Area-ratio optimization of granular pinned soil slopes in slope-stability analysis

Zone ratio optimisation des granulaires goupillé sol pentes dans l'analyse pente-stabilité

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ABSTRACT: The stone columns or granular piles are used to increase the stability of vulnerable soil-slopes. The response of the composite system in maintaining force and moment equilibrium of the reinforced slope and behavioral pattern of the soil-stone column 'unit' is different from case of soil-column vertical stress equilibrium. A mechanistic model of a soil-stone column 'unit cell' has been analyzed (Saha, 2001) to predict the increased resistance of the potential sliding mass of the 'pinned' slope due to higher shear strength and free-draining characteristics of the 'granular-pin'. Expressions for Factor of Safety (FOS) of the composite sliding mass and resultant of pair of inter-slice forces of the composite mass along the failure plane was derived (Saha, 2005). The results applied to a real life problem in river 'Bhagirathi-Hooghly' (*lower reach of river Ganges*), showed marked improvement in stability with increase in FOS by granular pinning effected by change of critical failure surface; for gradual and, more so for instantaneous drawdown conditions (Saha, 2009). This investigation ventures to mathematically derive the optimum area-replacement of granular-pinned slope from the strength and deformability characteristics of the composite ground and problem geometry that may be applied in fixing the optimum diameter of stone-column required to achieve the optimal improvement against minimum cost in field situation.

RÉSUMÉ : La pierre des colonnes ou des piliers sont granulaires utilisés pour augmenter la stabilité de sols vulnérables-pistes. La réponse du système composite dans le maintien de la force et l'équilibre des moments de la pente renforcé et modèle de comportement du sol-colonne en pierre "unité" est différent d'un cas de contrainte verticale de la colonne de sol l'équilibre. Un modèle mécaniste d'un sol en pierre de l'unité de la colonne 'cell' a été analysé (Saha, 2001) pour prédire la résistance accrue du potentiel de masse en mouvement la 'pente' épinglés en raison de l'augmentation de la résistance au cisaillement à drainage libre et caractéristiques de la 'pin'-granulaire. Les expressions de coefficient de sécurité (DS) de la masse en mouvement et composite résultante de paire de forces inter-coupe de la masse composite le long de l'échec plane est obtenue (Saha, 2005). Les résultats appliquée à un problème de la vie réelle dans la rivière 'Bhagirathi-Hooghly» (cours inférieur du fleuve Ganges), ont montré une nette amélioration de la stabilité avec augmentation de FOS par l'épinglage granulaire effectuée par le changement de surface de rupture critique ; de manière progressive et, plus pour les conditions de prélèvement instantané (Saha, 2009). Cette enquête ventures pour dériver la zone mathématiquement optimale-remplacement de charbon par axe pente de la force et de la déformabilité caractéristiques du terrain composite et la géométrie du problème qui peuvent être appliquées pour déterminer le diamètre optimal de la colonne de pierre nécessaires à l'amélioration optimale contre un coût minimum en situation sur le terrain..

KEYWORDS: slope stability, stone column reinforced slope, soil column unit-cell, area ratio optimization.

1 INTRODUCTION

The efficacy of granular pile installation in soft or loose soil beneath foundations to support vertical loads has been proved beyond doubt. The granular pile derives its axial capacity from the passive earth pressure developed due to the bulging effect of the column head and increased resistance to lateral deformation under sustained vertical stress on upper boundary. In parallel, it acts as vertical drains speeding up the process of consolidation and the initial compaction of ambient soil during the installation process helps in sorrounding soil densification. It also aids in mitigating liquefaction potential by preventing development of high pore pressure by way of providing drainage path.

