Soil-pile rows interaction under seismic transverse lateral loads

Interaction sol-rangées de pieux soumis à des charges latérales transversales sismiques

Rigoberto Rivera, Carmelino Zea, Germán López, Damián Vital

Geotechnical Department, Engineering School, UNAM, CDMX, Mexico, riverac@unam.mx

ABSTRACT: The problem of soil-pile interaction under lateral loads on soft soil in the case of Mexico City has been solved, for the case of an isolated pile, with the methodology developed by Professor Zeevaert (1980, 1983), which considers the soil as a continuous medium and the pile as a beam resting on interacting springs. Compatibility of deformations in the pile-soil interface is established to determine the displacements configuration and distribution of reactions generated by applying forces, static or dynamic. In this paper Zeevaert´s soil-foundation interaction procedure for an isolated pile is generalized to the case of piles rows under seismic transverse lateral loads, assuming that the base shear is not distributed equally between the total number of piles, but exist an interaction in the piles row, transverse to the lateral force, that cannot be ignored in the seismic design of these elements combined with the effects of torsion generated mainly in the corner piles. The methodology proposed here is applied to solve the case of a row of piles subjected to earthquake to compare the results with those of an isolated pile, finding important differences in the configuration of displacements and distribution of reactions which have an impact on the structural design element (rigidity, steel ratios, etc.), mainly in the slab union-pile foundation.

RÉSUMÉ : Le problème de sol-pieu interaction sous des charges latérales sur un sol mou dans le cas de la ville de Mexico, dans le cas d'une cellule isolée, peut être résolu avec la méthodologie développée par le professeur Zeevaert (1983, 1988). qui considère le sol comme un milieu continu et la pile comme une poutre reposant sur des ressorts en interaction. compatibilité de contrainte est définie sur l'interface pile-sol pour déterminer la configuration du mouvement et de la distribution des réactions engendrées par l'application de forces statiques ou dynamiques. Dans cet article, la procédure de l'interaction sol-fondation Zeevaert pour une seule pile, dans le cas de rangées de pieux sous charges transversales sismiques généralise, en supposant que le cisaillement de base est pas répartie uniformément entre le nombre total de batteries, mais il une interaction dans les rangées de pieux, qui ne peuvent être ignorés dans la conception sismique de ces éléments combinés avec les effets du couple généré principalement dans les batteries coin. La méthodologie proposée ici est appliquée pour résoudre le cas d'une rangée de pieux soumis au tremblement de terre de comparer les résultats avec ceux d'un pieu isolé, trouver des différences importantes dans la configuration des déplacements et la distribution des réactions qui ont un impact sur l'élément de conception structurelle (rigidité, relations d'acier, etc.), principalement à la tête d'articulation de la pile avec la dalle de fondation.

KEYWORDS: Soil-pile interaction, beam resting on interacting springs, Compatibility of deformations in the pile-soil interface.

1 INTRODUCTION.

The problem of soil-drawer-piles in soft soil under seismic lateral loads, very common in many deep foundations of Mexico City, can be solved for the case of a pile isolated with the methodology developed by Professor Leonardo Zeevaert (1973, 1980, 1983), which considers the soil as a continuous medium and the pile as a beam supported on interacting horizontal springs and with certain boundary conditions in both the head and the base of the pile. For the case of an earthquake in the structural model, it is assumed that at the level of the head of the piles, the lateral displacement is the same for all piles and whose magnitude is a function of the level of embedment of the foundation and of the range of seismic motion. Thus, the base shear caused by the earthquake is equally divided between piles number that form the foundation. Configuration of displacement and distribution of reactions, generated by acting static or dynamic forces, are determined by means of the compatibility of deformations in the soil-pile interface.

In this paper Zeevaert's soil-foundation interaction procedure for an isolated pile generalizes to the case of rows of piles under transversal seismic lateral loads, assuming that the base shear is not distributed evenly among the total number of piles, but in the row of piles located transversely to the horizontal seismic force, there is an interaction, a phenomenon that cannot be overlooked in the seismic design of these elements, in addition to the effects of torsion generated mainly in the corner piles.

