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# Numerical simulation of the consolidation characteristics considering the construction process for tailings

Simulation numérique des caractéristiques de consolidation en tenant compte du processus de construction des résidus

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**ABSTRACT:** The stability of tailings dams is important for the safety of residents living in the reservoir and downstream area, and is closely related to the consolidation of the dam during construction. In the present study, the consolidation behavior of fine tailings from Daxigou tailings dam in Shanxi Province, China was numerically analyzed through Geo-studio. A one-dimensional model coupling the seepage and stress-strain fields was developed to simulate the construction process of the tailing materials. The change of permeability coefficients and elastic modulus with effective stress was considered to simulate realistic situations and obtain reasonable predictions. The influence of the coefficient of consolidation was analyzed through the one-dimensional model. The results showed that the consolidation behavior of the tailing materials mainly depended on the coefficient of consolidation  $C_v$  rather than the elastic modulus and the permeability coefficient. The coefficient of consolidation significantly influenced the consolidation behavior in both construction and operation periods. Therefore, the change of  $C_v$  with effective stress should be considered in engineering design.

**RÉSUMÉ:** La stabilité des barrages à résidus est importante pour la sécurité des résidents vivant dans le réservoir et la zone en aval, et est étroitement liée à la consolidation du barrage pendant la construction. Dans la présente étude, le comportement de consolidation des résidus fins du barrage de résidus Daxigou dans la province du Shanxi, en Chine, a été analysé numériquement par Geo-studio. Un modèle unidimensionnel couplant les champs d'infiltration et de contrainte-déformation a été développé pour simuler le processus de construction des matériaux de résidus. Le changement des coefficients de perméabilité et du module élastique avec un stress effectif a été considéré pour simuler des situations réalistes et obtenir des prédictions raisonnables. L'influence du coefficient de consolidation a été analysée par le modèle unidimensionnel. Les résultats ont montré que le comportement de consolidation des matériaux de résidus dépendait principalement du coefficient de consolidation  $C_v$  plutôt que du module d'élasticité et du coefficient de perméabilité. Le coefficient de consolidation a considérablement influencé le comportement de consolidation dans les périodes de construction et d'exploitation. Par conséquent, le changement de  $C_v$  avec un stress efficace devrait être pris en compte dans la conception technique.

**KEYWORDS:** consolidation behavior; coupled analysis; coefficient of consolidation; simulation accuracy.

## 1 INTRODUCTION

Tailings production has increased rapidly during the last decades around the world, especially in countries rich in mineral resources including China, and this has caused lots of failures and damages (Zhang, et al., 2015). Most of the tailings are stored in tailings dams (Azam and Li, 2010). The stability of tailings dams is important for the safety of residents living in the reservoir and downstream areas (Klohn, 1997; Ferdosi et al., 2015).

Because of special construction methods, the consolidation behavior of tailings dam is quite different from that of conventional earth dam, although the rules and experiences gained from earth dam are used in engineering practice of tailings dam. The tailings are discharged into the dam with hydraulic filling technique, in a slurry-like original state. The high construction speed leads to high excess pore pressure remaining in the tailings layers. Furthermore, the material parameters changes during the consolidation period (Hu, 2016). All these characters are different from those of the earth dam, therefore further research on the unique consolidation behavior of tailings is needed for engineering practice (Wels, 2000; Jeeravipoolvarn, 2008; Bonin, et al., 2014).

Many researchers studied the consolidation of tailing dams through numerical methods. Some studies showed material parameters including elastic modulus and permeability coefficients had remarkable effects on the consolidation behavior of tailings (Ahmed and Siddiqua, 2014; Ito and Azam, 2013). Other studies illustrated the influence of coefficients of consolidation on this topic (Fahey et al., 2010). However, the change of these three parameters during the consolidation

process and their influence on the consolidation behavior of tailing dams still remain unclear.

This study aimed to conduct further analysis to investigate the consolidation characteristics of tailing dams through one-dimensional numerical models, and mainly focused on the influence of the changing of tailing parameters. In addition, the influence of the simulation accuracy on the consolidation behavior was also discussed.