These granular piles are also being used to increase the stability of natural vulnerable slopes and embankments constructed over soft or weak ground. The primary improvement in this case are attributed to the free-draining characteristics and increased shearing resistance along critical failure plane. A mechanistic model of a unit-cell was analyzed to predict the increase in Factor of Safety due to increased resistance of the potential sliding mass of the 'pinned' slope due to the higher shear strength and free draining characteristics of the 'granular-pin' (Saha, 2001, 2003b, 2005, 2007, 2009). The present paper attempts to mathematically derive the optimum area-replacement by stronger granular 'pin' material of vulnerable slope from the strength and deformability characteristics of the composite ground and problem geometry.

2 THE PROBLEM INVESTIGATED

It is considered that the columns are installed in a regular array of triangular, square or hexagonal grid in plan area over the vulnerable stretch of the slope. The unit-cell width of each slice is equal to the effective diameter (D_e) of equivalent circular area (A_r) for the chosen grid geometry and spacing (S) (Fig.1, top-left). The row of column is assumed to be an equivalent, continuous strip of stonewall (Fig.1, top-right). The free body diagram of unit-cell (Fig.1, bottom) defines the problem.

The method of analysis follows the same manner as for a normal slope stability problem except that each slice is considered of finite width equal to the effective diameter (D_e) of the unit-cell encapsulating the pin or column (of diameter d) and each row of granular column is considered to be converted into an equivalent, continuous strip or wall. In the analysis it is assumed that the length of this vulnerable stretch of reinforced slope is much greater than the failure width so that the complex 3-D problem may reasonably be simplified into a 2-D one.

Detailed analysis considering horizontal interslice forces in line with Bishop's (1955) as well as parallel interslice forces in line with Spencer's (1967) is done to quantify the improvement.

The optimal range of area-ratio has been mathematically quantified and in the process some startling facts regarding the increase in FOS of granular pinned slope (at *optimal area-ratio*) over virgin slope vis-a-vis degree of improvement with increasing slope angle has also been revealed.

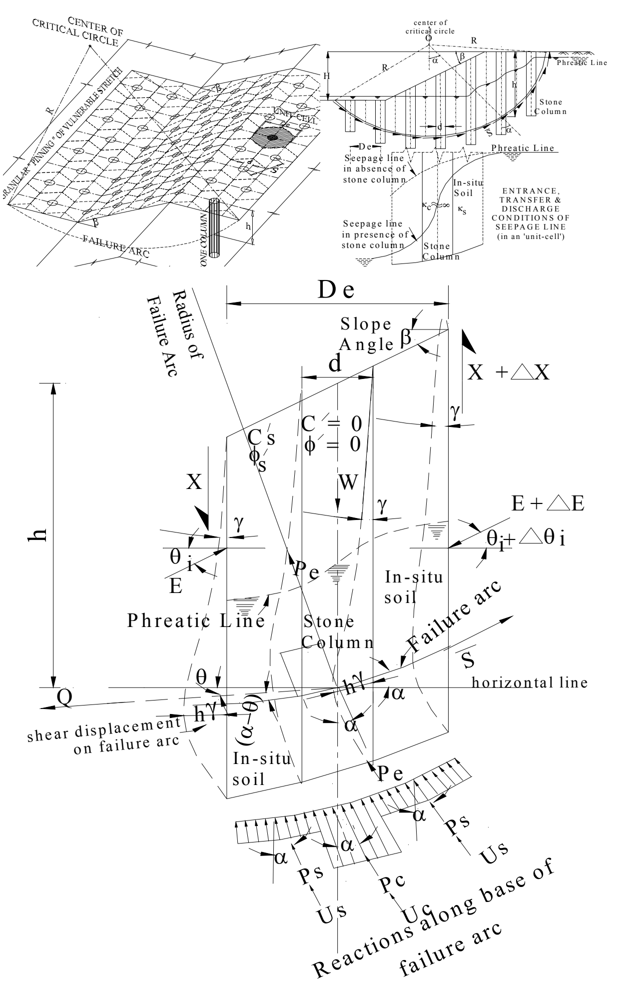


Figure 1. Potential sliding mass depicting problem geometry (top-left), The problem definition in a 2-D plane (top-right) & The free-body diagram of unit-cell for parallel interslice forces (bottom).

