Influence of the unsaturated soil property functions on numerical analyses of saturated and unsaturated water flow in embankments

Influence des fonctions des propriétés du sol non saturées sur les analyses numérique du flux d'eau saturée et non saturée dans les digues

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ABSTRACT: At present there are several specialized programs that allow performing numerical modeling of water flow in soils under saturated and unsaturated conditions. These programs, which apply unusual concepts in geotechnical engineering practice and classical soil mechanics, induce that incorrect unsaturated properties are assigned in several analyses that affect the solution of the problems. This raises the question whether it is really important to consider the concepts of the theory of unsaturated soils, and thus the importance of assuming the unsaturated soil property functions involved in the calculations (soil-water characteristic curve and hydraulic conductivity function). In this paper the influence of the unsaturated properties of a material on the results of water flow analyses in the section of a homogeneous embankment is evaluated. Different models for estimating unsaturated functions which provide solutions to these problems with sufficient approximation for practical purposes are also compared. The calculations are performed using the Finite Element Method (FEM) via Seep/W code. At the end, general comments concerning the performed calculations and recommendations for the study of such problems in geotechnical engineering practice are given.

RÉSUMÉ: Actuellement, il existe plusieurs programmes spécialisés qui permettent la modélisation numérique du flux d'eau dans les sols dans des conditions saturées et non saturées. Ces programmes, qui appliquent des concepts inhabituels en ingénierie géotechnique et mécanique des sols classique, permettent d'attribuer des propriétés non saturées incorrectes dans plusieurs analyses qui affectent la solution des problèmes. Cela soulève la question de savoir s'il est vraiment important de considérer les concepts de la théorie des sols non saturés et donc de l'importance d'assumer les fonctions ou propriétés des matériaux non saturés impliqués dans les calculs (courbe caractéristique sol-eau et fonction de conductivité hydraulique). Dans cet article, on évalue l'influence des propriétés non saturées d'un matériau sur les résultats des analyses d'écoulement d'eau dans la section d'un remblai homogène. On compare aussi différents modèles pour estimer des fonctions non saturées qui fournissent des solutions à ces problèmes avec une approximation suffisante à des fins pratiques. Les calculs sont effectués à l'aide de la méthode des éléments finis (FEM) via le code Seep/W. A la fin, des commentaires généraux sur les calculs effectués, ainsi que des recommandations pour l'étude de ces problèmes en ingénierie géotechnique sont donnés.

KEYWORDS: SWCC, hydraulic conductivity function, flow rate, hydraulic gradients, saturated and unsaturated flow.

1 THEORETICAL BACKGROUND

1.1 *Unsaturated soil property functions*

The water flow analyses in unsaturated soils depend on two basic functions: a) the water storage function or soil-water characteristic curve (SWCC), that describes the relationship between the volumetric water content of the soil and the soil suction, and it is directly associated with grain-size distribution and soil structure; and b) the hydraulic conductivity function, that represents the relationship between hydraulic conductivity and soil suction and can be expressed as a function of the degree of saturation or volumetric water content of soil.

There are field and laboratory procedures to determine these functions, however they are special tests which may require specilized personnel or be costly and complex. An alternative of experimental tests is the prediction or estimation of these functions using *estimation models* as a function of the index properties of soil (volume-mass and grain-size distribution relationships). In addition, when estimation models do not define a complete characteristic curve, it is possible to use *fitting models* in order to improve the representation of the characteristic curve in a complete range of suctions.

Some current *estimation models* that allow determining the soil-water characteristic curve (SWCC) include the following: Arya and Paris (1981), Scheinost et al. (1997), Fredlund and Wilson (1997). Regarding *fitting models* to represent the SWCC, some of them are: Brooks and Corey (1964), Gardner (1958), Van Genuchten (1980). With regard to the hydraulic conductivity function, although it can be obtained by means of laboratory tests, it is common to determine it from the characteristic curve by estimation models, some of them are: Kunze et al. (1968), Campbell (1973), Van Genuchten (1980), Brooks and Corey (1964), Campbell Modified (Fredlund, 1996), Mualem (1976), among others. Following one of the models that is used in this paper is described.

