Analysis investigating the applicability of Timoshenko, Euler-Bernoulli, and rigid beam theories in modeling laterally loaded monopiles

Analyse enquête sur l'applicabilité de Timoshenko, Euler-Bernoulli et théories de la poutre rigide pour modéliser les monopiles chargés latéralement

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ABSTRACT: A new method for analyzing large diameter hollow circular monopiles embedded in multilayered elastic soil deposits and subjected to lateral forces is developed based on the variational principle of mechanics. In the analysis, the soil is modeled as a continuum and the monopile as a one-dimensional Timoshenko beam. The analysis is progressively simplified to model the monopile as Euler-Bernoulli and rigid beams. Monopile responses are obtained using all the different beam theories and compared with the responses obtained from three-dimensional finite element analysis. Further, a systematic parametric study for a wide range of monopile-soil stiffness ratio and monopile slenderness ratio is performed to establish which beam theory is the most appropriate to model laterally loaded monopiles, and the range of applicability of the different beam theories is discussed.

RÉSUMÉ: Une nouvelle méthode d'analyse de grand diamètre creux circulaire chargés latéralement monopiles incorporé dans un dépôt de plusieurs couches de sol élastique est développée, basée sur le principe variationnel de la mécanique. Dans l'analyse du sol est modélisé comme un continuum et le monopile comme un faisceau de Timoshenko unidimensionnel. L'analyse est progressivement simplifiée afin de modéliser la monopile d'Euler-Bernoulli et poutre rigide. Monopile réponses sont obtenues à l'aide de toutes les théories de faisceau différent et comparées aux trois analyses par éléments finis dimensionnelles pour vérification et validation. En outre, une étude paramétrique systématique pour un large éventail de rigidité du monopile-sol et monopile élancement est effectuée pour établir quelle théorie des poutres est la plus appropriée modéliser les monopiles chargés latéralement et la gamme de l'applicabilité de chacune des théories faisceau est discutée.

KEYWORDS: Monopile, Timoshenko beam, Euler-Bernoulli beam, rigid beam, lateral load, elastic analysis, continuum

1 INTRODUCTION

Monopiles are hollow steel piles supporting offshore wind turbines and are subjected to lateral loads from wind, waves, and water currents. These piles typically have a diameter of 4-6 m with a low slenderness ratio (length/diameter) of 5-6 (Klinkvort and Hededal 2013). Currently, design of laterally loaded monopiles is based on the *p-y* method in which the soil is modeled as a series of uncoupled non-linear springs, and the pile as an Euler-Bernoulli (EB) beam (API 2011). Modeling the large-diameter monopiles using the EB beam theory is however questionable because the EB theory does not account for the shear deformations that might be significant for large pile cross sections. The Timoshenko beam theory may be better suited to model these piles. On the other hand, three dimensional (3D) finite element (FE) studies on monopiles (Abdel-Rahman and Achmus 2005, Haiderali et al. 2013) have shown that monopiles may undergo rigid-body rotation when subjected to lateral loads. Thus, it is necessary to investigate whether laterally loaded monopiles should be modeled using Timoshenko, EB or rigid beam theories.

The purpose of this paper is to present a framework for analysis of laterally loaded monopiles using the variational principles of mechanics in which the monopile is modelled as a Timoshenko beam and the soil as a multilayered elastic continuum. Successive simplifications made in the analysis framework leads to the pile being modeled as EB and rigid beam theories — thus, all the different beam theories that might be applicable to monopiles are incorporated in the framework. A parametric study is performed using the framework to investigate the range of applicability of each beam theory on monopiles with varying slenderness ratio and monopile-soil stiffness ratio.

