Department of Civil Engineering, University of Coimbra, Portugal, amgpedro@dec.uc.pt

D.M.G. Taborda
Department of Civil and Environmental Engineering, Imperial College London, UK

ABSTRACT: The importance of variability in soils and rocks has been widely acknowledged and the object of numerous studies. In this paper, the impact of modelling rock variability is assessed by performing numerical analysis of the excavation of a deep circular tunnel in a rock mass characterised by a uniform and isotropic stress state. The variability is introduced in the analyses by considering random fields of variable parameters generated using the Local Average Subdivision (LAS) method. The results of the performed analyses are compared against the results of a traditional deterministic approach where uniform properties are considered, both in terms of displacements around the excavation and forces in the tunnel lining, so that the influence of introducing variability can be assessed. Finally, a parametric study is carried out in order to evaluate the influence of the parameters adopted in the generation of random fields, such as the standard deviation, the spatial correlation distance and the ratio of anisotropy.

RÉSUMÉ : L'importance de la variabilité des sols et roches est largement reconnue et a déjà été l'objet de nombreuses études au cours des dernières années. Cet article évalue l'impact de la modélisation de la variabilité des roches, en effectuant une analyse numérique du creusement d'un tunnel dans une masse rocheuse caractérisée par un état de contrainte uniforme et isotrope. La prise en compte de la variabilité est possible à moyen de la considération de champs aléatoires de paramètres variables générés par la méthode en subdivision de la moyenne locale (LAS). Les résultats de ces analyses sont ensuite comparés avec les résultats d'approches traditionnelles déterministes, dans lesquelles on considère un champ uniforme, aussi bien en termes de déformations du contour du creusement qu'au niveau des forces sur le revêtement du tunnel. Enfin, une étude paramétrique est exécutée dans le but d'évaluer l'influence des paramètres adoptés sur les déformations et les forces sur le revêtement do tunnel au niveau de la génération de champs aléatoires, tels que l'écart type, la distance de corrélation et le rapport d'anisotropie.

KEYWORDS: soils variability; random fields; numerical analysis; tunnelling.

1 INTRODUCTION

It is widely acknowledged that soil and rocks are materials whose properties are heterogeneous in space and variable in magnitude (Zhang, 2013). Nevertheless, the majority of geotechnical analysis assume uniform properties, which are usually determined based on a limited number of laboratory tests carried out on samples collected at a few pointwise locations in the field (Bourdeau & Amundaray, 2005). Consequently, it is clear that a single deterministic calculation based on those properties cannot translate completely the real behaviour of the material and can only be considered as an approximation.

In order to overcome the aforementioned issues, analyses where variability is considered have become increasingly popular. These analyses are by nature probabilistic and involve the study, for a given geotechnical structure, of multiple variations of a given property over the soil/rock mass, which are designated random fields (RFs). This is required as it is difficult to know and simulate the real pattern of variability within soil or rock masses.

Several numerical methods were developed in order to generate representative RFs of soil conditions (Vanmarcke, 2010), with one of the most broadly employed being the Local Average Subdivision (LAS) method (Fenton & Vanmarcke, 1990). LAS has already proved to be efficient in different types of geotechnical problems (Hicks & Samy, 2002; Hicks & Onisiphorou, 2005; Cai, 2011; Pedro et al., 2012; Ferreira, 2016) and, given its potentialities, was employed in the present work.

So that the implication of introducing variability when modelling geotechnical structures could be clearly assessed, a simple, well-defined application was studied: the excavation of a deep circular tunnel in a rock mass subjected to a uniform isotropic stress state. Given its boundary conditions and geometric symmetry, it is possible to isolate the effect of the introduction of variability when comparing the results against those obtained in a deterministic analysis.

An extensive parametric study will be presented and the influence of the input parameters adopted in the generation of the fields, namely the standard deviation, the isotropic spatial correlation distance and the ratio of anisotropy, on the tunnel lining deformations and forces is assessed.

