Predicting the behaviour of non-circular, curved-in-plan retaining walls using the trial load method

Prédire le comportement des murs de soutènement non-circulaires incurvés en utilisant la méthode de la charge d'essai (trial load method)

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ABSTRACT: The trial load method is a long established method to predict the behaviour, and design of arch dams. This paper presents the first implementation of this method to predict the behaviour of non-circular, curved-in-plan, deep excavations. The results of the method were compared to a curved excavation, observed in literature and results of models in PLAXIS 3D. The results show that the trial load method can be successfully used to predict the behaviour of curved walls, and provides insights on further research to validate its use more widely for design purposes and its potential to be applied to the design of typical (i.e. straight) retaining walls.

RÉSUMÉ: La méthode de charge d'essai (trial load method) est une méthode établie depuis longtemps pour prédire le comportement et la conception des barrages d'arc. Cet article présente la première mise en œuvre de cette méthode pour prédire le comportement des excavations profondes non circulaires, courbées dans le plan. Les résultats de la méthode ont été comparés à une excavation courbe, observée dans la littérature et les résultats des modèles dans PLAXIS 3D. Les résultats montrent que la méthode de la charge expérimentale peut être utilisée avec succès pour prédire le comportement des murs incurvés et fournit des informations sur des recherches plus poussées pour valider son utilisation plus largement à des fins de conception et son potentiel à appliquer à la conception d ' Murs de soutènement.

KEYWORDS: Retaining wall, PLAXIS 3D, Numerical method

1 INTRODUCTION

A designer can use a variety of different methods to design retaining walls. These mainly fall under two categories: numerical methods using computer programmes and analytical methods such as Terzaghi or Peck design charts and other limit equilibrium solutions as described for instance in CIRIA C580 (Anderson, 2012). Both methods have their advantages and disadvantages. Analytical methods are easy to analyse but give conservative designs (evidenced by the excessively high stiffness in cantilever retaining walls resulting in very small displacements) (Long, 2001). Numerical methods are able to model complex loading and excavation sequences but the inputs and analysis options in the software need to be carefully and correctly applied or the consequences can either lead to very conservative designs (O'Brien, 2010) or, in extreme situations, even collapse (Whittle and Davies, 2006).

The design of retaining walls using analytical methods mainly relies on either 2D section designs (e.g. Coulomb's or Rankine's theory) or on empirical evidence (e.g. Clough and O'Rourke (1990) design charts). These design methods whilst having some advantages as stated above also have a few disadvantages such as not incorporating the effect of the horizontal stiffness of the retaining wall. This disadvantage can lead to very conservative designs as shown by Long (2001) resulting in higher material usage, higher economic and environmental cost and wasted personnel hours. Its use for more innovative designs of the plan of the retaining wall such as the "peanut" shaped retaining walls is impossible. For such shapes, designers need to resort to using numerical methods (Puller et al, 2015). Having an analytical method or process which includes the additional stiffness of the horizontal dimension of the retaining wall has the potential to allow for more economic design whilst still using simple calculations.

A non-circular, curved-in-plan retaining wall (referred to as an arched retaining wall) is a proposed variation on the use of straight retaining walls (as shown in Figure 1) for deep excavations. Such shape helps to avoid/reduce the amount of props or excavation supports required by using the theory of arches to distribute the load from the centre of the wall to the corners of the excavations: hence reducing the displacements. This reduction in displacements and props has a series of benefits, including; economic, production rates, material use and health and safety (Gilmore, 2015). Research on this topic has been focused on diaphragm walls as done by the author (Gilmore, 2015 and 2016), and on arched secant pile walls by Yi-ping and Tu-qiao (2000).

This paper will present a new analytical method for designing curved-in-plan cantilever retaining walls using a method which has been used extensively to design arch dams (The trial load method). Case study data are used to show its effectiveness. It is thought that this method can be used as both a stand-alone design method and also as a checking tool for numerical models.

2 TRIAL LOAD METHODOLOGY

The trial load method is to estimate the behaviour of an arched dam by using the relative stiffness of the vertical and horizontal sections to predict its behaviour (Fanelli, 1999). The method divides the arch dam into vertical (or inclined members) and horizontal members as shown in Figure 2 (which for an arched retaining wall are a cantilever and arch beam respectively). The deflections of each of the members are then calculated in isolation, adjusting the applied loading (shown in Figures 3 and 4) in iterative manner until the displacement at points where the vertical and horizontal sections cross are the same.