2 THEORETICAL DEVELOPMENT

A monopile with hollow circular cross section of radius r_p and wall thickness t_p = (6.35 + $2r_p$/100) mm (API 2011), and modeled as a Timoshenko beam is considered to be embedded in a multilayered elastic soil deposit with $n + 1$ layers, as shown in Figure 1, where H_i is the vertical distance from the ground surface to the bottom of any soil layer *i*. Any soil layer *i* is characterized by the Lame's constants λ_{si} and G_{si}. The goal of the analysis is to obtain monopile head displacement and rotation because of the applied force F_a and/or moment M_a acting at the monopile head (Figure 1).

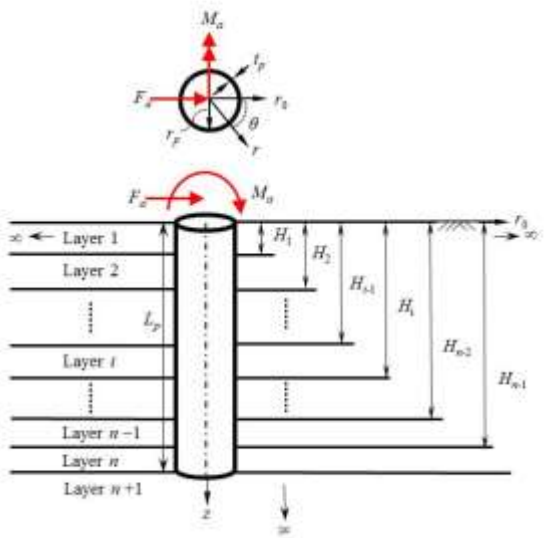


Figure 1. Laterally loaded monopile embedded in a multilayered soil

2.1 *Potential energy and displacement field*

The total potential energy of the monopile-soil system, considering the Timoshenko beam theory, is given by

$$\Pi = \int_0^{L_p} \frac{1}{2} E_p I_p \left(\frac{d\psi}{dz}\right)^2 dz + \int_0^{L_p} \frac{1}{2} \kappa G_p A_p \left(\frac{dw}{dz} - \psi\right)^2 dz + \int_{\Omega_0} U_D \, r dr d\theta dz - F_a \, w\big|_{z=0} + M_a \psi\big|_{z=0} \quad (1)$$

where L_p = monopile length, E_p = monopile Young's modulus, I_p = second moment of inertia of monopile cross section, ψ = bending slope of monopile axis, z = depth, κ = correction factor to account for the non-uniform shear strain distribution of a hollow circular cross-section (Cowper 1966), G_p = monopile shear modulus, A_p = monopile cross sectional area, U_D = strain energy density of soil in a right-handed cylindrical (r-θ-z) coordinate system with its origin at the center of monopile head and z axis pointing downward and coinciding with the monopile axis, Ω_0 = soil domain that participates in the monopile-soil interaction, and w = horizontal monopile displacement. Neglecting vertical displacements in both monopile and soil, and expressing the horizontal soil displacements as (Basu et al. 2009)

$$u_r = w(z)\phi_r(r)\cos\theta \quad (2)$$

$$u_\theta = -w(z)\phi_\theta(r)\sin\theta \quad (3)$$

the soil strain energy density U_D (= $\sigma_{pq}\varepsilon_{pq}/2$ where σ_{pq} and ε_{pq} are soil stress and strain tensors, respectively) is expressed in terms of the functions w, ϕ_r and ϕ_θ, and the Lame's constants λ_{si} and G_{si} of the soil layers that relate the soil stress and strain tensors.

Equations (2) and (3) simplify the soil displacement field such that, at any radial distance r from the monopile axis, the variation of horizontal soil displacement with depth is a scaled down variation of monopile displacement $w(z)$, with the scaling factor being decided by the functions ϕ_r and ϕ_θ, and the sine and cosine functions of the tangential angular distance θ (measured from a vertical plane that contains the applied force F_a) ensure that the 3D soil displacements conform to the monopile displacement w. The functions ϕ_r and ϕ_θ are chosen dimensionless varying between 1 at the monopile-soil interface (which ensures no slip between soil and monopile) and 0 at infinite radial distance from the monopile shaft (which ensures a decrease in soil displacement with increase in radial distance r from the pile axis), but their actual variations with radial distance r are unknown (i.e., to be determined).