2 LOCAL AVERAGE SUBDIVISION (LAS) METHOD

By acknowledging the fact that most properties in engineering practice are represented by the average value within the analysis domain Fenton & Vanmarcke (1990) developed the LAS method. This method allows the generation of three-dimensional RFs, with the advantage of keeping the statistics consistent with the field resolution at each level. The method was originally developed assuming the RFs have a stationary Gaussian distribution, although other statistical distributions can be employed through transformation functions. In addition to the input parameters of the distribution – average and standard deviation – this method requires the definition of the auto-correlation function, which correlates the properties of any two points of the domain (Ching et al., 2015).

LAS starts from a given initial average and standard deviation, both constant throughout the domain, and divides it equitably from level to level until the required precision is reached. The number of divisions is an input parameter and is dependent on the intended discretisation. The values of the subsequent levels have a correct standard deviation according to averaging theory and are correlated with the precedent level and with each other. The correlation between values on the same level is accomplished through a covariance matrix, obtained from the spatial correlation function. In this work, the Gauss-Markov function shown in Equation 1 was employed, as suggested by Fenton & Griffiths (2008). In this equation σ is the standard deviation of the field, θ_x and θ_y the spatial correlation distances and $\tau_{ij,x}$ and $\tau_{ij,y}$ the distances between points i and j in the x and y directions, respectively. The spatial correlation distances establish the length interval over which properties are expected to be largely correlated. A small correlation distance implies that significant variations of a properties' value occur within a smaller space, while the opposite is expected for a higher correlation distance. If the spatial correlation distances in both directions have the same value, the RF is isotropic, with an anisotropic RF being generated if θ_x is different from θ_y. For small anisotropy ratios (θ_x/θ_y), an almost vertically layered RF is generated, while the opposite is observed (i.e. horizontally layered RF) for high ratios.

$$\rho(\tau_{ij}) = \sigma^2 \times exp\left[-2\left(\frac{|\tau_{ij,x}|}{\theta_x} + \frac{|\tau_{ij,y}|}{\theta_y}\right)\right] \quad (1)$$

3 NUMERICAL MODEL

The influence of introducing variability was assessed by simulating the excavation of a deep tunnel with a circular section on a rock mass characterised by a uniform isotropic stress state. The 2D mesh used is presented in Figure 1 with dimensions 120m x 120m and an 8m diameter centred tunnel for a single RF of the 1000 analysed. The mesh has a total of 812 solid elements, each with 8 nodal points and 4 Gauss points for displacement and stress evaluation, respectively. The lining was simulated using 0.20m thick solid elements and modelled as a linear elastic material, with a Young's modulus of 20 GPa and a Poisson's ratio of 0.25. The boundary conditions were set so that no displacements were possible at the limits of the mesh. A uniform and isotropic stress state of 1 MPa was assumed, meaning that the effect of gravity forces was neglected.

The three-dimensional effects normally associated with excavations of tunnels were reproduced by employing the stress relaxation method (Potts & Zdravković, 2001). Firstly, the elements of the tunnel are removed and a percentage of existing stresses (α) are applied to the contour of the excavation, then, on a second stage, the lining is installed, and the remainder of the unbalanced stresses is applied ($1-\alpha$) so that a final equilibrium state is achieved. In the present analysis a stress relief factor (α) of 80% was considered to be adequate given the dimensions of the tunnel, the stress state and the characteristics of the rock mass.

For the rock mass the Hoek-Brown failure criterion (Hoek et al., 2002) was adopted (Equation 2). The parameters for this model, together with those characterising the deformability of the rock mass, can be obtained through correlations with a single parameter (*GSI*) (Equations 3 and 4). These relationships, which require five input parameters – the constant (m_i), the Young's modulus (E_i) and the compression strength (σ_{ci}) of the intact rock and the disturbance factor (D) – simplify the introduction of variability since only one RF is needed. Table 1 presents the reference values adopted for each parameter, together with that of Poisson's ratio (Pedro et al., 2012).

