Validity of the method of fragments for seepage analysis in double-wall cofferdams

Validité de la méthode du fragment pour l'analyse de l'infiltration dans batardeaux double paroi

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ABSTRACT: The method of fragments is a simple analytical technique for solving confined seepage problems. Design charts for determining the flow rate and exit gradient, have been proposed in literature for a double-wall cofferdam. The accuracy of the above method depends on the assumption that equipotential line at the tip of the cut-off wall is vertical, dividing the flow domain into two fragments. This paper discusses the validity of this assumption, and its effect on the accuracy of the solutions. Extensive numerical simulations were conducted by varying the cofferdam width ($2L$), embedded depth of the cut-off wall (s) and excavation depth (αs) with a constant height of the soil domain (T). Numerical simulations show that the equipotential lines at the tip of the cut-off wall are far from vertical for low values of L/T and s/T ratios and become closer to the vertical with increasing values of L/T and s/T. Also the values of flow rate and exit gradient estimated from the method of fragments become very close to the values computed from the full numerical solutions when increasing the values of L/T and s/T. Nevertheless, for low values of L/T and s/T, estimates from the method of fragments are on the conservative side and are still within acceptable limits.

RÉSUMÉ: La méthode de fragmentation est une technique analytique simple pour résoudre les problèmes d'infiltration confinés. Plusieurs méthodes basées sur des graphiques ont été proposées dans la littérature afin de déterminer le débit et le gradient de sortie pour les batardeaux double paroi. La précision de ces méthodes dépend de l'hypothèse que la ligne équipotentielle à l'ext rémité de la paroi isolée est verticale, divisant le domaine de flux en deux fragments. Cet article discute de la validité de cette hypothèse, et son effet sur la précision des solutions. Des simulations numériques approfondies ont été menées en faisant varier la largeur du batardeau ($2L$), la profondeur enterrée de la paroi isolée (s) et de la profondeur d'excavation (αS) avec une haute ur constante de sol (T). Les simulations numériques montrent que les lignes équipotentielles à l'extrémité de la paroi isolée son t loin d'être verticales pour de faibles valeurs des rapports L/T et s/T et tendent à se rapprocher de la verticale lorsque les vale urs de L / T et s/T augmentent. De plus, en augmentant les valeurs de L/T et s/T, les valeurs de débit et de gradient de sortie estimés par la méthode de fragmentation deviennent très proches des valeurs obtenues à partir d’analyses numériques . Toutefoi s, pour les faibles valeurs de L/T et s/T, les estimations faites a partir de la méthode de fragmentation restent à la fois conserv atrices et dans des limites acceptables.

KEYWORDS: Method of fragment, Double-wall cofferdam, Flow rate, Exit gradient

1 INTRODUCTION

A cofferdam is a commonly used seepage control structure used with excavations. However, water seepage into the excavation can induce hydraulic failure of the excavation base and change the passive earth pressure distribution; hence it influences the stability of the cofferdam (Bouchelghoum and Benmebarek 2011). Piping is a common type of hydraulic failure mode, and factor of safety (F_s) against piping failure can be defined as (Harza 1935):

$$F_s = \frac{\text{Critical hydraulic gradient}}{\text{Maximum exit gradient}} \tag{1}$$

Consequently, flow rate and maximum exit hydraulic gradient are two of the key parameters required for cofferdam designs.

Double-wall cofferdam is long and narrow (length is larger with respect to its width); therefore a two dimensional seepage analysis is adequate (King and Cockroft 1972, Banerjee 1993). Even though various two dimensional solution methods are available such as flow nets, analytical solutions or numerical simulations, the method of fragments still has a place due to its simplicity and ability to provide a quick estimate of flow rate and exit gradient (Griffiths 1984, Sivakugan and Alaghbari 1993, Madanayaka and Sivakugan 2016).

Method of fragments is a simple analytical solution method developed by Pavlovsky (1956), and Harr (1962) brought it to the attention of the western world. This is an approximate method where the flow domain is divided into few fragments by assuming vertical equipotential lines at the critical points (e.g., equipotential lines at the cut-off wall tip in double-wall cofferdams). Sivakugan and Alaghbari (1993) found excellent agreement between method of fragments and flow net solutions for seepage beneath concrete dams and sheet piles. The flow rate through any given fragment can be given as:

$$q = \frac{kh_i}{\Phi_i} \tag{2}$$

where, k is the soil permeability, h_i and q are the head loss and flow rate through the fragment i. Φ_i is the dimensionless form factor of the fragment i, which depends only on the geometry of the fragment. In any flow domain, each fragment should have the same flow rate q and hence it can be written as:

$$q = \frac{kh_1}{\Phi_1} = \frac{kh_2}{\Phi_2} = \dots = \frac{kh_i}{\Phi_i} = \frac{kh}{\sum \Phi} \tag{3}$$

Here, h is the total head loss over the entire flow domain. Accordingly, the flow rate and head loss across any fragment can be calculated if all form factors are known.