Within the trial load method there are also different forms of analysis each with their own levels of accuracy and complexity. These are: the *Crown-Cantilever Analysis*, *Radial Deflection Analysis* and *Complete Adjustment Analysis*. These methods increase in accuracy and also in complexity (i.e. the C*rown-Cantilever analysis* is the least accurate but is the simplest method) (Ghanaat, 1993 § 2.3). The results of all three methods were compared by the United States Department of the Interior (1977) when analysing Yellowtail dam which confirmed the accuracy of the methods.

For the non-circular curved-in-plan cantilever retaining wall, the *Radial Deflection Analysis* method will be used. This method will be used to enable an accurate model of the retaining wall to be made (particularly midway along the retaining wall) without requiring in-depth analysis of the tangential shear (which is what is required for complete adjustment analysis) (Ghanaat, 1993). For the *Radial Deflection Analysis* the wall was split into the sections that can be seen in Figure 2. The arch section (as shown in Figure 3) was chosen based on the areas of most interest (the displacement in the retained height is higher than in the embedded depth hence three of the arches are located in the retained height). The cantilever sections (as shown in Figure 4) were chosen based on the method (which suggests to locate the cantilever sections at quarter lengths along the wall) (Ghanaat, 1993)). For the analysis it is assumed that the deflection at the corner and at the base of the retaining wall is at zero. The horizontal elements will be a two pinned arch (as the wall is curved-in-plan) using the same arch section as Gilmore (2015), and the vertical elements will be cantilever sections (which will be rotating about a centre of rotation as used in a straight cantilever retaining wall design).

To simplify the analysis and to make the analysis more similar to typical retaining wall designs, the vertical elements take the entire load of the earth and the arches act as props on the vertical elements (with the prop force distributed along a set length of the wall as a uniformly distributed load (UDL)). This avoids the complex analysis of modelling both the arch and the vertical wall with a variable load.

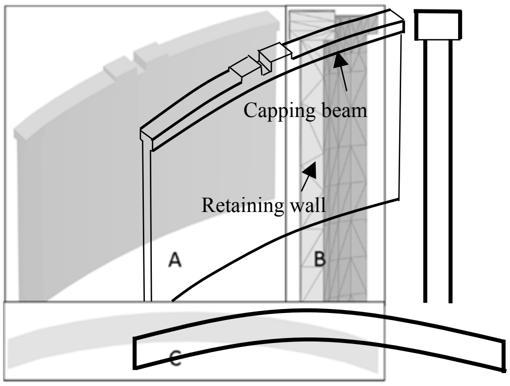

Figure 1. A 3D (A), section (B) and plan (C) image of an arched cantilever retaining wall. *Source:* Gilmore (2016)

3 NUMERICAL MODELLING

Using the model setup from section 2 the vertical and horizontal element equations were created to determine the displacements as can be seen in Equations 1 and 2. The equation for the vertical element used the Euler–Bernoulli beam theory (McCormac, 2007) as a cantilever is a statically determinate structure. For the horizontal element the structure is statically indeterminate, as can be seen in Figure 3, and requires the use of the method of virtual work and the Simpson's rule of approximation to determine the displacement (James, 2010).

To test the validity of the method for the use in retaining walls, the results were compared to a model of a retaining wall in London clay developed using PLAXIS 3D. The properties of the retaining wall and the soil can be seen in Table 1.

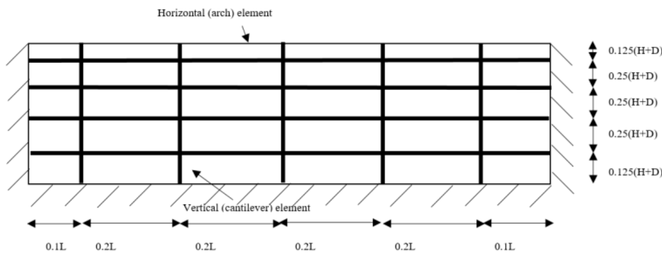

Figure 2. The location of the vertical and horizontal elements used in this example

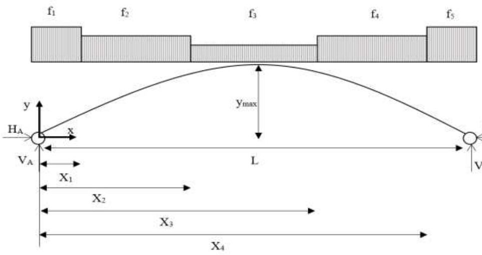

Figure 3. The loading diagram for the horizontal (arch) member ($f_{1\text{-}5}$ denote the forces from the vertical (cantilever) elements)..