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \cdot \left(m_b \cdot \frac{\sigma'_3}{\sigma_{ci}} + s\right)^a \quad (2)$$

$$m_b = m_i \cdot e^{\left(\frac{GSI-100}{28-14D}\right)} \; ; \; s = e^{\left(\frac{GSI-100}{9-3D}\right)} \; ; \; a = \frac{1}{2} + \frac{1}{6} \cdot \left(e^{GSI/15} - e^{20/3}\right) \quad (3)$$

$$E_m = E_i \cdot \left(0.02 + \frac{1-D/2}{1+e^{\left(\frac{60+15D-GSI}{11}\right)}}\right) \quad (4)$$

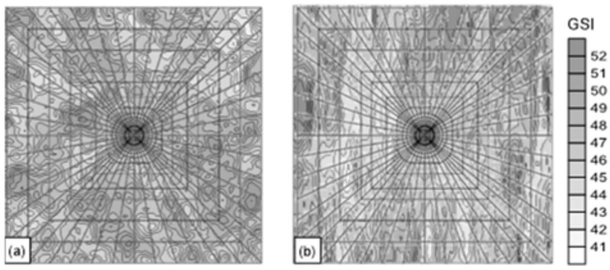

Figure 1. Finite element mesh (120m x 120m): (a) STD=7, θ_x=8m, θ_x/θ_y=1 (b) STD=7, θ_x=8m, θ_x/θ_y=1/8

Table 1. Rock mass parameters

GSI	E_i (MPa)	σ_{ci} (MPa)	ν	m_i	D
46	8100	27	0.25	9	0

4 DETERMINISTIC ANALYSIS

In order to establish a reference analysis, a deterministic calculation was performed with the parameters presented in Table 1 and a GSI of 46. The results of the total displacements for the final stage of the excavation are illustrated in Figure 2(a). As expected, given the boundary and initial stress conditions the calculation originates uniform radial displacements, with a maximum value of 2.5 mm obtained at the intrados of the tunnel. A similar behaviour was also observed in the hoop forces on the lining, where a uniform value of 338 kN was determined.

5 RANDOM FIELD ANALYSES

5.1 *General considerations*

For all the performed random analyses, a Gaussian distribution of the *GSI* with a mean value of 46 was considered. In the first set of analyses the influence of the standard deviation and of the spatial correlation distance was evaluated. Values of standard deviation (STD) of 5, 7 and 9 were considered. Nine values for the spatial correlation distance, which was considered to be equal in both directions (θ_x/θ_y=1), were analysed (3, 4, 5, 8, 10, 12, 15, 18 and 20 m) in order to verify whether its influence is related with the dimensions considered for the tunnel.

In the second parametric study, anisotropy was introduced in the RF by varying the ratio θ_x/θ_y. Three values of θ_x were tested (4, 8 and 12 m) for anisotropy ratios of 1/8, 1/4, 1/2, 1, 2, 4 and 8, while maintaining a constant standard deviation value of 7. The extreme values of anisotropy used, 1/8 and 8, were chosen to simulate scenarios where the rock mass was approximately layered in the vertical and horizontal directions, respectively.

Given the random nature of the fields a total of 1000 realisations, twice the minimum suggested by Fenton & Griffiths (2008), were performed for all scenarios of each case in order to establish a reliable pattern. An arbitrary example of a RF generated with an STD=7, θ_x=8m and θ_x/θ_y=1/8 is presented in Figure 1b).

5.2 *Influence of the standard deviation and isotropic correlation distance*

Figure 2 presents the contours of the total displacements around the excavation for the last stage modelled and for 3 different analyses, the deterministic one (a) and two arbitrary random cases (b, c), generated with the same input parameters (STD=7, θ_x=4m, θ_x/θ_y=1). The differences between the results are both quantitatively and qualitatively noticeable. The contours of the total displacements in the random analyses are no longer radial, being oriented according to the weak zones introduced in the massif. The effects of this are also observed in the magnitude of the displacements, which differ significantly in all analyses.