2.2 *Potential energy minimization and differential equations*

Minimizing the total potential energy (i.e., setting $\delta\Pi = 0$), the governing differential equations and boundary conditions of the unknown functions w, ψ, ϕ_r and ϕ_θ are obtained. The governing differential equations of $w(z)$ and $\psi(z)$ are coupled and, for the i^{th} soil layer, these equations are given by

$$E_p I_p \frac{d^2\psi_i}{dz^2} + \kappa G_p A_p \left(\frac{dw_i}{dz} - \psi_i\right) = 0 \quad (4)$$

$$\kappa G_p A_p \left(\frac{d\psi_i}{dz} - \frac{d^2 w_i}{dz^2}\right) + k_i w_i - 2t_i \frac{d^2 w_i}{dz^2} = 0 \quad (5)$$

where k_i and t_i are the soil parameters expressed in terms of the Lame's constants and soil displacement functions ϕ_r and ϕ_θ (see Basu et al. 2009, Gupta and Basu 2016). A mathematical transformation is made to express w and ψ in terms of an auxiliary function $F(z)$ to obtain an uncoupled form of the Eqs. (4) and (5) governing the monopile displacement, given by

$$\left(1 + \frac{2t_i}{\kappa G_p A_p}\right)\frac{d^4 F_i}{dz^4} - \left(\frac{k_i}{\kappa G_p A_p} + \frac{2t_i}{E_p I_p}\right)\frac{d^2 F_i}{dz^2} + \frac{k_i}{E_p I_p} F_i = 0 \quad (6)$$

The above equation corresponding to each soil layer is solved analytically using the associated boundary conditions at the layer interfaces that ensure the continuity of displacement, slope, bending moment and total shear force at the layer interfaces. At the pile head, the boundary conditions ensure that force and moment equilibriums are satisfied.

The differential equations of $\phi_r(r)$ and $\phi_\theta(r)$ are obtained as

$$\frac{d^2\phi_r}{dr^2} + \frac{1}{r}\frac{d\phi_r}{dr} - \left[\left(\frac{\gamma_1}{r}\right)^2 + \left(\frac{\gamma_2}{r_p}\right)^2\right]\phi_r = \frac{\gamma_3^2}{r}\frac{d\phi_\theta}{dr} - \left(\frac{\gamma_1}{r}\right)^2 \phi_\theta \quad (7)$$

$$\frac{d^2\phi_\theta}{dr^2} + \frac{1}{r}\frac{d\phi_\theta}{dr} - \left[\left(\frac{\gamma_4}{r}\right)^2 + \left(\frac{\gamma_5}{r_p}\right)^2\right]\phi_\theta = -\frac{\gamma_6^2}{r}\frac{d\phi_r}{dr} - \left(\frac{\gamma_4}{r}\right)^2 \phi_r \quad (8)$$

These equations are also coupled and is solved using a iterative finite difference scheme using the boundary conditions $\phi_r(r_p) = \phi_\theta(r_p) = 1$ and $\phi_r(\infty) = \phi_\theta(\infty) = 0$. The dimensionless constants γ_1-γ_6 are functions of the soil Lame's constants, the soil parameters k_i and t_i, and the monopile displacement w. Further, to obtain a complete solution of monopile response an iterative solution scheme is used as described in Basu et al. (2009) and Gupta and Basu (2016).

2.3 *Simplifications for Euler-Bernoulli and rigid beam theories*

The EB beam theory neglects shear deformation, which is implemented by assuming $G_p \rightarrow \infty$, and using the relationships $F(z) = E_p I_p w(z)$ and $\psi(z) = dw(z)/dz$. Thus, Eq. (6) simplifies to the differential equation governing monopile displacement w that follows the EB beam theory, given by

$$E_p I_p \frac{d^4 w_i}{dz^4} - 2t_i \frac{d^2 w_i}{dz^2} + k_i w_i = 0 \quad (9)$$

along with the corresponding boundary conditions.