3 'UNIT-CELL' GEOMETRY & STRENGTH FACTORS

The subscripts s, c & e stands for soil, stone-column and equivalent parameters respectively.

3.1 *Area ratio & spacing ratio*

The area ratio $A_r = A_c/(A_s+A_c) = k_1(d/S)^2 = (d/D_e)^2 = (1/S_r)^2$. For an equilateral triangular pattern, $k_1=\pi/(2\sqrt{3})$, for a square pattern, $k_1=\pi/4$ and for hexagonal pattern, $k_1=\pi/(3\sqrt{3})$.

3.2 *Equivalent unit weight*

The equivalent unit weight of soil, γ_e is defined by,

$$\gamma_e = \gamma_s\left(1-\frac{1}{S_r}\right)+\gamma_c\left(\frac{1}{S_r}\right)=\gamma_s\left(1-\sqrt{A_r}\right)+\gamma_c\sqrt{A_r} \quad (1)$$

3.3 *Equivalent pore water pressure*

The equivalent pore water pressure u_e is defined by,

$$u_e = u_s\left(\frac{S_r-1}{S_r}\right)^2+u_c\left(\frac{1}{S_r}\right)^2 = u_s\left(1-\sqrt{A_r}\right)^2+u_cA_r \quad (2)$$

3.4 *Equivalent pore pressure ratio*

The equivalent pore pressure ratio r_{ue} is defined by,

$$r_{ue}=r_{us}\left\{\frac{1}{\dfrac{1}{\left(1-\sqrt{A_r}\right)}+\left(\dfrac{\gamma_c}{\gamma_s}\right)\left\{\dfrac{\sqrt{A_r}}{1-\sqrt{A_r}}\right\}}\right\}+r_{uc}\left\{\frac{1}{\dfrac{1}{\sqrt{A_r}}+\left(\dfrac{\gamma_s}{\gamma_c}\right)\left\{\dfrac{1-\sqrt{A_r}}{A_r}\right\}}\right\} \quad (3)$$

Since stone column is free-draining, hence, $r_{uc} \cong 0$.

3.5 *Weighted average SF mobilized along base of unit cell*

$$\bar{S} = \left\{c_s^{/}(D_e-d)\sec\alpha + P_s^{/}\tan\varphi_s^{/}\right\}l + \left\{P_c^{/}\tan\varphi_c^{/}\right\}n \quad (4)$$

Where, l and n are mobilization factors in respect to soil & stone column respectively and may be regarded as reciprocal of partial material safety factors. The absolute values of l and n vary for different slices, and $0 < l, n < 1$, & $l/n = m = F_c/F_s$.

3.6 *Local composite factor of safety*

The local composite factor of safety, $F=S/\bar{S}=$
Weighted average S.F. available along base of unit-cell
Weighted av. S.F. mobilized along base to maintain equilibrium

$$F = \left(\tau_s+\tau_c\right)\Big/\left(\frac{\tau_s}{F_s}+\frac{\tau_c}{F_c}\right) = \frac{F_c}{S_r}\left\{\frac{\tau_s}{\tau_c}+1\right\} \quad (5)$$

Further,

$$F = F_s\left(1-\frac{1}{S_r}\right)+F_c\left(\frac{1}{S_r}\right) = F_s\left(1-\sqrt{A_r}\right)+F_cA_r$$

(6)

Or,

$$F = F_s\left\{\left(1-\frac{1}{S_r}\right)+m\left(\frac{1}{S_r}\right)\right\} = F_s\left\{\left(1-\sqrt{A_r}\right)+mA_r\right\} \quad (7)$$