1.2 *Models of Fredlund and Xing (1994) for SWCC and Fredlund et al. (1994) for hydraulic conductivity function*

Fredlund and Xing (1994) developed a fitting model of the SWCC based on the shape of grain-size distribution of the studied material. This model considers three fitting parameters that are closely related to the shape of the SWCC. The model of Fredlund and Xing (1994) is applicable to a wide variety of materials, it also allows defining the SWCC for a complete range of suction with a maximum value of 1.0×10^6 kPa (Fredlund and Xing 1994):

$$\theta_w(\psi)=\theta_s\left[1-\frac{\ln\left(1+\frac{\psi}{h_r}\right)}{\ln\left(1+\frac{10^6}{h_r}\right)}\right]\left\{\frac{1}{\ln\left[e+\left(\frac{\psi}{a_f}\right)^{n_f}\right]^{m_f}}\right\} \quad (1)$$

where: $\theta_w(\psi)$ = volumetric water content of the soil for a given suction; θs = saturated volumetric water content; a_f= fitting parameter derived from the air-entry value of the soil;

n_f = fitting parameter related to the rate of desaturation of the soil once the air-entry value has been exceeded; m_f = fitting parameter related to the residual water content of soil; h_r = suction value when residual water content is reached; and ψ = soil suction.

According to research conducted by Kunze et al. (1968), Fredlund et al. (1994) developed an estimation model that, based on SWCC, allows defining the hydraulic conductivity function:

$$k_r(\psi) = \frac{\int_{\ln \psi}^{b} \frac{\theta(e^y) - \theta(\psi)}{e^y} \theta'(e^y)\, dy}{\int_{\ln \psi_{aev}}^{b} \frac{\theta(e^y) - \theta_s}{e^y} \theta'(e^y)\, dy} \tag{2}$$

where: b = ln (1 000 000); y = dummy variable of integration that represents the logarithm of the suction; and ψ_{aev} = soil suction at the air-entry value.

1.3 *General water flow equation*

Based on the principle of flow continuity and the Darcy's law generalized to problems under unsaturated conditions (Fredlund and Rahardjo 1993), the general equation governing the flow of water in saturated and unsaturated soils can be deduced:

$$\frac{\partial}{\partial x}\left[k_x(h_m)\frac{\partial h_m}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y(h_m)\frac{\partial h_m}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z(h_m)\left(\frac{\partial h_m}{\partial z} + 1\right)\right]$$

$$= C(h_m)\frac{\partial h_m}{\partial t} \tag{3}$$

where: h_m = matric suction head; h = hydraulic head; $k(h_m)$ = hydraulic conductivity function; $C(h_m)$ = specific storage capacity as a function of the suction head; and t = time.

2 INFLUENCE OF FITTING PARAMETERS ON DEFINITION OF THE SWCC

2.1 *Considerations for the analyses*

Experimental information of a typical material is assumed and the magnitude of its fitting parameters is varied. The experimental data are obtained from the SoilVision Database (SoilVision LTD, 2009). The fitting model of Fredlund and Xing (1994) is used to define a complete range of suctions (0.01 kPa – 1 000 000 kPa). Figure 1 shows the set of experimental data, as well as the data adjusted by the model of Fredlund and Xing (1994), whose fitting parameters are: a_f=40.917 kPa, n_f=0.919, m_f=1.140 and h_r=868.078 kPa.

2.2 *Evaluation of the influence of parameters a_f, n_f y m_f on the shape of the SWCC*

Theoretical and experimental researches have demonstrated that fine soils are gradually desaturated (Pérez-García 2008), so that their SWCC are usually extended and with slight slopes. In the case of coarse or granular materials, desaturation occurs rapidly (Pérez-García 2008), resulting in narrow SWCC with steep slopes.

Figure 2 illustrates the fitting SWCC of the experimental data (experimental fitting) previously obtained by the Fredlund and Xing model (1994) (Fig. 1). Additionally four SWCC obtained from the variation of parameters a_f and h_r are shown. From Figure 2 can be deduced that: a) an increase in the magnitude of the fitting parameters a_f and h_r causes curves which present typical characteristics of fine materials, and b) in contrast, a decrease in the magnitude of the parameters a_f and h_r provokes curves which exhibit characteristics of coarse materials. In addition, Figure 3 shows the influence of fitting parameter n_f on the definition of SWCC. In this case, it can be appreciated that: a) an increase in the magnitude of n_f generates steep slopes representative of a sudden desaturation, a typical characteristic of coarse or granular materials, and b) a reduction of n_f causes light slopes that represent slow desaturations, a typical feature of fine materials.