## 2 NUMERICAL SIMULATION

### 2.1 Non-linear change of tailing properties

The governing equation of the simulation is Biot's consolidation theory (Biot, 1941). In this theory, the consolidation coefficient expression under one-dimensional condition is shown as follows:

$$C_{v1} = \frac{kE'(1-\nu')}{\gamma_w(1+\nu')(1-2\nu')} \quad (1)$$

where  $C_{v1}$  means the coefficient of consolidation under one-dimensional condition;  $k$  means the permeability coefficient;  $E'$  means the elastic modulus;  $\gamma_w$  means the unit weight of water and  $\nu'$  means the passion's ratio. Because  $\nu'$  is constant in this study,  $C_v$  mainly depends on  $k$  and  $E'$ .

Plenty of experimental studies have shown the regression fitting relationship between void ratio and permeability coefficient. Meanwhile, a regression fitting relationship also

exists between effective stress and void ratio, as shown in the study by Mesri and Rokhsar (1974):

$$e = e_0 + C_k \log\left[\frac{k}{k_0}\right] \quad (2)$$

$$e = e_0 - C_c \log\left[\frac{\sigma'}{\sigma'_0}\right] \quad (3)$$

in which  $e$  means the void ratio and  $e_0$  is the initial void ratio;  $k_0$  is the initial permeability coefficient;  $C_c$  is the compression index and  $C_k$  is permeability constant;  $\sigma'$  is effective stress and  $\sigma'_0$  is its initial value. Therefore the relationship between  $k$  and  $\sigma'$  can be represented as:

$$k = k_0 \left[\frac{\sigma'}{\sigma'_0}\right]^{-(C_c/C_k)} \quad (4)$$

The relationship between the oedometric modulus and void ratio is similar to that between the permeability coefficient and void ratio. The expression is as follows:

$$e = e_0 - N \times \log(E_s / E_{s0}) \quad (5)$$

in which  $E_{s0}$  is the initial oedometric modulus and  $N$  is a factor to describe the relationship between  $E_s$  and  $e$ . In the same way, the relationship between compressibility modulus and effective stress can be represented as:

$$E_s = E_{s0} \left[\frac{\sigma'}{\sigma'_0}\right]^{C/N} \quad (6)$$

Then by considering the conversion relation between the oedometric modulus  $E_s$  and elastic modulus  $E$ , we can derive the relationship between elastic modulus and effective stress, which can be expressed as:

$$E = E_0 \left[\frac{\sigma'}{\sigma'_0}\right]^{C/N} \quad (7)$$

## 2.2 Numerical model

In this study, a one-dimensional model was developed to simulate the consolidation of tailings process. The final height of the model was 200 m. There were two time phases in this simulation, named construction period and operation period separately. The construction period lasted for 300 days, and during this period, the height of the tailings increased and the excess pore water pressure accumulated gradually. The operation period lasted for 10 years, and in this period, the height of the model reached the final value and the tailings consolidated with the dissipation of the excess pore water pressure.

The material in the model was the fine tailings from the Daxigou tailings dam in Shanxi province, China. The geotechnical parameters were measured from experiments, as shown in Table 1.

Table 1. Geotechnical Parameters of the fine tailings in the numerical simulation.

Material Parameters	$\gamma$ (kN/m <sup>3</sup> )	$\nu$	$\omega_{sat}$	$k_{sat}$ (m/sec)
Value	21.5	0.35	0.44	$3.42 \times 10^{-7}$

In Table 1,  $\gamma$  is the unit weight of tailings;  $\omega_{sat}$  means the saturated water content and  $k_{sat}$  means the saturation permeability coefficient.

As shown before, the permeability coefficient and the elastic modulus were related to the effective stress. According to the results of consolidation test and permeability test illustrated in Fig. 1 and Fig. 2, the fitting relationship could be expressed as follows:

$$E' = 27.74 \times \sigma^{0.881} \text{ (kPa)} \quad (8)$$

$$k = 3.4159 \times \sigma^{-0.137} \text{ (} 10^{-7} \text{ m/s)} \quad (9)$$

According to the two equations,  $C_v$  increased with effective stress because the sum of the indexes was larger than zero. In following discussions, other conditions including constant and decreasing  $C_v$  were also discussed to analyze the impact of  $C_v$ .