3.7 *Global composite factor of safety*

Considering horizontal interslice forces and examining in line with Bishop's (1955) analysis, the composite (or global) Factor of Safety (F_e) of reinforced slope was given by (Saha 2001):

$$F_e = \frac{\displaystyle\sum_{Cell,i=1}^{i=n}\frac{\left\{\left(1-\sqrt{A_r}\right)^2\left(\dfrac{c_s^{/}}{\gamma_eH}\right)\left(\dfrac{D_e}{H}\right)+\left(\dfrac{D_e}{H}\right)\left(\dfrac{h}{H}\right)\left(1-r_{ue}\right)\tan\varphi_s^{/}\right\}\sec\alpha_i}{F\left\{\dfrac{\left(1-\sqrt{A_r}\right)^2+m_iA_r\left(\dfrac{\tan\varphi_s^{/}}{\tan\varphi_c^{/}}\right)}{\left(1-\sqrt{A_r}\right)+m_i\sqrt{A_r}}\right\}+\tan\alpha_i\,\tan\varphi_s^{/}}}{\displaystyle\sum_{Cell,i=1}^{i=n}\left(\frac{D_e}{H}\right)\left(\frac{h}{H}\right)\sin\alpha_i} \quad (8)$$

Where, m_i is the ratio of mobilization potentials of soil and stone column (*the local stiffness factor*), is solved from the soil-stone column boundary strain compatibility within unit-cell by applying strain energy principle under the assumption of small change in geometry at limiting equilibrium. Equating the average strain energy in shear {$=\bar{U}= g\tau/2$, where g = shearing strain (*small*) at soil-stone column boundary at failure & τ = shear stress mobilized along the failure plane} stored in ambient subsoil and stone column per unit volume at limiting equilibrium, one gets,

$$\frac{F_c}{F_s}=\frac{1}{n}=m=\frac{\left(S_r-1\right)\left(P_c^{/}\tan\varphi_c^{/}\right)}{\left\{c_s^{/}\left(S_r-1\right)d\sec\alpha+P_s^{/}\tan\varphi_s^{/}\right\}} \quad (9)$$

Further, at limiting equilibrium, the active and passive stress states coincide when the conjugate stress ratio K has a unique value and is equal to, $K=\cos 2\beta+\sin 2\beta\tan\phi^{/}$. At soil-stone column boundary the shear stress τ_s (or τ_c) on a plane inclined at an arbitrary inclination α to the horizontal is given by,

$\tau_s=\{c^{/}_s(S_r-1)d\sec\alpha+P^{/}_s\tan\varphi^{/}_s\}/\{(S_r-1)d\sec\alpha\}$
$=0.5\gamma_s h\{\sin 2\alpha+K_s\sin 2(\beta-\alpha)\}$, and (10)

$\tau_c = P'_c \tan\varphi'_c / d\sec\alpha = 0.5\gamma_c h\{\sin 2\alpha + K_c \sin 2(\beta-\alpha)\}$ (11)

Where, K_s (or K_c) are the initial stress parameters of any small element at the soil-stone column interface and γ_s & γ_c are the in-situ bulk densities of soil and stone column respectively.

Limiting equilibrium of any small element at the soil-stone column interface calls for, $K_s\gamma_s h = K_c\gamma_c h$ or, $K_s/K_c = \gamma_c/\gamma_s$ (12)

Equation (9) in association with equations (10) to (12) yields,

$$m_i = \left(\frac{\cos 2\beta + \sin 2\beta \tan\varphi'_s}{\cos 2\beta + \sin 2\beta \tan\varphi'_c}\right)\left\{\frac{1+\tan 2(\beta-\alpha_i)\tan\varphi'_c}{1+\tan 2(\beta-\alpha_i)\tan\varphi'_s}\right\} \quad (13)$$