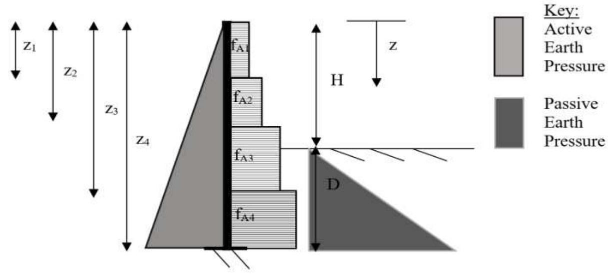

Figure 4. Arrangement of the vertical (cantilever) element (f_{A1-4} Denote the arch interaction forces). *Source:* Gilmore (2016).

$$U_{arch}(x) = \frac{\int_0^L Mm'\,dx}{EI} \quad (1)$$

Where:

$$M = V_a x - H_a y - \sum_{x=0}^{x=L} f_n \left[\frac{\langle x-x_{n-1}\rangle^2}{2} * \frac{\langle x_n - x\rangle}{x_n - x} + \frac{\langle x-x_n\rangle}{x-x_n}(x_n - x_{n-1})\left(x - \frac{x_n + x_{n-1}}{2}\right)\right]$$

$$m' = \left(1 - \frac{x}{L}\right)x' - H'y - \langle x - x'\rangle$$

$$V_a = \sum_{x=0}^{x=L} f_n (x_n - x_{n-1})\left(1 - \frac{x_n + x_{n-1}}{2L}\right)$$

$$H_a = \frac{\int_0^L (M - H_a y) y(x)}{\int_0^L y(x)^2}$$

$$y(x) = Ax^2 + Bx$$

$$A = -\frac{4y_{max}}{l^2}$$

$$B = \frac{4y_{max}}{L}$$

$$H' = \frac{\int_0^L \left(\left(1 - \frac{x}{L}\right)x' - \langle x - x'\rangle\right) y(x')\,dx'}{\int_0^L y(x')^2\,dx'}$$

x' = Alternate x axis for virtual work (where x is fixed)

$$y(x') = Ax'^2 + Bx$$

$$U_{cantilever}(z) * EI = \frac{\gamma}{18}\left(K_o z^4 - K_p\langle z-H\rangle^4\right) - \sum_{z=0}^{z=z} f_{an}\left[\frac{\langle z_n - z\rangle}{z_n - z} * \frac{\langle z - z_{n-1}\rangle^4}{24} + \frac{\langle z-z_n\rangle}{z-z_n} * \frac{(z_n - z_{n-1})\left(z - \frac{z_n+z_{n-1}}{2}\right)^3}{6}\right] + c_1 z + c_2 \quad (2)$$

Where:

$$c_1 = \sum_{z=0}^{z=H+D} f_{an}\left[\frac{(z_n - z_{n-1})}{2}\left(H + D - \frac{z_n+z_{n-1}}{2}\right)^2\right] + \frac{2\gamma}{9}\left(K_p D^3 - K_0(H+D)^3\right)$$

$$c_2 = \sum_{z=0}^{z=H+D} f_{an}\left[\frac{z_n - z_{n-1}}{6}\left(H + D - \frac{z_n+z_{n-1}}{2}\right)^3\right] + \frac{\gamma}{18}\left(K_p D^4 - K_0(H+D)^4\right) - c_1(H+D)$$

Table 1. Properties of the trial load method and PLAXIS 3D analysis.