Harr (1962) defined form factor values for six types of confined flow fragments. Griffiths (1984) condensed them into two types, namely, fragment A and fragment B and introduced a new fragment, named as fragment type C. Further, he developed design charts for form factors of all three fragments and one exit gradient chart for fragment C which represents the exit gradient. For all the charts, he incorporated the soil anisotropy R, defined as $R=\sqrt{k_V/k_H}$ where k_V and k_H are the vertical and horizontal permeabilities, respectively.

2. METHOD OF FRAGMENTS FOR DOUBLE-WALL COFFERDAMS

Figure 1 shows the cross sectional view of a double-wall cofferdam. Here, two sets of parallel cut-off walls (sheet piles) $2L$ distance apart are driven into the ground to a depth of s, and the thickness of the soil layer is T. The excavation depth is given by the αs ($0 < \alpha < 1$), and head loss over the domain is h. Due to symmetry, analysis of half the section along the centerline is adequate to estimate the flow rate q per unit length. Also, the centerline acts as an impermeable boundary, and total flow rate into the cofferdam is $2q$. For a half section of cofferdam, Griffiths (1984) proposed a method of fragments solution by dividing the domain into fragment A and fragment C at the tip of the cut-off wall (see Figure 1 for right half section). Hence, flow rate q can be estimated as:

$$q=\frac{kh}{(\Phi_A+\Phi_C)} \quad (4)$$

where Φ_A and Φ_C are the form factors of fragments A and C, respectively.

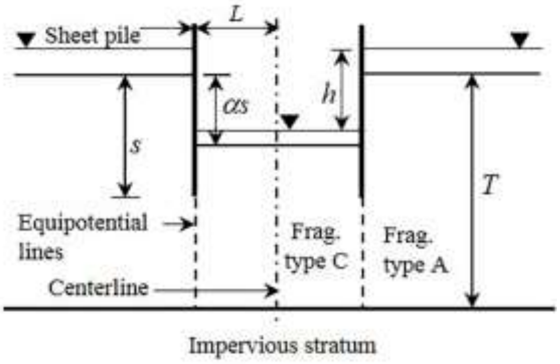

Figure 1. Cross sectional view of a double-wall cofferdam

Thus, flow rate can be estimated using fragment A (when the apron width is equal to zero) and fragment C form factor charts. Also, fragment C exit gradient chart provides exit gradient at the most critical point (i.e., the point on the excavation base adjacent to the cut-off wall). Madanayaka and Sivakugan (2016) also developed form factor and exit gradient charts for fragment C within the range of $0.1 \le s/T \le 0.9$ and $0.1 \le LR/T \le 1.0$. Here also R represents the soil anisotropy similar to the definition by Griffiths (1984) mentioned in the above section. Note that their form factor chart covers some additional geometries which are not considered by Griffiths (1984) chart. Also, they proposed a simple chart to estimate fragment A form factors (only for the situation where apron width is equal to zero). Further, they simplified the method of fragments solution for the double-wall cofferdam by defining simple expressions for form factors and exit gradient, which enable one to implement method of fragments in spreadsheets. The direct applicability of these expressions have been limited to the geometries bounded by $0.1 \le s/T \le 0.9$ and $0.1 \le LR/T \le 1.0$. Their expression for fragment C form factor is given as:

$$\Phi = \ln\left(1+\frac{s}{T}\right)+0.85\left(\frac{s/T}{LR/T}\right)+0.4e^{-0.5(LR/T)} \quad (5)$$

where s and T are the embedded depth of cut-off wall and thickness of soil layer, respectively, within the fragment C region. The expression for fragment A form factor is (Madanayaka and Sivakugan 2016):

$$\Phi = 0.43e^{5/3(s/T)} \quad (6)$$

Here s and T are the cut-off wall depth and thickness of soil layer within the fragment A. The exit gradient expression for the fragment C is given by (Madanayaka and Sivakugan 2016):

$$i_E\left(\frac{s}{h}\right)=1.5\left[\frac{1-e^{(-s/T)}}{(s/T)^{(0.3LR/T+0.65)}}\right](1-0.5\,LR/T) \quad (7)$$

where h is the head loss within the fragment C. However the accuracy of these methods [Griffiths (1984) and Madanayaka and Sivakugan (2016)] depend on the assumption that the equipotential line at the tip of the cut-off wall is vertical. Hence, the focus of this study is to analyze the validity of this assumption, and the effect of any deviation on the estimated flow rate and exit gradient.