3.8 *Resultant of inter-slice forces*

Analyzing in line with Spencer's (1967), the resultant of interslice forces of reinforced slope was given by Saha (2001):

$$Q=\gamma_c H D_c \frac{\left[\dfrac{(1-\sqrt{A_r})^2\left(\dfrac{c'_s}{\gamma_c H}\right)}{F\left\{\dfrac{(1-\sqrt{A_r})^2+mA_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m\sqrt{A_r}}\right\}}+\left(\frac{1}{2}\right)\left(\frac{h}{H}\right)\left(\dfrac{\tan\varphi'_s(1-2r_{ue}+\cos 2\alpha)}{F\left\{\dfrac{(1-\sqrt{A_r})^2+mA_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m\sqrt{A_r}}\right\}}\right)-\left(\frac{1}{2}\right)\left(\frac{h}{H}\right)\sin 2\alpha\right]}{\left[\cos\alpha\cos(\alpha-\theta)\left(1+\dfrac{\tan\varphi'_s\tan(\alpha-\theta)}{F\left\{\dfrac{(1-\sqrt{A_r})^2+mA_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m\sqrt{A_r}}\right\}}\right)\right]} \quad (14)$$

When the external forces on the slope are in equilibrium, the vectorial sum of the inter-slice forces is zero (Spencer, 1967). i.e., $\Sigma[Q\cos\theta]=0$ (15) & $\Sigma[Q\sin\theta]=0$ (16)

Furthermore, if the sum of the moments of the external forces about the center of rotation is zero, the sum of the moments of the inter-slice forces about the center of rotation must also be zero, i.e., $\Sigma[QR\cos(\alpha-\theta)]=0$ (17)

As the slip surface is assumed to be circular, the radius of curvature (R) is constant and $\Sigma[Q\cos(\alpha-\theta)]=0$ (18)

In a given problem, there are thus three equations to be solved, two in respect of forces {Equations (15) & (16)} and one in respect of moments {Equation (17)}. Values of F and θ must be found which satisfy all three equations. Also, it must be noted that although, for a given slice, the value of θ would be the same in each equation, the inter-slice forces might not be parallel throughout (Spencer, 1967). Moreover, the point of application of an inter slice force is not far outside the middle third of the vertical boundary on which the force acts. In order to simplify the problem without appreciable loss in accuracy, it is assumed that the inter-slice forces are parallel, i.e., θ is constant throughout, when Equations (15) & (16) become identical, i.e., $\Sigma Q=0$ (19).

4 MATHEMATICAL OPTIMISATION

It may be observed from equations (8) & (19) that the expression for F & Q are the same as that of Bishop's (1955) & Spencer's (1967) respectively, excepting the factor:

$$\sum_{\text{Cell},\,i=1}^{i=n}\left\{\frac{(1-\sqrt{A_r})^2+m_i A_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m_i\sqrt{A_r}}\right\}=\Re\,(\text{say}) \quad (20)$$

that gets multiplied with the "F" term and the factor $(1-\sqrt{A_r})^2$ that gets multiplied with the "c'_s" term.

For 'FOS' F to be minimum, $\Re$ has to be minimum.

To locate the optima, differentiating $\Re$ w.r.t. A_r,

$$\frac{d}{dA_r}(\Re)=\frac{d}{dA_r}\sum_{\text{Cell},\,i=1}^{i=n}\left\{\frac{(1-\sqrt{A_r})^2+m_i A_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m_i\sqrt{A_r}}\right\} \quad (21)$$

On simplification, $d\Re/dA_r$ =

$$\sum_{\text{Cell},\,i=1}^{i=n}\frac{\left\{A_r\left(m_i\dfrac{\tan\varphi'_s}{\tan\varphi'_c}+1\right)(m_i-1)+2\sqrt{A_r}\left(m_i\dfrac{\tan\varphi'_s}{\tan\varphi'_c}+1\right)-(m_i+1)\right\}}{2\sqrt{A_r}\left\{(1-\sqrt{A_r})+m_i\sqrt{A_r}\right\}^2} \quad (22)$$