Property	Value
Length of wall (L)	30m
Retained height (H)	10m
Embedded depth (D)	10m
Arch rise (y_{max})	2m
Thickness of wall	1m
Elastic Modulus of wall (E)	30GPa
Soil	London Clay
Weight Density (γ)	21kN/m^3
Angle of internal resistance (□)	30º
At rest earth pressure coefficient (K)	1.4
Elastic Modulus of the soil (E_{soil})	1250C_u (kN/m^2)
Cohesion of soil (C_u)	80+8z (kN/m^2)

To refine the solution, the earth pressure coefficient was calculated as a function of the wall displacement using Equation 3. This was done within this paper only for the passive side as the active was considered to be included within the calculated loading. Further work is needed to cater for this.

After running the results in MICROSOFT EXCEL using its built-in solver (Solver Add-in) to adjust the UDL forces to get the lowest overall magnitude of error in displacements, the solution with the lowest magnitude of error is shown below in Figure 5 in terms of displacements (the total error included thought to be due to the estimation of Simpson's rule).

The distribution of displacements in Figure 5 shows the displacements increasing towards the centre of the wall (see x=15m) and reducing near the corners (see x=3m), this is in a similar way to the FE model shown in Figure 6. The distribution of arch forces as shown in Table 2 is almost symmetrical (about the mid-point of the arch (f_3)) with the highest forces near the supports with the force reducing about the centreline of the arch (except for the top horizontal element, it is in line with predictions and helps create the near symmetrical displacement profiles shown in Figure 5).

Observing Figure 5 the method needs to be amended to include a factor for increasing the earth pressure above the embedment depth as is shown Figure 5 the displacement of x =15m between z = 0-2m goes into the soil which would increase the soil's earth pressures to a passive state, which is not taken into account currently in the model.

The displacement profile from Figure appears similar to the displacement profile from the PLAXIS 3D analysis as shown in section 4 so comparisons can be made between the two different analyses and so the PLAXIS 3D analysis will be used to show proof of the hypothesised load distribution for the trial load method.

$$K_p(\delta) = \frac{\delta}{N}\left(\frac{E_{soil}}{F_v}\right) + K_O \qquad K_P(\delta) \geq K_O \qquad (3)$$

Where:

$\delta = U_{cantilever}(H)$

N = Average length of soil in passive wedge $\left(N = \frac{D}{2}\tan\left(45° + \frac{\varphi}{2}\right)\right)$

E_{soil} = 250,000 kN/m^2

$F_v = \frac{\gamma_{soil}}{2}(D)^2$

4 PLAXIS 3D ANALYSIS

PLAXIS 3D is a 3D FE (Finite element) programme which specialises in the modelling of geotechnical structures (Brinkgreve et al, 2013). This programme was used to verify the results of the trial load method using the properties and layout in Table 1 and Figure 6 respectively and also to show the benefits of an arched retaining wall.

The displacement results of the analysis of an arched cantilever retaining wall are given in Figure 6 and show that the overall displacement behaviour is similar to that predicted by the trial load method along the horizontal axis (i.e. the displacements increase as you get towards the centre of the wall along the x axis).The location of maximum displacement for both methods is the same (with both methods showing the height of maximum displacement at around 0.25-0.5 times the retained height on the z axis along the centre of the retaining wall). The height of maximum displacement is due to the higher relative stiffness of the arch element compared with the cantilever causing a more noticeable 'prop' effect at the top of the wall and reducing displacements more at the top of the wall than further down causing the maximum displacement to move further down the wall.

The relationship between the FE analysis and the trial load method shown in this paper is reinforced by observations shown in Gilmore (2016) which showed that the behaviour of the retaining wall is similar to that predicted in the trial load method. However, as Figure 7 shows which compares the displacements of the FE results, (top lines) and the trial load method (bottom lines), there are significant differences in the results. It is thought that these differences are due to the errors discussed in section 3 and the overall error (which is shown in the error bars). This difference and error in the trial load method (particularly as it under predicts the FE results) means that further work is required to make it appropriate to use as a design tool.

From the analysis of the non-circular, curved-in-plan wall there was also a number of benefits and disadvantages found against straight retaining walls, these benefits are shown in Table 3 with their description and analysis shown in Gilmore (2016 & 2015).

5 CONCLUSION

This paper showed a new limit equilibrium method for determining the displacements of retaining walls and applied it to a deep excavation in London clay and verified the result using PLAXIS 3D. Whilst there are errors in the equations and method used as highlighted in section 3, resulting in differences in observations between the FE analysis and the trial load method overall, it is shown that the trial load method gives similar results to that shown in the FE analysis. This similarity shows that the trial load method could potentially be developed into a limit equilibrium method to predict the displacements of retaining walls.