For rigid piles, the displacement profile is assumed to be linear such that the monopile displacement is given by (Gupta and Basu 2016)

$$w(z) = w_h - \psi_h z \quad (10)$$

where w_h is the monopile head displacement and ψ_h is the clockwise rotation of the monopile axis. For rigid beam theory, Eq. (10) is used in place of Eq. (6).

2.4 *Equivalent soil shear modulus*

Randolph (1981) found that soil Poisson's ratio ν_s has a negligible contribution to lateral soil resistance of slender piles and that the effect of ν_s can be taken into account by defining an equivalent shear modulus G_s^* given by

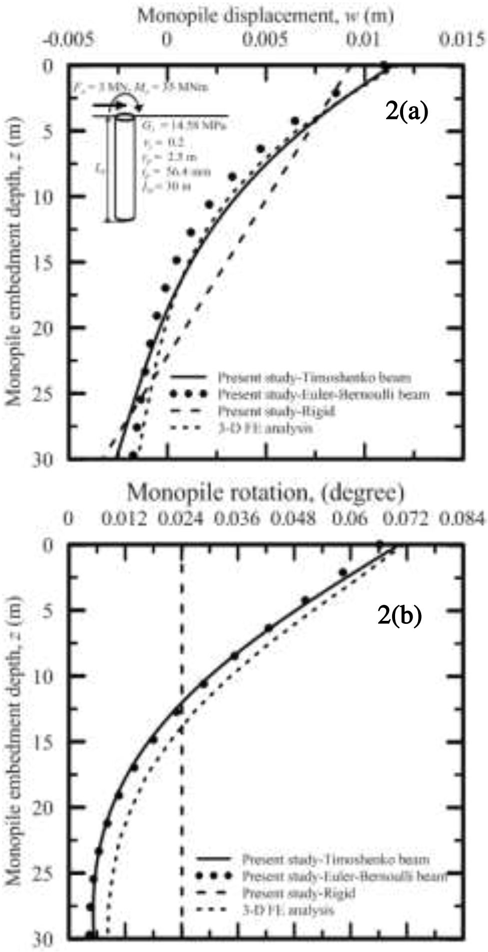
$$G_s^* = G_s\left(1+0.75\nu_s\right) \qquad (11)$$

The advantage of Eq. (11) is that the monopile response can be investigated in terms of a single soil parameter that integrates the effect of both the elastic constants. Guo and Lee (2001) found that, for assumed soil displacement fields such as that given in Eqs. (2)-(3), monopile response is excessively stiff for ν_s close to 0.5. In order to avoid this excessive stiffness, ν_s is set equal to zero (following Guo and Lee (2001)) irrespective of its actual value, and its effect is indirectly taken into account through Eq. (11).

3 RESULTS

3.1 *Verification of analysis*

Pile responses obtained from this analysis are compared with those of 3D FE analyses performed using ABAQUS for hollow steel monopiles embedded in homogeneous and three-layer soil profiles, respectively, as shown in Figs. 2(a)-(d). The details of the monopile and soil properties are given in the figures.