In order to obtain the values of the influences between piles in horizontal sense, the fundamental Mindlin´s solutions for horizontal punctual loading was integrated to the case of a rectangular area uniformly loaded, assuming that the soil has a linear elastic behavior (Zea et al, 2010). The methodology was applied to solve the case of an isolated pile and a row of piles under earthquake, with important differences in the configuration of displacements and the distribution of reactions, which has an impact on the structural design of the element (rigidity, quantities of steel, etc.), mainly in the joint foundation-pile slab.

2 STRUCTURAL MODELING

Consider a foundation formed by a drawer and piles subject to seismic lateral loads, a solution frequently adopted in soft soils, as in the case of Mexico City. The seismic movements in the ground cause interacting horizontal reactions on the walls of the drawer and on the shaft of the piles which, combined with the external forces, keep the foundation system in balance. Because the soil can be considered as a continuous medium, at the interface of the foundation system and the soil reacting laterally, the compatibility of deformations must be fulfilled. Structural model used here considers that lateral displacement, magnitude of which depends on deep of embedment of the foundation and amplitude of the seismic movement at the level of their head is the same for all piles.

In order to solve the structural problem, it is usual to idealize the soil through horizontal springs, which provides lateral support to the foundation system and whose final stiffness will depend on the compatibility of deformations in the soil-foundation interface.

Modeling the foundation system is achieved by transforming it into a mesh or grid of bars with equivalent properties to the actual foundation (Fig. 3). Nodes of this mesh will therefore have six degrees of freedom, namely: displacements in direction of the "*x*", "*y*" and "*z*" axes and rotations around these axes.

In this model, resistance to displacement and rotation of nodes is due to both rigidity of the bars of the structure and rigidity of the springs that simulate the soil.

Equilibrium of the physical model thus proposed leads to a system with 6xN equations (where N = number of nodes), with the same number of unknowns (displacements and rotation of the nodes), which is represented by following matrix equation (Zeevaert 1980, 1983):

$$\left[\left[\overline{S}\,\text{[illegible]}_{ij}\right]+\left[\overline{S}?_{ij}\right]+\left[\ /K_i\right]_D\right]\cdot\{X_i\}=\{\Delta_{i0}^{P}\}+\{\Delta_{i0}^{A}\}+\{\Delta\delta_{si}\} \quad (2.1)$$

Donde:

$\left[\overline{S}_{ij}\right]$ Flexibility Matrix of structural system formed by the rows of piles, assuming finite stiffness, fixed but spring base support, guided in the head and virtual unit loads in each node.

$\left[\overline{S}\,\text{[illegible]}_{ij}\right]$ Flexibility matrix of structural system formed by the rows of piles, assuming infinite stiffness, elastic support in the base, guided in the head and virtual unit loads in each node.

$\left[1/K_i\right]_D$: Diagonal flexibility matrix of soil (Section 3).

$\{\Delta_{i0}^{P}\}$: Relative displacements and rotations vector of the piles nodes, assuming finite stiffness, fixed but spring base support, guided in the head and basal shear force on head.

$\{\Delta_{i0}^{A}\}$: Relative displacements and rotations vector of the piles nodes, assuming infinite stiffness, elastic base support, guided on the head and basal shear force on head.

$\{\Delta\delta_{si}\}$: Relative horizontal displacements vector, produced in soil mass by the seismic movement (Section 4).

Equation 2.1 can be rewrite as follow:

$$\left[F_{ij}\right]_T\cdot\{X_i\}=\{\delta_i\}_T \quad (2.2)$$

Where:

$\left[F_{ij}\right]_T=\left[\overline{S}\,\text{[illegible]}_{ij}\right]+\left[\overline{S}?_{ij}\right]+\left[\ /K_i\right]_D$: Total flexibility matrix

$\{\delta_i\}_T=\{\Delta_{i0}^{P}\}+\{\Delta_{i0}^{A}\}+\{\Delta\delta_{si}\}$: Total vector displacements

Solving equation 2.2 yields the reactions, from which it will be possible to calculate the bending moments and the shear forces. However, because springs that simulate soil interacting each other, the solution of the system of equations 2.1 has to be done iteratively to ensure compatibility of deformations in the soil-pile interface.

3. SOIL RIGIDITY

In this part of the analysis, the soil is assumed to be a continuous medium which will simultaneously be represented by a series of "*m*" elastic springs of horizontal stiffness K_i, interrelated to each other, while the pile rows will be divided into a finite number of "*n*" plates of area a_i.