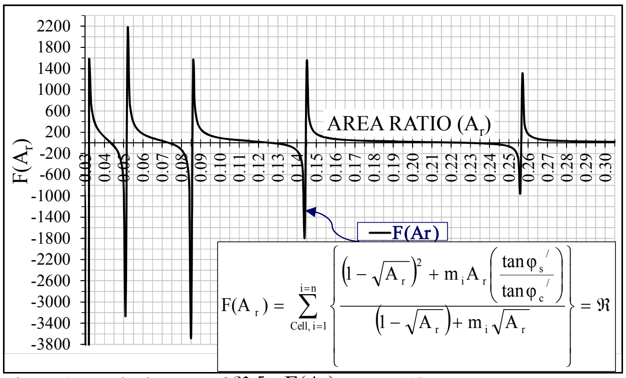
At optimization, the numerator, N =

$$\mathrm{N}=\sum_{Cell,\,i=1}^{i=n}\left\{A_r\left(m_i\frac{\tan\varphi'_s}{\tan\varphi'_c}+1\right)(m_i-1)+2\sqrt{A_r}\left(m_i\frac{\tan\varphi'_s}{\tan\varphi'_c}+1\right)-(m_i+1)\right\}\to 0 \quad (23)$$

Further, to check whether this represents a minimum, differentiating $\Re$ again and simplifying,

$$\frac{d^2\Re}{dA_r^2}=\sum_{\text{Cell},\,i=1}^{i=n}\left\{\frac{\{(1-\sqrt{A_r})+m_i\sqrt{A_r}\}(m_i+1)-3(m_i-1)\left(m_i\dfrac{\tan\varphi'_s}{\tan\varphi'_c}+1\right)A_r}{4A_r\sqrt{A_r}\left\{(1-\sqrt{A_r})+m_i\sqrt{A_r}\right\}^3}\right\} \quad (24)$$

Equation (23) and (24) has been solved numerically considering same as well as different critical failure surfaces for virgin and reinforced soil, and presented in Table-1 (Col.14, Col.23 and Col.18, Col.27), for different values of slope geometry and strength parameters of host soil and installed column, to arrive at the optimal area and spacing ratios, $\mathbf{A_{r,opt}}$ & $\mathbf{S_{r,opt.}}$. of stone column arrangement (Col.7 & 8). The factor $\Re$ (Equation 20), which is a function of A_r and Σm_i, the sum of local stiffness factors (Col.13 & 22), accounts for the primary difference between virgin and reinforced slope, is tabulated in Col.17 & 26. A typical graph of $\Re$ (*corresponding to Sl.-2 of Table-1*) is presented in Figure-2. Slope angles from about 25^0 to 70^0 (Col.3) have been investigated. Realistic range of ϕ'_s (Col.5) & ϕ'_c (Col.6) values has been assumed. Typical results of stone column reinforced slope over virgin slope reveal that ***FOS decreases with increasing slope angle for virgin slope*** (Col.12), ***though for reinforced slope at optimum area ratio, FOS increases significantly*** (Col.15 & 24) ***and percentage increment increases appreciably as slope becomes steeper*** (Col.16 & 25). It is found that the ***optimal range of area ratio (A_r) lies between 0.11 (column spacing=3d) to about 0.30 (column spacing=1.8d)***. Further, the second order derivative of $\Re$ at this optimal range of area ratio (Col.18 & 27) is found to be positive (i.e., > 0) indicating that it is the minimal value corresponding to minimum FOS of stone column reinforced slope.



$$F(A_r)=\sum_{\text{Cell},\,i=1}^{i=n}\left\{\frac{(1-\sqrt{A_r})^2+m_i A_r\left(\dfrac{\tan\varphi'_s}{\tan\varphi'_c}\right)}{(1-\sqrt{A_r})+m_i\sqrt{A_r}}\right\}=\Re$$

Figure 2. Typical curve of $\Re$ [= $F(A_r)$, Eqn.-20]