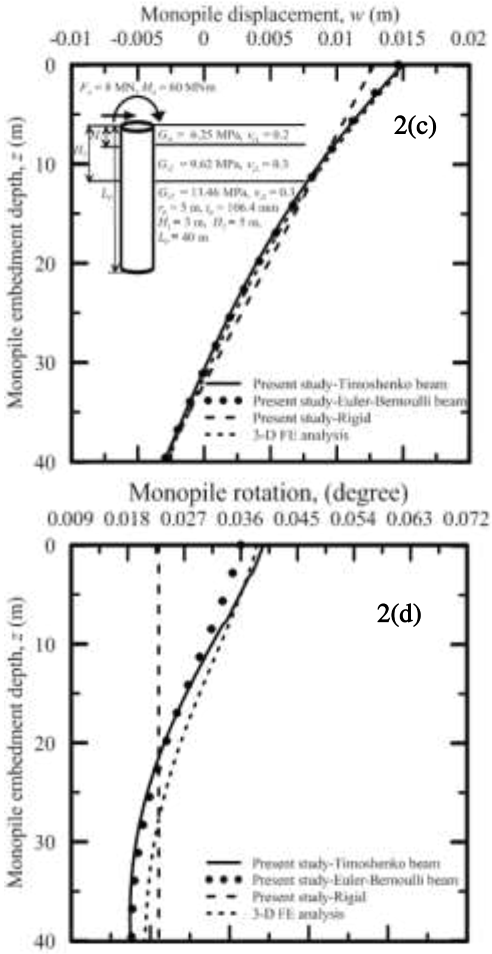


Figure 2. Comparison of monopile response obtained from present and FE analyses for monopiles subjected to a lateral force and moment at the head: (a) displacement, and (b) rotation profiles in homogeneous soil; and (c) displacement, and (d) rotation profiles in three-layer soil

A reasonable match with the results of FE analysis is obtained using the Timoshenko and EB theories for both the problems analyzed while the rigid theory underestimates the pile response. In addition to the problems shown in Fig. 2, a few additional problems were analyzed with different pile lengths and diameters in homogeneous, two-layer, and three-layer soil profiles. The differences in the results between the present and FE analysis are found to be in the range of 0.4-3.4% (for head displacement), and 1.3-2.5% (for head rotation) using Timoshenko beam theory, 1.9-5.1% (for head displacement) and 4.6-8.1% (for head rotation) using EB beam theory, and 4.1-7.8% (for head displacement) and 12.0-54.1% (for head rotation) using rigid beam theory.

3.2 *Effect of monopile-soil stiffnesss*

To investigate the range of applicability of Timoshenko and EB beam theories, normalized head displacement ($w_hG_s^*r_p/F_a$ and $w_hG_s^*r_p^2/M_a$) and rotation ($\psi_hG_s^*r_p^2/F_a$ and $\psi_hG_s^*r_p^3/M_a$) versus monopile-soil stiffness ratio E_p/G_s^* are plotted for hollow steel monopiles for different values of pile slenderness ratio L_p/r_p. Figures 3(a) and (b) show the normalized head displacement and rotation for the applied force F_a. It is observed that Timoshenko and EB beam theories produce different responses for $E_p/G_s^* < 10^4$.

3.3 *Effect of monopile slenderness ratio*

Normalized head displacement ($w_hG_s^*r_p/F_a$ and $w_hG_s^*r_p^2/M_a$)

and rotation ($\psi_h G_s^* r_p^2 / F_a$ and $\psi_h G_s^* r_p^3 / M_a$) versus pile slenderness ratio (L_p/r_p) are plotted for applied force and moment for different values of monopile-soil stiffness E_p/G_s^*. Figures 4(a) and (b) show the plots of normalized head displacement and rotation for the applied force F_a. It is evident that the rigid beam theory can be used with reasonable accuracy for cases when $E_p/G_s^* \geq 10^5$.