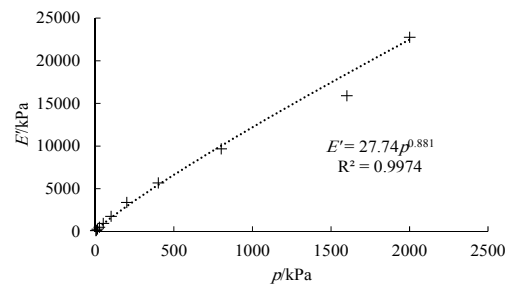


Figure 1. Fitting curves of  $E$ - $\sigma$

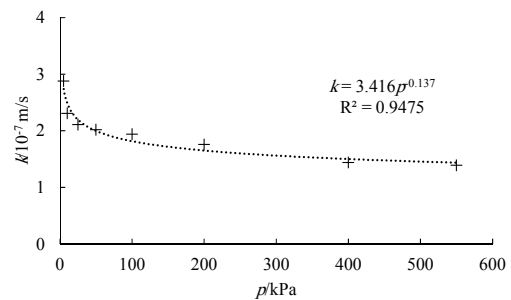


Figure 2. Fitting curves of  $k$ - $\sigma$

## 2.3 Excess pore water pressure ratio

The average degree of consolidation  $\bar{U}$  is usually used to represent the degree of consolidation, which can be calculated by both the value of pore water pressure and the settlement. However, as for the condition discussed in this study, since the height of the model keeps changing during the construction period, it is not proper to use  $\bar{U}$  to represent the consolidation degree here.

A non-dimensional index, named excess pore water pressure ratio  $\alpha$ , is therefore defined to describe the degree of consolidation. It can be calculated by the following formula:

$$\alpha = \frac{\int_0^h Q_t dh}{\int_0^H Q_0 dh} \quad (10)$$

where  $Q_t$  means the excess pore water pressure value for each point on the present moment;  $h_t$  means the total height of tailings at this moment;  $H$  means the final height of the tailings. If all the tailings were stacked up to the height of  $H$  instantaneously, the initial excess pore water pressure value for each point is  $Q_0$ .

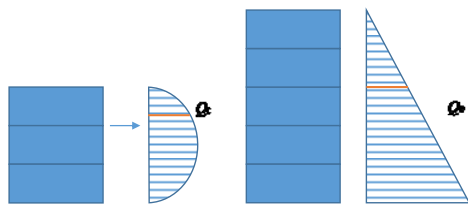


Figure 3. Sketches of the calculation of  $\alpha$

As shown in Fig. 3, the numerator of the formula above is the area of the distribution curve of excess pore water pressure along the height on the present moment. The denominator of the formula equals to the area of the distribution curve of excess pore water pressure if the height of the materials increased to the final height for a moment. This area is defined as the area of a triangular distribution along final height, which could be calculated as follows:

$$\int_0^H Q_0 dh = \frac{1}{2} \gamma_{sat} H^2 \quad (11)$$

By definition, the value of  $\alpha$  ranges from 0 to 1. The following discussion in this paper will use  $\alpha$  for comparison.

### 3 RESULTS AND DISCUSSION

#### 3.1 Influence of the coefficient of consolidation $C_v$

Previous numerical simulations in engineering usually considered a constant coefficient of consolidation, but the experiments show that the coefficient of consolidation of tailings changes with the effective stress. In this section, the influence of different relationships between  $C_v$  and  $\sigma$  on the consolidation behavior is discussed. The working conditions of numerical simulations are shown in Table 2.

The final height of the model is 200 m, with a unit weight of 21.5 kN/m<sup>3</sup> for the tailings, and the maximum effective stress is about 4300 kPa. The variation range of the coefficient of consolidation for each working condition is also shown in Table 2, which was calculated in the model considering the change of the effective stress from 50 kPa to 4300 kPa. The initial value of the coefficient of consolidation (when the effective stress is equal to 50 kPa) is constant in each working condition.

Table 2. Working conditions of different relationship between  $C_v$  and  $\sigma$

Working condition	$E$ (kPa)	$k$ ( $\times 10^{-7}$ m/s)	$C_v$ (m <sup>2</sup> /s)	Variation range of $C_v$
1	$E=27.7 \times \sigma^{0.881}$	$k=62.8 \times \sigma^{0.881}$	$2.79 \times 10^{-5}$	Constant (1)
2	$E=510 \times \sigma^{0.137}$	$k=3.42 \times \sigma^{0.137}$	$2.79 \times 10^{-5}$	Constant (2)
3	$E=27.7 \times \sigma^{0.881}$	$k=1.15 \times 10^3 \times \sigma^{1.63}$	$5.13 \times 10^{-4} \times \sigma^{0.744}$	$\div 28$
4	$E=27.7 \times \sigma^{0.881}$	$k=269 \times \sigma^{1.25}$	$1.20 \times 10^{-4} \times \sigma^{0.372}$	$\div 5$
5	$E=27.7 \times \sigma^{0.881}$	$k=3.42 \times \sigma^{0.137}$	$1.52 \times 10^{-6} \times \sigma^{0.744}$	$\times 28$
6	$E=119 \times \sigma^{0.509}$	$k=3.42 \times \sigma^{0.137}$	$6.52 \times 10^{-6} \times \sigma^{0.372}$	$\times 5$

Figs. 4 and 5 show the relationship between the excess pore water pressure ratio and time for different working conditions in the construction period and operation period. The curve named

' $\times 28$ ' corresponds to the realistic material parameters obtained from experiments.