Calculation of the reaction modules or equivalent spring constants of the subsoil can be performed by mean of a soil-foundation interaction analysis, solving the following equation called HEMA (Zeevaert 1980 and 1983):

$$\{\delta_i\}=\left[\overline{\delta}_{ji}\right]\{q_i\} \quad (3.1)$$

Where:

$\{\delta_i\}$: Vector of horizontal displacements.

$\{q_i\}=\left\{\dfrac{X_i}{a_i}\right\}$: Vector of horizontal reactions uniformly distributed in the contact area, a_i, in the soil-foundation interface. Hence $X_i = q_i a_i$.

$\left[\overline{\delta}_{ji}\right]$: Matrix of horizontal displacements where each column of it, $\{\overline{\delta}_{ji}\}$, is obtained as follows:

$$\{\overline{\delta}_{ji}\}=\left[I_{ji}\right]^T\left[\alpha_n\right] \quad (3.2)$$

Where:

$\left[I_{ji}\right]$: Matrix of influences obtained by applying a uniformly distributed virtual unit load to each plate of pile rows with tributary area, a_i, calculated from the integration of Mindlin's fundamental solution (Zea et al, 2010) at different horizontal distance along direction analyzed.

$\left[\alpha_n\right]$: Matrix of horizontal compressibility of the different soil strata involved in the analysis.

Equation 3.1 can be written as:

$$\{\delta_i\}=\left[\overline{\delta}_{ji}\right]\left\{\frac{X_i}{a_i}\right\} \quad (3.3)$$

Where $\{X_i\}$ vector of horizontal reactions on the plates of rows of piles.

4. SEISMIC ACTION IN THE SUBSOIL

In the case of a stratified subsoil, we can estimate the configuration of relative distortions and shear stresses generated by the shear waves by mean of the following procedure (shear soil-beam model, Zeevaert, 1973,1980).

Figure 1 show a soil element in a soil column under dynamic maximum amplitude generated by the shear waves.

From elastic-dynamic equilibrium of the soil element we obtain the algorithms using the computation of the maximum horizontal displacement and the shear stresses. These equations are:

$$\delta_{i+1}=A_i\delta_i-B_i\tau_i \quad (4.1)$$

$$\tau_{i+1}=C_i(\delta_i+\delta_{i+1})+\tau_i \quad (4.2)$$

In which the coefficients A_i, B_i y C_i have the following values:

$$A_i=\frac{1-N_i}{1+N_i} \quad (4.3)$$

$$B_i = \frac{1}{1+N_i}\left(\frac{d_i}{G_i}\right) \tag{4.4}$$

$$C_i = \frac{1}{2}\rho_i d_i \omega_s^2 \tag{4.5}$$

Where:

$$N_i = \frac{\rho_i d_i}{4G_i}\omega_s^2 \tag{4.6}$$

And:
ω_s: Circular frequency in the stratified deposit.
G_i: Dynamic soil rigidity for the stratum i.
ρ_i: Density of stratum i.
d_i: Thickness of stratum i.

Knowing the maximum surface acceleration (a_o), surface boundary conditions ($(\delta_s)_{máx}$; τ_i=0) and firm ground basis (δ_b=0 and $(\tau_{sb})_{máx}$), it is possible to calculate the final configurations of the horizontal displacements and shear forces using equations 4.1 and 4.2 in an iterative manner. Configurations produced by the first and second modes of vibrations can be studied in the most unfavorable form, using for this purpose a coefficient of mode participation.