The above results demonstrate that the fitting parameters directly affect the representation of the material type. An incorrect assumption of these parameters may overestimate or underestimate the results of groundwater flow analyses.

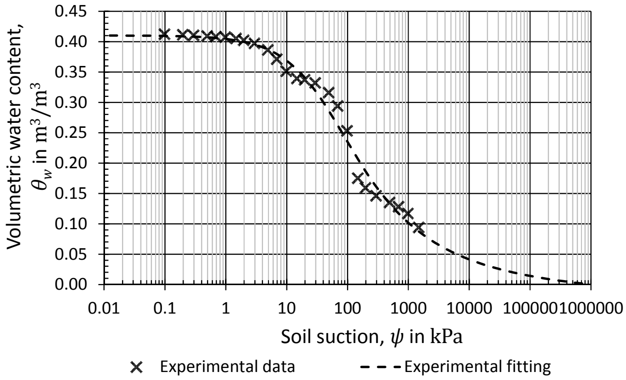


Figure 1. Experimental data and SWCC adjusted by the model of Fredlund and Xing (1994), assumed in calculations.

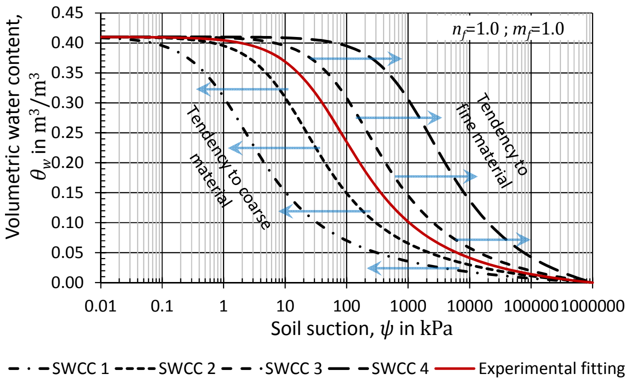


Figure 2. Influence of fitting parameters a_f and h_r on definition of the characteristic curves. SWCC 1: a_f =1 kPa, h_r =10 kPa; SWCC 2: a_f=10 kPa, h_r=100 kPa; SWCC 3: a_f=100 kPa, h_r=1000 kPa; SWCC 4: a_f=1000 kPa, h_r=10000 kPa.

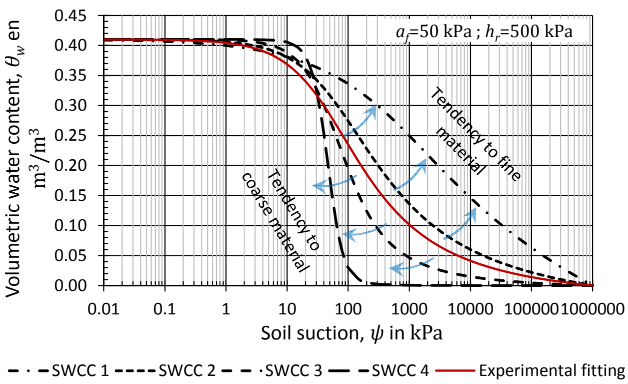


Figure 3. Influence of fitting parameter n_f on definition of the characteristic curves. SWCC 1: n_f =0.5, m_f =0.5; SWCC 2: n_f =0.9, m_f =0.9; SWCC 3: n_f=1.4, m_f=1.4; SWCC 4: n_f=3.0, m_f=3.0.

3 INFLUENCE OF SATURATED AND UNSATURATED SOIL PROPERTIES ON WATER FLOW ANALYSIS

3.1 *General considerations for the analyses*

The assessment of the influence of saturated and unsaturated soil properties on water flow analysis is carried out considering the following criteria:

a) Experimental data relating the volumetric water content of the soil to suction (SWCC) are assumed. The fitting SWCC of the experimental data (experimental fitting) considered in calculations is illustrated in Figure 1.
b) A comparison between different estimation models with respect to the experimental fitting is performed. Figure 4 and Table 1 show the index properties (volume-mass relationships and grain-size distribution) required for the estimation of unsaturated soil property functions.
c) The unsaturated properties of the material are neglected. In order to perform these calculations, a constant function that represents the saturated condition (constant hydraulic conductivity and constant water content) is defined.

Table 1. Properties of the material assumed in calculations.