The coefficient of consolidation  $C_v$  exhibits significant impact on the consolidation behavior in both construction period and operation period. In the construction period, the excess pore water pressure ratio increased with time. The curve was higher when  $C_v$  decreased with effective stress and lower when  $C_v$  increased with effective stress. Larger change in  $C_v$  causes larger differences between the curves. In the operation period, the excess pore water pressure ratio decreases with time.

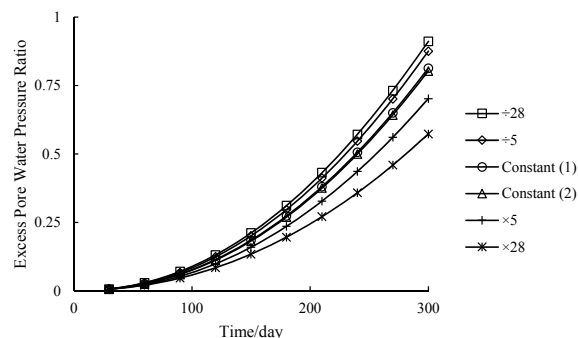


Figure 4. Excess pore water pressure ratio-time history of different working condition in the construction period (different relationship between  $C_v$  and  $\sigma$ )

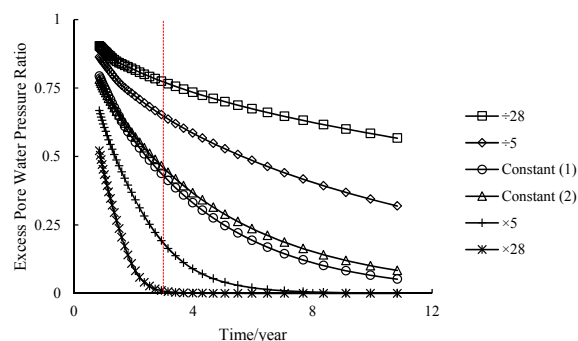


Figure 5. Excess pore water pressure ratio-time history of different working condition in the operation period (different relationship between  $C_v$  and  $\sigma$ )

When  $C_v$  remains constant during the simulation, the relationships between the excess pore water pressure ratio and time are almost the same, although both the permeability coefficient and the elastic modulus changed with the effective stress.

The red dash line in Fig. 5 shows when  $t$  equals to 3 years, the value of  $\alpha$  is quite different for different curves. The value of  $\alpha$  of the curve marked ' $\times 28$ ', which corresponds to the actual material parameters gained from experiments, is almost equal to zero (below 0.01), although the value of the curves marked 'Constant 1' and 'Constant 2' is about 0.45. It shows that in the condition of Daxigou tailings, using a constant  $C_v$  might lead to a lower degree of consolidation (higher  $\alpha$ ) compared with the realistic situation, therefore it is necessary to consider the change of  $C_v$  with effective stress in the consolidation simulation of sequential deposition tailings.

To further investigate the impact of the coefficient of consolidation, we calculated more working conditions with constant  $C_v$ . The results are shown in Table 3.

Table 3. Working conditions with constant  $C_v$

Working condition	$E$ (kPa)	$k$ ( $\times 10^{-7}$ m/s)	$C_v$ (m <sup>2</sup> /s)	Variation range of $C_v$
1	$E=27.7 \times \sigma^{0.881}$	$k=62.8 \times \sigma^{0.881}$	$2.79 \times 10^{-5}$	Constant (1)
2	$E=510 \times \sigma^{0.137}$	$k=3.42 \times \sigma^{0.137}$	$2.79 \times 10^{-5}$	Constant (2)
3	$E=871$	$k=2.00$	$2.79 \times 10^{-5}$	Constant (3)
4	$E=2.25 \times 10^4$	$k=0.0775$	$2.79 \times 10^{-5}$	Constant (4)
5	$E=4.59 \times 10^4$	$k=0.0379$	$2.79 \times 10^{-5}$	Constant (5)

For all the working conditions listed in Table 3,  $C_v$  is constant and equal, but the variations of elastic modulus and permeability coefficient are different. The first two conditions are same with the first two conditions in Table 2. In the last three conditions, the elastic modulus and permeability coefficient are both constant.