The introduction of variability causes a noticeable asymmetry in the deformation of the tunnel and, consequently, on the hoop forces of the lining. Naturally, such asymmetry cannot be predicted using a deterministic analysis (Figure 2a)).

The distribution of the vertical convergences obtained in the 1000 realisations are depicted in Figure 3 for different standard deviations of *GSI* and isotropic spatial correlation distances of (a) 4 m and (b) 8 m. The figure shows that both the increase of the spatial correlation distance and of the standard deviation imply a wider range of displacements. It is also possible to observe that the vertical convergences appear to follow a distribution akin to a lognormal distribution with its peak being close to the value calculated in the deterministic analysis, as expected.

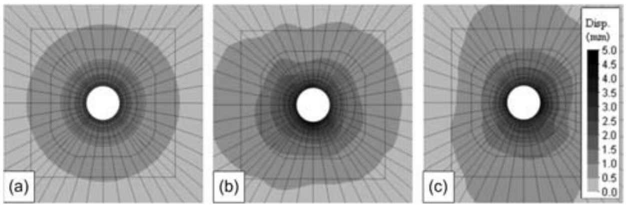

Figure 2. Detail of the total displacements around the excavation: (a) deterministic; (b) 1: STD=7, θ_x=4m, θ_x/θ_y=1; (c) 2: STD=7, θ_x =4m, θ_x/θ_y=1

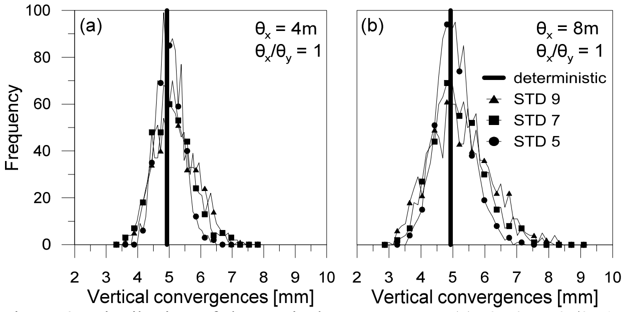

Figure 3. Distribution of the vertical convergence: (a) θ_x=4m, θ_x/θ_y=1; (b) θ_x=8m, θ_x/θ_y=1

These results are also visible in Figure 4, where the percentage change relatively to the deterministic value of the average value (AVG) and of the coefficient of variation (COV=STD/AVG) of the fitted lognormal distributions are presented against the spatial correlation distance. The results show that the vertical convergence AVG appears to be almost independent of the spatial correlation distance value, but influenced by the standard deviation of the *GSI*, increasing with it. However, its influence can still be considered reduced with a maximum value of about 8% obtained for the highest standard deviation tested.

In contrast, the increase of both the standard deviation of the *GSI* and of the spatial correlation distance originate a steadily increase of the COV. From the figure it is not possible to detect any particular relation between the spatial correlation distance and the dimensions considered for the tunnel. The increase of variation observed is directly associated with the width of the fitted distribution and variations from 6 up to 26% were observed for the different scenarios tested. Naturally, the increase of the COV corresponds to a wider distribution and, consequently, to a higher probability of the vertical convergence exceeding a given value.

The envelopes of the hoop force acting along the lining of the tunnel are depicted in Figure 5 for isotropic spatial correlation distances of (a) 4 m and (b) 8 m. Superimposed in the figure are also the results of the deterministic case (solid line) and of two arbitrary random analyses (dashed lines), which clearly show that when variability is introduced the hoop forces become asymmetric. The difference in magnitude is even more substantial if the deterministic analysis is compared with the envelopes of the 1000 realizations. In this case an increase of up to 25% in the hoop force is predicted, even when a small standard deviation is considered in the analysis. Naturally, a similar decrease is also observed in the minimum force, leading to asymmetries in the hoop force of up to 50%, which can have a major impact in the lining design.