Variable	Value
Specific gravity *Gs* (dimensionless)	2.65
Void ratio *e* (dimensionless)	0.70
Volumetric water content θ_w (m³/m³)	0.41
Saturated permeability $k_{w\,sat}$ (m/s)	1.0×10^{-6}

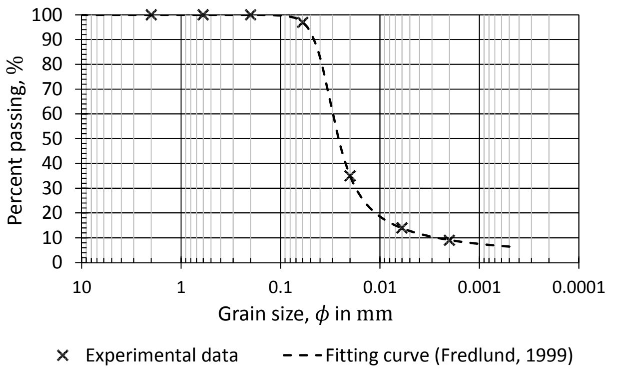


Figure 4. Grain-size distribution of the material assumed in calculations.

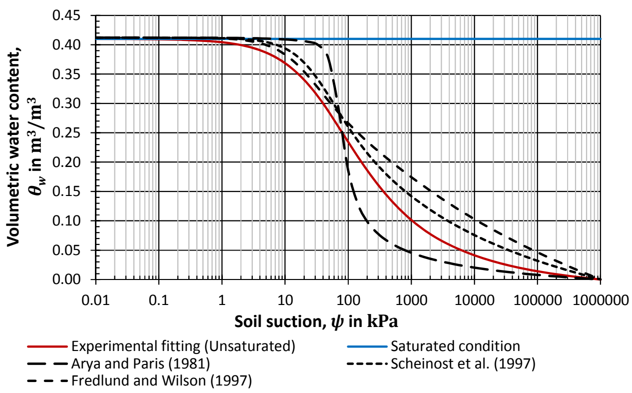


Figure 5. SWCC assumed for the analyses.

In all performed analyses the assumed SWCC were estimated by the following estimation models: Arya and Paris (1981), Scheinost et al. (1997) and Fredlund and Wilson (1997), however due to the dispersion of the curves and to get a complete range of suctions, all previous SWCC were adjusted by the fitting model of Fredlund and Xing (1994) (Fig. 5). The definition of the corresponding hydraulic conductivity functions was performed using the model of Fredlund et al. (1994). These functions were estimated from the fitting of the SWCC obtained by estimation models and experimental data mentioned above (Fig. 6).

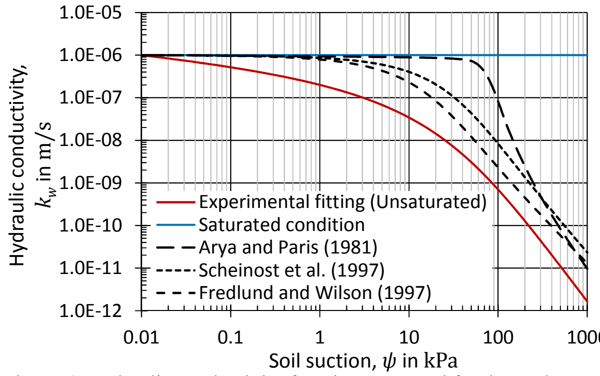


Figure 6. Hydraulic conductivity functions assumed for the analyses.

3.2 *Geometry for the analyses*

The cross-section of the analyzed embankment is 10.0 m height, 10.0 m width of the crown, 50.0 m width of the base and 2:1 slope ratio (Fig. 7). In the calculations three hydraulic head conditions (water levels in the reservoir) are proposed: (a) H1=9.0 m, (b) H2=5.0 m, and (c) H3=1.0 m). H1 represents the condition in which saturated flow governs, while H3 governs the unsaturated flow, and H2 represents an intermediate condition (Fig. 7). The numerical modeling is carried out by the finite element method (MEF) via the Seep/W code (GeoSlope LTD. 2014). Figure 8 illustrates the finite element mesh (1228 elements and 685 nodes) and boundary conditions assumed in calculations.

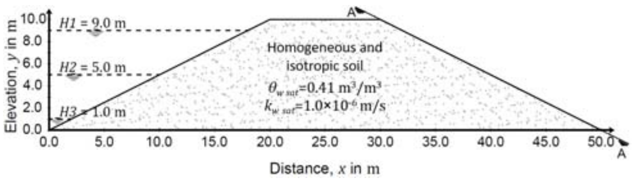


Figure 7. Cross-section of the homogeneous embankment evaluated.