Figs. 6 and 7 illustrate that for all the conditions with the same  $C_v$ , the relationship between the excess pore water pressure ratio and time are almost the same, although the variations of elastic modulus and permeability coefficient are different. The consolidation behavior mainly depends on the coefficient of consolidation, rather than the elastic modulus and permeability coefficient.

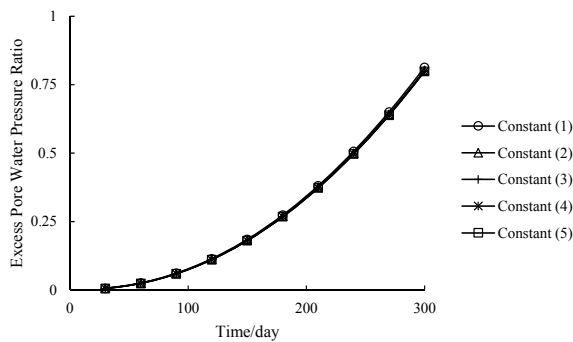


Figure 6. Excess pore water pressure ratio-time history of different working condition in the construction period (constant  $C_v$ )

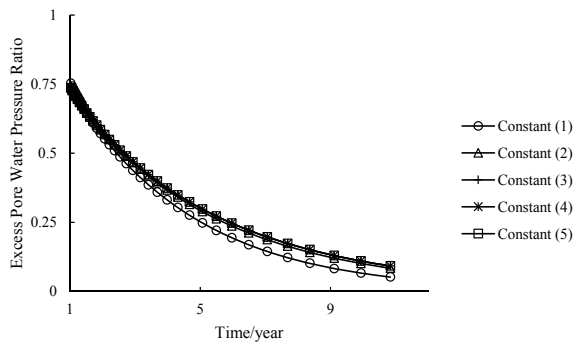


Figure 7. Excess pore water pressure ratio-time history of different working condition in the operation period (constant  $C_v$ )

### 3.2 Influence of simulation accuracy

Because tailings are piped into the area of the dam through the pipeline continuously, the construction process of tailings dam is actually a continuous process, which is different from that of earth dams. Therefore, using the construction grading method in this paper to simulate the process of the accumulation of tailings dam is in fact an approximate approach. Theoretically

speaking, more calculation grades will lead to more accurate results when the final height of the tailings is fixed.

In this section, the influence of simulation accuracy on the behavior of consolidation is examined. We developed models with different grades. The construction period and the final height of the tailings are the same for each model, which means the speed of construction is the same for each model. The number of grades and time for each grade are shown in Table 4 and the figures of the models are shown in Fig. 8.

Table 4. Working conditions of different simulation accuracy

Number of grades	1	2	3	4	5
Length of time for each grade	300 d	150 d	60 d	30 d	15 d

Figs. 9 and 10 compared the relationship between the excess pore water pressure ratio and time curves for different simulation accuracy. Fig. 9 showed the curves for construction period and Fig. 10 showed the curves for operation period. When there was only one grade in the model, the accumulation of the pore water pressure in the construction period could not be displayed in the  $\alpha$ -time curve because there was only one point in the figure. For other models with the number of grades over two, the accumulation tendency could be displayed clearly.

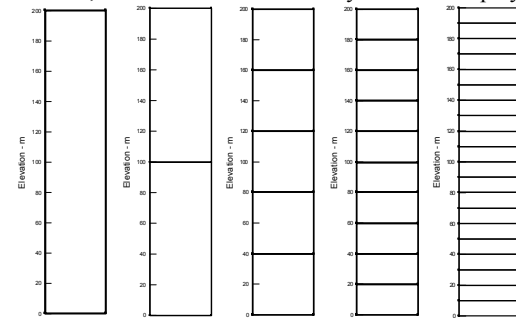


Figure 8. Models of different simulation accuracy

The curves gradually approached to each other with the increase in the numbers of the grades. The curve with ten grades and twenty grades were almost the same, indicating that it was appropriate to use construction grading method to simulate the continuous process of the accumulation of tailings dam. For the calculation conditions of Daxigou tailings dam, the models of over ten grades are precise enough for engineering design.