Similar to the convergences it can be observed that an increase of both the standard deviation and of the spatial correlation distance corresponds to slightly higher forces applied to the lining. However, it appears that for standard deviations of the *GSI* higher than 7 the increase in the hoop force tends to stabilise, regardless of the isotropic spatial correlation distance adopted.

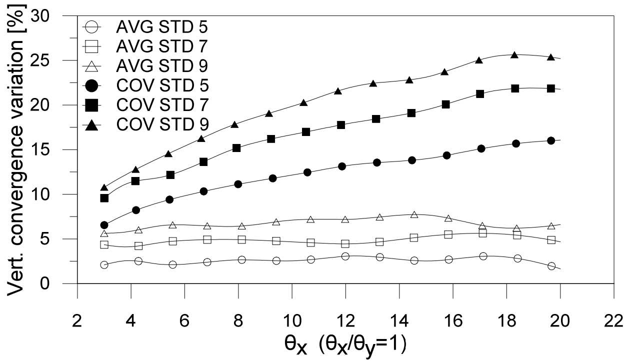

Figure 4. Variation of the fitting parameters of the vertical convergence distribution with the spatial correlation distance

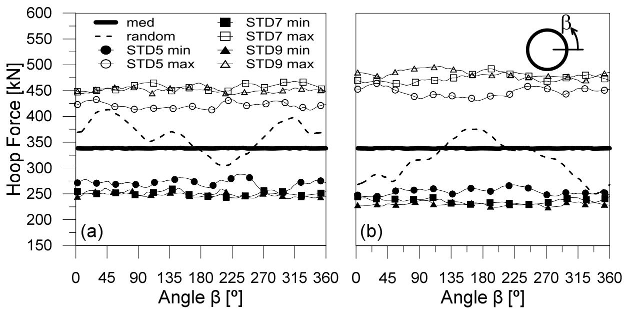

Figure 5. Hoop force envelopes for different STD: (a) θ_x=4m, θ_x/θ_y=1; (b) θ_x=8m, θ_x/θ_y=1

5.3 *Influence of the ratio of anisotropy*

The influence of anisotropy of the RF was analysed by varying the ratio θ_x/θ_y. The effect of this factor is clearly visible in Figure 1 where an example of one (a) isotropic and (b) a highly (θ_x/θ_y=1/8) anisotropic RF are displayed. While the isotropic RF presents no particular orientation of the *GSI*, the anisotropic RF shows a preferential vertical orientation due to θ_y being considerable larger than θ_x. This orientation can have a significant impact on the excavation since layers of weak rock mass can be formed in the vicinity of the excavation. The width of these layers is directly linked with the spatial correlation distance adopted in the perpendicular direction (θ_x) with its increase resulting in wider layers.

The impact of anisotropy on the convergence of the tunnel can be assessed through the analysis of Figure 6, where (a) the average value and of (b) the COV of the lognormal distributions fitted to the convergences (vertical and horizontal) obtained in the 1000 realisations performed are presented against the ratio of anisotropy. In comparison with the corresponding isotropic value, the variation observed on the average value of the distribution is less than 4% and appears to increase with the increment of the spatial correlation distance. As for the COV of the distributions of convergence values, it is possible to observe a significant increase of this quantity with the increase in spatial correlation. However, the increase of the COV is related to the ratio of anisotropy and presents a minimum value for isotropic conditions (θ_x/θ_y=1). For anisotropic conditions the COV value tends to increase. As expected, the COV of the vertical convergence tends to increase with the decrease of the ratio of anisotropy (vertical layering) while it appears to stabilize when higher ratios (horizontal layering) are adopted. This behaviour is naturally mirrored in the horizontal convergence, with the highest COV observed for high anisotropy ratios.