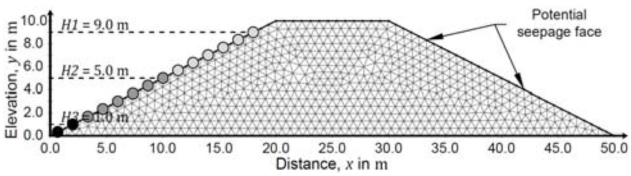
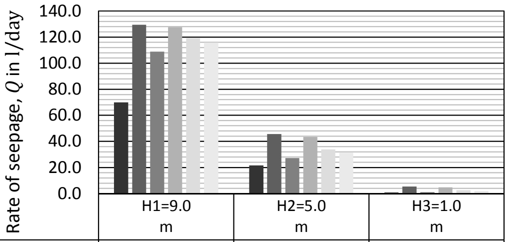


Figure 8. Finite element mesh and boundary conditions assumed in calculations for the homogeneous embankment.

3.3 *Assessment of discharge or rate of seepage*

Figure 9 summarizes the rates of seepage calculated through the homogeneous cross-section of the evaluated embankment. The results refer to the three analysis criteria considered in calculations (experimental fitting, estimates of different unsaturated soil property functions, and saturated conditions). In addition, the Casagrande method (1932) is used to calculate the flow only in the saturated zone of the soil. The evaluation of the obtained rates of seepage shows that their estimation mainly depends on the flow condition that governs the medium. That is, when the unsaturated zone predominates, the definition of constant hydraulic functions (representing a saturated condition) causes an overestimation of the rate of seepage. Conversely, if the system is governed by a saturated condition, the ommision of the unsaturated soil property functions does not have a significant influence on the calculated discharge. It is therefore convenient to define *a priori* the flow condition governing the studied medium. Additionally, classical theories of water flow (e.g. Casagrande method) provide different results, since they neglect the flow in the unsaturated zone.



	H1=9.0 m	H2=5.0 m	H3=1.0 m
Casagrande (1932)	69.98	21.60	0.86
Saturated condition	129.48	45.61	5.35
Experimental fitting (Unsaturated)	108.86	27.35	1.12
Arya and Paris (1981)	127.49	43.50	4.80
Scheinost et al. (1997)	119.05	33.81	2.59
Fredlund and Wilson (1997)	115.76	31.30	1.99

Figure 9. Variation of rates of seepage through the homogeneous embankment for the three water levels evaluated.

3.4 *Assessment of hydraulic gradients*

Figure 10 exhibits the magnitude of the resulting exit hydraulic gradients in section A-A of Figure 7, for the three hydraulic head conditions assumed in calculations (H1, H2 and H3). It can be seen that for condition H1 in which the saturated flow governs, there are important differences between the magnitude of the obtained gradients with the unsaturated properties and the obtained gradients with constant functions. In general terms, the gradients obtained from constant functions show zones of overestimation and underestimation of their magnitude. The boundary defining the previous condition is directly associated with the position of the phreatic surface (PS) (in this case for H1, Fig. 11), such that the overestimation occurs above the PS, while below it the gradients tend to be smaller. On the other hand, in the condition where unsaturated flow governs (H3=1.0 m), the differences between considering or not the unsaturated properties are insignificant in the analysis.

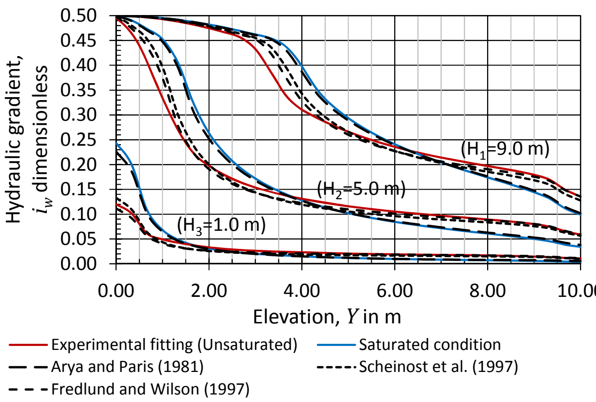


Figure 10. Exit hydraulic gradients (resulting magnitude) in section A-A from the Fig. 7, for the three water levels evaluated.