This paper also presented a new form of cantilever retaining wall (non-circular, curved-in-plan) and showed the potential benefits and disadvantages of using such a retaining wall.

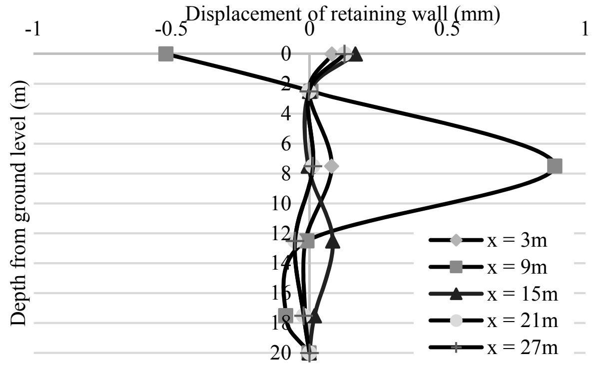

Figure 5. Displacement of retaining wall at different distances along the wall from the left hand support

Table 2. Distribution of UDL forces after multiple iterations

f_n (kN/m^2)	f_1	f_2	f_3	f_4	f_5
f_{a1}	41.0	38.3	51.9	38.9	40.4
f_{a2}	28.6	35.7	0.0	34.8	29.9
f_{a3}	6.5	0.0	30.6	0.0	5.8
f_{a4}	-0.1	0.4	0.3	0.2	0.1
Total difference in displacements between vertical and horizontal elements (mm)					1.41

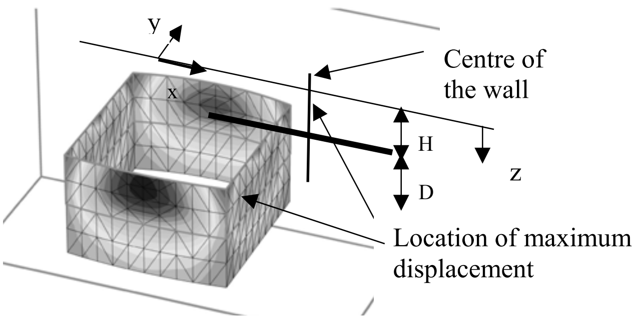

Figure 6. Horizontal displacement of an arched cantilever retaining wall from PLAXIS 3D (the darker colours indicate increased displacements).

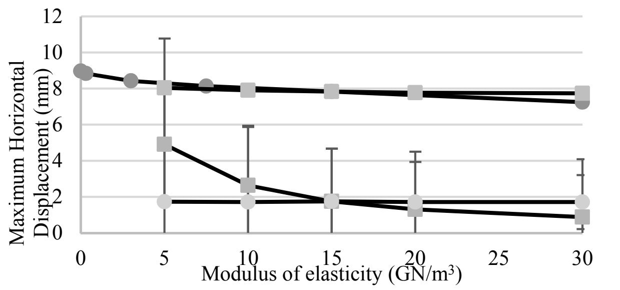

Figure 7. The maximum horizontal displacement in an arched retaining wall as the named modulus of elasticities where varied (circle data points are where the horizontal modulus was varied and the square points are for the vertical) (as each modulus of elasticity was varied, the other axis modulus was kept constant at 15GN/m^3). *Source:* Gilmore (2016).

Table 3. A summary of the advantages and disadvantages of the arched retaining wall compared to a straight retaining wall *Source:* Gilmore (2015) and Gilmore (2016).

Advantages	Disadvantages
There is a 15-25% reduction in the maximum horizontal wall displacement (y_{max} = 1 and 2m respectively)	Higher bending moments and shear forces in the wall
Uses construction techniques and technology used for the construction of straight and circular diaphragm walls	Wall type limited to diaphragm walls as lateral stiffness of wall is fundamental to the arch effect
Reduces or eliminates props used in a deep excavation (leading to health and safety and productivity improvements)	Prop arrangements will generate complex joints in excavation
Lower effective embedment depth	Increased land take
Displacements can be further reduced by increasing the arch rise	

6 ACKNOWLEDGEMENTS

If you are interested in this topic we are planning to release a journal paper in the future.

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