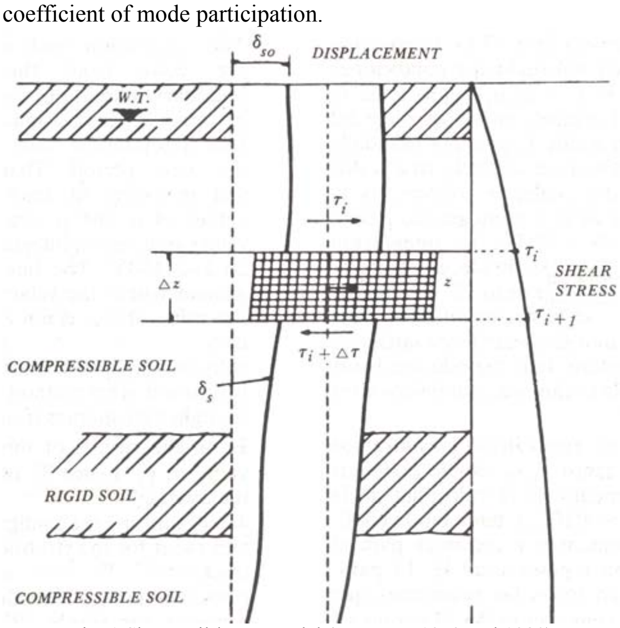

Fig. 1 Shear soil-beam model (Zeevaert, 1973 and 1980)

5. EXAMPLE

In order to illustrate proposed methodology a foundation with a set of six friction piles, of square section 40x40 cm, joined in their head by a foundation rigid slab is adopted; geometry of the foundation and adopted soil properties in which the piles are embedded are shown in Figure 2. It is a typical stratigraphy of Mexico City composed of very compressible clay deposits and low shear strength. Structure that will support the foundation in this example generates a basal shear of 13500 KN, associated with a maximum acceleration at the soil surface of 1.0 m/s^2. Dynamic modulus of concrete is considered to be 22240 MPa, while the Poisson ratio of the clayey soil strata was considered to be 0.5. For torsion phenomenon, 100% acceleration in the most unfavorable direction was considered and 30% in the other direction (Gobierno de la Ciudad de México, 2005). Vector $\{\Delta\delta_{si}\}$ for 90% of first vibrate mode plus 10% for second mode, is:

$$\{\Delta\delta_{si}\}^T_{33x1} = (0.115 \quad 0.115 \quad . \quad . \quad 0.000 \quad 0.000)\,\text{m}$$

For calculation of matrixes $\left[\overline{S}_{ij}\right]$, $\left[\overline{S}^{T}_{ij}\right]$, $\left[\frac{1}{K_i}\right]_D$ and vectors $\left\{\Delta^P_{i0}\right\}$, $\left\{\Delta^A_{i0}\right\}$ model of analysis shown in Figure 3 was adopted. Final total flexibility matrix for the most unfavorable direction is equal to:

$$[F_{ji}]_T = \begin{bmatrix} 1130.33 & 1124.88 & . & . & 175.41 & 175.42 \\ 1124.88 & 1130.67 & . & . & 176.59 & 175.42 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 175.41 & 176.59 & . & . & 49.59 & 27.58 \\ 175.42 & 175.42 & . & . & 27.58 & 49.59 \end{bmatrix}_{30x30} \quad (\text{KN/m})$$

While total vector of independent terms is equal to:

$$EI(\delta_i)^T_{30x1} = (154239 \quad 154241 \quad . \quad . \quad 23985 \quad 23985)$$

Hence the following reaction vector:

$$2(X_i)^T_{30x1} = (32.22 \quad 30.40 \quad . \quad . \quad 1.65 \quad 1.67)\ \text{KN}$$

Relative displacements, bending moment and shear force diagrams derived from the reactions forces, X_i, are presented in figures 4, 5 and 6 respectively. Results include the analysis for the pile in the center row and in the edge row, without and considering the torsion. For comparison purposes, results corresponding to the analysis proposed by Zeevaert (1973, 1980) for an individual pile are also presented.