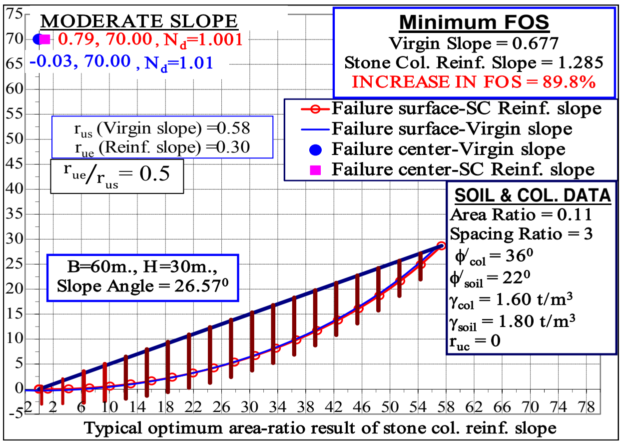


Figure 3. Stone col. reinf. slope profile showing critical failure surfaces of virgin & reinf. slope (with optimum A_r) (*Sl.-2 of Table-1*)

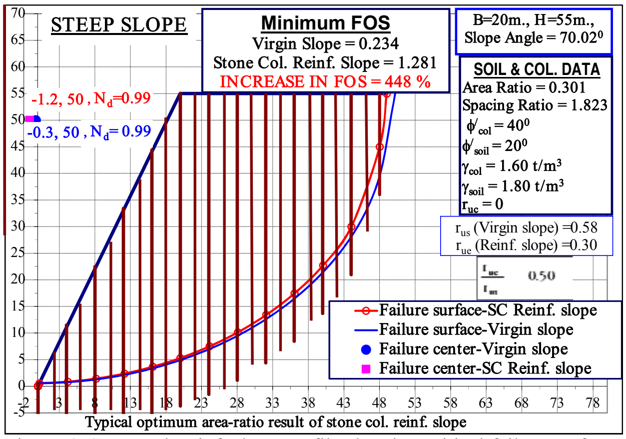


Figure 4. Stone col. reinf. slope profile showing critical failure surfaces of virgin & reinf. slope (with optimum A_r) (*Sl.-8 of Table-1*)

Table-1. Typical results of increase in FOS of stone column reinforced slope over virgin slope, considering same critical surface and different critical surfaces.