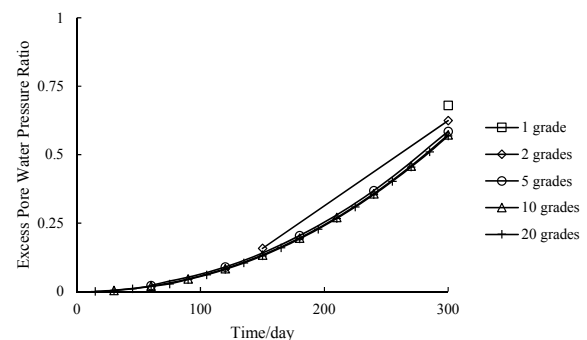


Figure 9. Excess pore water pressure ratio-time history of different working condition in the construction period (different simulation accuracy)

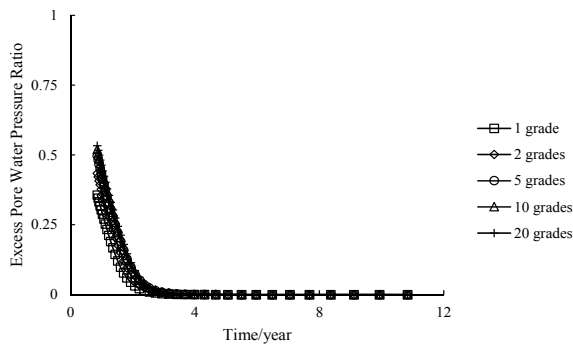


Figure 10. Excess pore water pressure ratio-time history of different working condition in the operation period (different simulation accuracy)

The maximum excess pore water pressure for the models were different, which was reached at the end of the construction period and the beginning of the operation period. This value also showed a convergent tendency, according to Fig. 11.

The shape of the curves in operation period was similar to each other. Through a horizontal translation of the curves, the curves almost became the same (Fig. 12). The curve marked '1 grade' was chosen as a baseline and moved other curves to the left by different years, which were shown in the legend. The translational time shows the error between the one grade model and models with higher accuracy. It could be seen that the error also shows a convergent tendency, which is possible to be controlled.

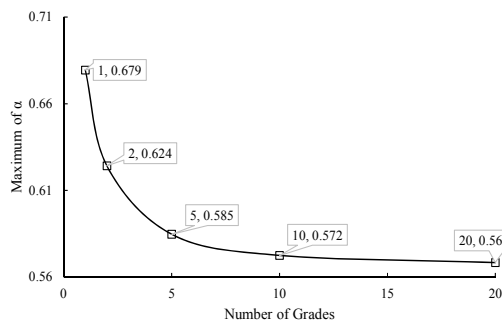


Figure 11. Maximum of excess pore water pressure for the models

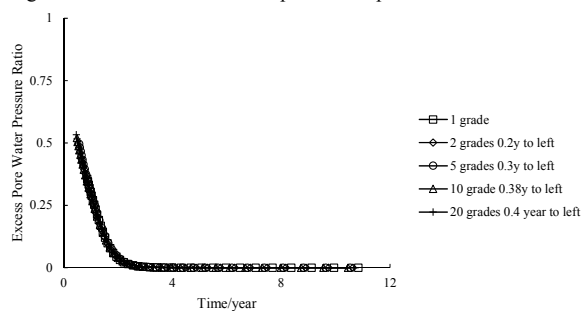


Figure 12. Partial coincident relations of the curves through a horizontal translation

#### 4 CONCLUSION

In this study, we investigated the consolidation behavior of tailings by considering the construction process in the numerical models. The influence of the non-linear variation of tailing parameters on the consolidation behavior was analyzed. Furthermore, we also analysed the effect of the simulation

accuracy of the construction procedure. The following conclusions were drawn:

(1) The excess pore water pressure ratio,  $\alpha$ , which is defined in this study, could represent the degree of the consolidation of tailings in the construction period and the operation period.

(2) The consolidation behavior of tailings mainly depends on the coefficient of consolidation rather than the elastic modulus and permeability coefficient.

(3) The coefficient of consolidation ( $C_v$ ) exhibits significant influence on the consolidation behavior in both the construction period and operation period. It is necessary to consider the change of  $C_v$  with effective stress in engineering calculation.

(4) The accuracy of simulation improves rapidly with the increase in the grades of the models. It is appropriate to use construction grading method to simulate the continuous process of the accumulation of tailings dam. For the calculation conditions of Daxigou tailings dam, the models of over ten grades are precise enough in engineering.

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