In the case of hoop forces (Figure 7), a similar tendency can be observed. The average values (Figure 7(a)) present some fluctuation that is related to the ratio of anisotropy employed. For ratios above 1 the lining presents higher hoop forces located at 0º and 180º (horizontal direction) and smaller at 90º and 270º (vertical direction), while the exact opposite occurs for ratios below 1. In Figure 7(b), the results of the COV show a minimum for the isotropic conditions. With the increase of anisotropy, the variation of the COV reaches a maximum value of about 13.5%. However, similarly to the observed in the convergences, the pattern of the COV depends on ratio of anisotropy. For ratios higher than 1 the maximum values are observed at 90º and 270º (vertical direction), while for ratios below 1 the highest values occur at 0º and 180º (horizontal direction).

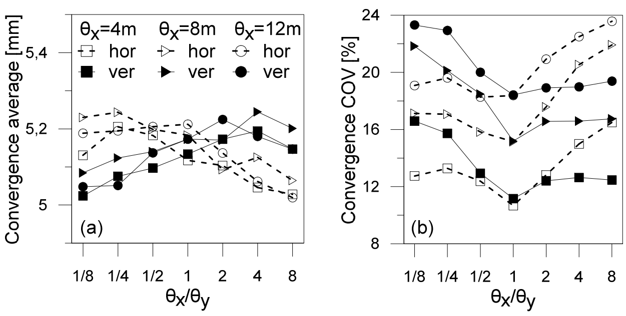

Figure 6. Influence of the ratio of anisotropy on the fitting parameters of the vertical and horizontal convergence distribution: (a) average; (b) COV

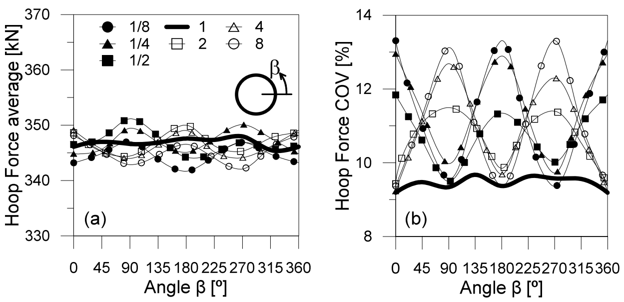

Figure 7. Influence of the ratio of anisotropy on the fitting parameters of the hoop force distribution along the lining: (a) average; (b) COV

6 CONCLUSIONS

The existence of variability in soil and rock massifs is known to have implications in geotechnical and structural behaviour. In this paper the influence of introducing variability in numerical analysis was assessed through the modelling of the excavation of a deep circular tunnel in a rock mass characterised by a uniform and isotropic stress state. Multiple random fields were generated using the LAS method and the variability was introduced by assuming a Gaussian distribution for the *GSI*, which was then related to the strength parameters of the Hoek-Brown failure criterion and to the stiffness of the rock mass. The influence of the standard deviation of the distribution, of the spatial correlation distance and of the ratio of anisotropy of the random field was analysed by performing 1000 realisations for each case. The following conclusions can be drawn from the obtained results:

- The introduction of variability induced an asymmetric behaviour on both displacements around the excavation and forces acting on the lining, which was absent from the results of the deterministic analysis. Based on the results of the 1000 realisations performed for each case, it was possible to establish a reliable distribution of the displacements and of the envelopes of the hoop forces acting on the lining, which were considerably different from those calculated in the deterministic analysis.
- The increase of the standard deviation and of the isotropic spatial correlation distance causes wider displacement distributions and higher hoop force envelopes, suggesting an increase in the uncertainty of the results.
- Variations of the ratio of anisotropy (θ_x/θ_y) affect the maximum horizontal and vertical convergences. The COV of the horizontal convergences was shown to increase with the θ_x/θ_y while that of the vertical convergence decreases with it. These results are justified by the vertical/horizontal layers of weak rock mass that are formed when the anisotropy increases and that accentuate the displacements in those areas.

The study performed with this methodology provided results, both in magnitude and behaviour, which a deterministic analysis could not predict. Consequently, the introduction of variability in the numerical analysis can improve and optimise the design of geotechnical structures, allowing for a reliability-based design.

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