	GEOMETRY & STRENGTH PARAMETERS								CRITICAL SLIP CIRCLE PARAMETERS: VIRGIN & STONE COLUMN REINFORCED SLOPE (considering same critical failure surface)										CRITICAL SLIP CIRCLE PARAMETERS: STONE COLUMN REINFORCED SLOPE (considering different critical failure surfaces)								
Sl. No.	B = Slope Width	H = Slope Height	Slope Angle = β = $\tan^{-1}$(H/B)	No. of Slices	ϕ'_{soil}	ϕ'_{col}	$A_{r,\ opt.}$ = Optimum Area Ratio	$S_{r,\ opt.}$ = Optimum Spacing Ratio	CX (=x-co-ord. of critical circle centre)	CY (=y-co-ord. of critical circle centre)	N_d=Depth Factor	Min. FOS of Virgin Slope (Bishop's)	Σm_i= Sum of local stiffness factor	Value of Expression N	Min. FOS of Stone Col. Reinf. Slope (Author's)	Increase in Min. FOS of St. Col. Reinf. Slope over Virgin Slope	ℜ	$d^2ℜ/dA_r^2$	CX (=x-co-ord. of critical circle centre)	CY (=y-co-ord. of critical circle centre)	N_d=Depth Factor	Σm_i= Sum of local stiffness factor	Value of Expression N	Min. FOS of Stone Col. Reinf. Slope (Author's)	Increase in Min. FOS of St. Col. Reinf. Slope over Virgin Slope	ℜ	$d^2ℜ/dA_r^2$
Col.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	65	30	24.78	20	26	34	0.212	2.172	-0.50	70	0.98	0.868789	24.467	-0.0002	1.434760	1.651448	84.7	6714.5	-0.16703	70	0.980	23.759	-0.0003	1.374885	1.582531	32.7	7098.1
2	60	30	26.57	20	22	36	0.110	3.015	-0.03	70	1.01	0.677021	7.038	-0.0004	1.398758	2.066049	-1.5	63236	0.79377	70	1.001	29.599	-0.0004	1.284815	1.897747	-30	1E+08
3	60	35	30.26	20	24	40	0.141	2.663	0.00	70	1.00	0.528514	11.107	-0.0006	1.363222	2.579351	2.89	9271.7	-0.62178	70	1.000	11.108	-0.0012	1.360578	2.574349	42	7922.5
4	35	30	40.60	20	25	36	0.250	2.000	-0.30	30	0.98	0.439451	17.319	-0.0007	1.285020	2.924150	43.2	1439.9	-1.02000	30	0.980	8.213	-0.0004	1.261947	2.871647	0.51	5272.9
5	30	30	45.00	10	30	40	0.258	1.969	-0.25	65	0.90	0.568390	11.775	-0.0004	1.515885	2.666980	21.8	371.06	-1.03146	65	0.900	10.465	-0.0002	1.309223	2.303388	7.64	258.27
6	40	45	48.37	20	18	36	0.255	1.980	-0.50	50	0.99	0.214849	17.171	-0.0002	1.111291	5.172420	26.4	984.75	-0.97229	50	0.990	14.261	-0.0007	1.075824	5.007342	12.5	1066.3
7	30	60	63.43	10	26	38	0.222	2.122	-0.50	70	0.95	0.309685	4.862	-0.0002	1.400194	4.521343	1.4	577.42	-5.03880	70	0.950	12.310	-0.0004	1.236136	3.991587	7.65	143.66
8	20	55	70.02	12	20	40	0.301	1.823	-0.30	50	0.99	0.233916	13.887	-0.00001	1.321349	5.648827	20.1	251.62	-1.19824	50	0.990	11.949	-0.0001	1.281082	5.476682	5.84	166.5

Figure 3 & 4 depict two typical slope profiles reinforced with granular pinning with optimum area-ratio.

Interestingly, similar results for mathematical optimization were obtained for vertical loading on stone column reinforced soil also. For vertical stress equilibrium case, as the ultimate strength of the column is governed by the maximum lateral resistance of host soil around the zone of column bulge, it was mathematically deduced (Saha, 2005) that the normalized horizontal soil stress maximizes at a minimum area-ratio within the range of A_r values of 0.1275 to 0.0875 (S_r =2.80 to 3.38), for a ϕ_c' variation of 38⁰ to 44⁰. This fact was further corroborated when field test results of rammed stone columns in a wide variety of alluvial soils (ranging from loose to medium dense sands/silty sands and clayey silt/silty clays with and without fills) reported by Chandra Prakash et al (2005) were analyzed (Saha, 2010). It emerged that the range of optimum area ratio for vertical loading is between A_r = 0.08 to A_r = 0.25 (S_r=3.5 to S_r=2.0), the optimal being around A_r=0.13 (S_r=2.75).

5 CONCLUSION

It followed from the foregoing analysis that for a techno-economic solution, only a small portion of the in-situ soil needs to be replaced. Computational analytics of stone column reinforced slope mechanistic modelling revealed that the range of optimum area-ratio lies between A_r=0.11 to A_r=0.30 (that is, a column spacing of 3.0d to 1.8d) of regular array of stone column plan arrangement, for soil slopes varying from 25⁰ to 70⁰ with the horizontal. The key factor playing the pivotal role in area-ratio optimisation of composite ground is the local stiffness factor defined to be the ratio of mobilization potentials of shear force components of virgin soil and installed column along the critical plane at incipient failure.

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