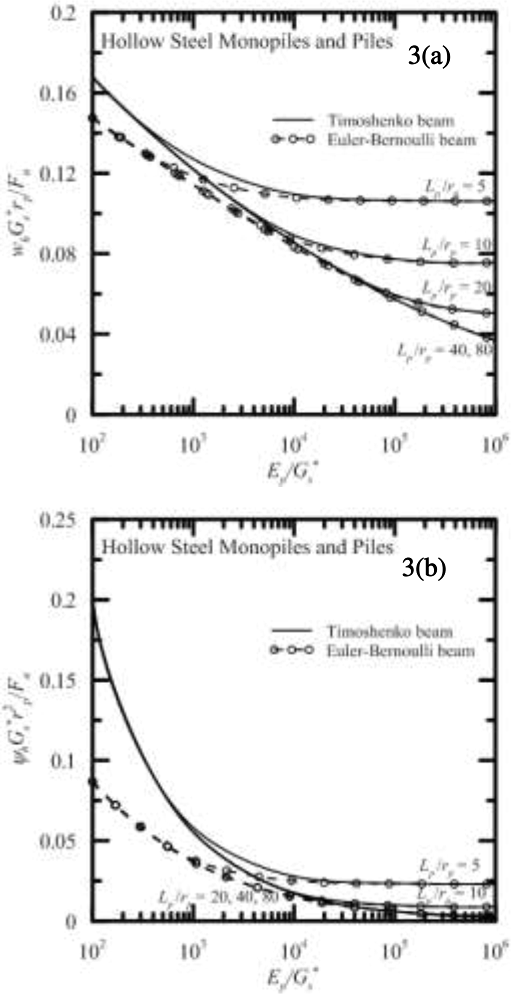
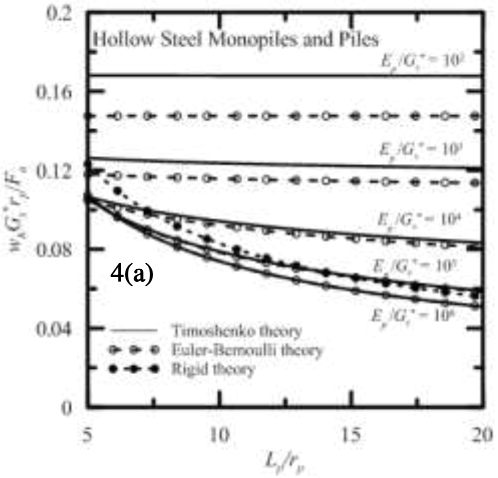


Figure 3. Dimensionless (a) head displacement, and (b) head rotation versus monopile-soil stiffness ratio for applied force on hollow steel monopiles using Timoshenko and Euler-Bernoulli beam theories



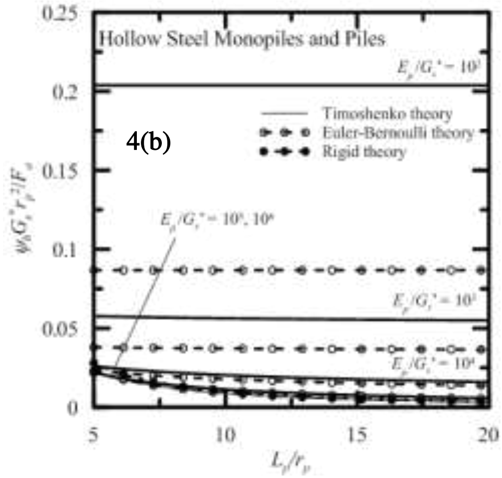


Figure 4. Dimensionless (a) head displacement and (b) head rotation for applied force versus slenderness ratio on hollow steel monopiles using Timoshenko, Euler-Bernoulli, and rigid beam theories

4 CONCLUSIONS

An analysis framework for laterally loaded monopiles based on the variational principles of mechanics is presented. The soil surrounding the monopile is modelled as an elastic continuum and the monopile is modelled as a Timoshenko beam. The governing differential equations obtained are solved following an iterative algorithm.

It is shown that successive simplifications of the differential equations representing the Timoshenko beam theory resulted in equations representing the Euler-Bernoulli and rigid beam theories, respectively. Several example problems are solved using the analysis, and comparisons with 3D FE analysis show that the difference with the FE results in predicting monopile head displacement and rotation is the least for the Timoshenko beam theory and the greatest for the rigid beam theory. A parametric study based on normalized monopile head displacement and rotation showed that, for circular hollow monopiles, the Timoshenko and EB beam theories produced different results when $E_p/G_s^* < 10^4$. Further, the rigid theory can be used with reasonable accuracy when $E_p/G_s^* > 10^5$.

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