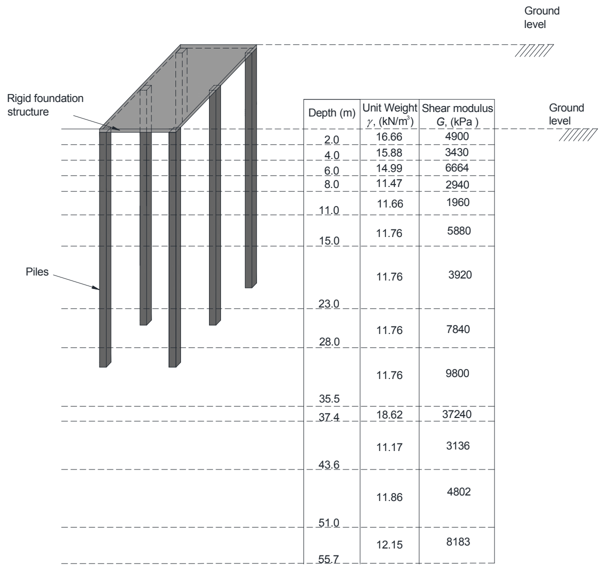

Fig. 2 Geometry of foundation and properties of soil

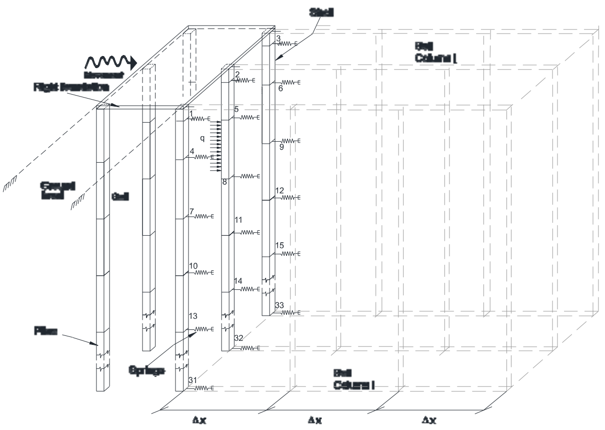

Fig. 3 Plates and soil columns of the model and structural model of beams for the example.

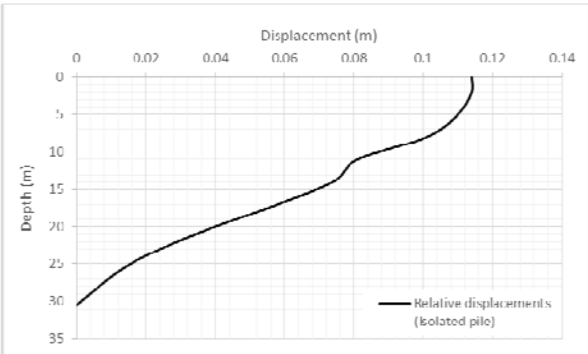

Fig. 4 Relative displacement (Isolated pile)

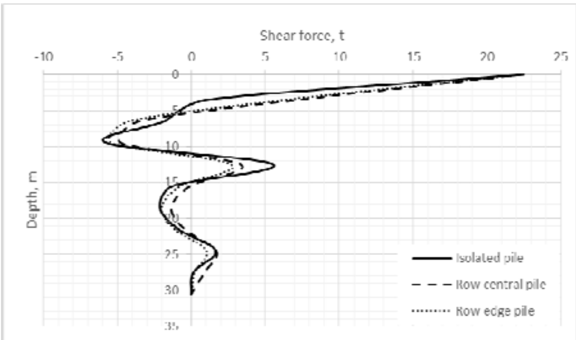

Fig. 5 Shear force diagram

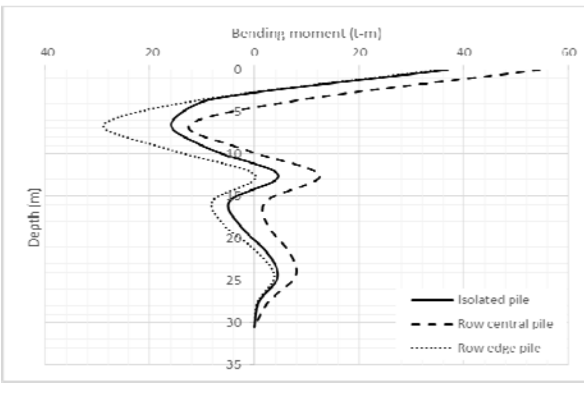

Fig. 6 Bending moment diagram

6. CONCLUSIONS

Proposed method, based on the Zeevaert´s interaction methodology, provides a good alternative for the calculation of soil-pile rows interaction from practical engineering point of view.

Analysis results of example compared with the methodology developed by Zeevaert for an individual pile indicate importance of taking account three-dimensional effect in interaction analyzes, since mechanical elements present important differences; in central pile the shear force resulted of more than double in some points, joined to torsion effects.

Generalization of the Zeevaert´s method to the three-dimensional interaction case allows to analyze influence of the rows and effect of torsion on the pile behaviour.

Finally, authors of this paper recommend comparing analytical results derived from this methodology with field measurements to validate proposed method.

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