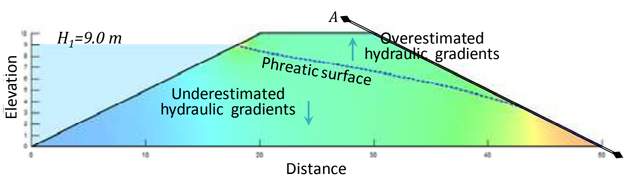


Figure 11. Exit hydraulic gradients (resulting magnitude) and zoning of overestimates and underestimates of their magnitude according to PS.

4 GENERAL CONCLUSIONS

From results of analyses performed here, some conclusions and recommendations can be drawn:

- The obtaining of representative results in the water flow analyses mainly depends on the correct definition of the unsaturated soil property functions.
- Considering or omitting the unsaturated soil property functions mainly affects the hydraulic gradients, discharges and flow velocities.
- In homogeneous media, the simplification or omission of the unsaturated soil property functions depends on the state of the material that governs the system (saturated or unsaturated condition).
- Materials with high water content or high water retention capacity can be assumed under saturated conditions.

5 REFERENCES

Arya L.M. and Paris J.F. 1981. A physicoempirical model to predict the soil moisture characteristic from particle-size distribution and bulk density data. *Soil Science Society of America Journal* (45), 1023-1030.

Brooks R.H. and Corey A.T. 1964. Hydraulic properties of porous media. Colorado State University Hydrology Paper, Fort Collins, Colorado, eds. A.T. Corey, R.E. Dils y V.M. Yevdjevich; (3), 37 p.

Campbell J.D. 1973. *Pore pressures and volume changes in unsaturated soils*. Ph.D. Thesis. University of Illinois at Urbana-Champaign, Illinois, USA.

Casagrande L. 1932. *Naeherungsmethoden zur Bestimmung von Art und Menge der Sickerung durch geschuettete Daemme*. Thesis, Technische Hochschule, Vienna.

Fredlund M.D. 1996. *Design of a Knowledge-Based System for unsaturated soil properties*. M.Sc. Thesis. University of Saskatchewan, Canada.

Fredlund M.D. 1999. *The role of unsaturated soil property functions in the practice of unsaturated soil mechanics*. PhD Thesis. University of Saskatchewan, Saskatoon, Saskatchewan, Canada: 293 p.

Fredlund D.G. and Rahardjo H. 1993. *Soil mechanics for unsaturated soils*. New York: John Wiley and Sons.

Fredlund D.G. and Xing A. 1994. Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(3), 521-532.

Fredlund D.G., Xing A. and Huang S. 1994. Predicting the permeability function for unsaturated soil using the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(3), 533-546.

Fredlund M.D., Wilson G.W. and Fredlund D.G. 1997. Prediction of the soil-water characteristic curve from grain-size distribution and volume-mass properties. Proceedings of the 3rd Symposium on Unsaturated Soil, NSAT '97, Rio de Janeiro, Brazil, (1), 13-23.

Gardner W. R. 1958. Some steady state solutions of the unsaturated moisture flow equation with application to evaporation from a water-table. *Soil Science Journal*, 85(4), 228-232.

Geo-Slope International LTD. 2014. *Seep/W. GeoStudio 2012*. Geo-Slope International, Calgary, Alberta, Canada: Version 8.14.2.11317.

Kunze R.L., Uehara G. and Graham K. 1968. Factors important in the calculation of hydraulic conductivity. *Soil Science Society of America Journal*, (32), 760-765.

Mualem Y. 1976. A new model for predicting hydraulic conductivity of unsaturated porous media. *Water Resources Research*, (12), 513-522.

Pérez-García N. 2008. *Determinación de curvas características en suelos no saturados con celdas de presión*. Publicación No. 313, IMT, SCT, Safandila, Queretaro: 66 p.

Scheinost A.C., Sinowski W. and Auerswald K. 1997. Regionalization of soil water retention curves in a highly variable soilscape, I. Developing a new pedotransfer function. *Geoderma*, (78), 129-143.

SoilVision Systems LTD. 2009. *Soilvision: A Knowledge-Based Database System For Saturated/Unsaturated Soil Properties*. SoilVision Systems LTD, Saskatoon, Saskatchewan, Canada: V 4.

Van Genuchten M.T. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, (44), 892-898.