3. METHODOLOGY

A series of numerical runs were carried out using the finite element software RS2 (Rocscience 2011) for half section of double-wall cofferdam for different combinations of s/T, LR/T and α while keeping the soil as isotropic (R = 1). Here, flow rate q and exit gradient i_E were computed for the geometries with s/T ratio changing from 0.1 to 0.9 while LR/T ratio varied from 0.1 to 1 for a constant permeable layer thickness T (20 m) and head loss h (10 m). The values of α were selected as 0, 0.4, and 0.8. Also, equipotential line at the tip of cut-off wall was drawn for each geometry and its behavior was studied. In addition, flow rate and exit gradient were computed by full numerical modelling of the half section of cofferdam, and the results were compared with the solutions given by Griffiths (1984) method and Eqs. 5, 6, and 7 proposed by Madanayaka and Sivakugan (2016). The design charts provided by Madanayaka and Sivakugan (2016) and Griffiths (1984) are not compared because they are similar with the former covering a wider range of values.

4. VALIDITY OF THE ASSUMPTION

To evaluate the validity of the assumption, the shape of the equipotential lines starting at the tip of the cut-off wall were studied for a series of geometries.

4.1 *Effect of the cofferdam width and depth of the cut-off wall*

Numerical simulations show that the equipotential lines at the tip of the cut-off walls are far from being vertical for low values of LR/T and s/T and become closer to vertical with increasing the values of LR/T and s/T. This behavior of the equipotential lines for LR/T = 0.1, 0.4 and 0.8 are shown in Figure 2, where s/T increases from left to right and LR/T increases downwards.

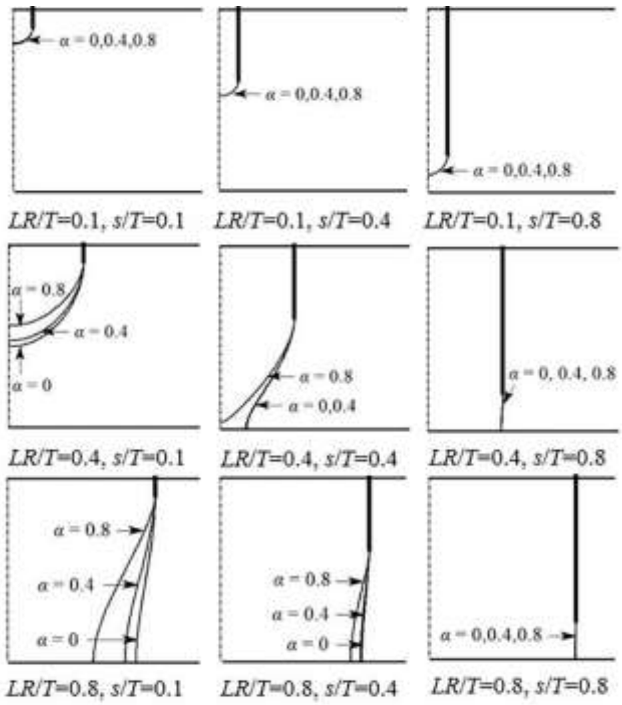

Figure 2. Behavior of the equipotential lines with cofferdam geometry.

4.2 *Effect of the excavation depth*

When increasing the excavation depth (increasing α) for any geometry, the deviation of the equipotential line at the tip of the cut-off wall from vertical is more pronounced, thus violating the assumption. However, there is no major difference between equipotential lines when α increases from 0 to 0.4. Hence, the effect of the excavation depth is not significant on the behavior of equipotential lines for moderate excavation depths (α = 0 to 0.4) and, the validity of the assumption is mostly controlled by the cofferdam width and depth of the cut-off wall.

When α changes from 0.4 to 0.8, the difference between the relevant equipotential lines is considerable. However, with increasing the depth of cut-off wall, this difference decreases. Thus, when the excavation depth is relatively large ($\alpha > 0.4$), the equipotential line being vertical is jointly controlled by the cofferdam width, depth of the cut-off wall and excavation depth; however, effect of the excavation depth decreases with increasing the depth of cut-off wall.

5. EFFECTS OF NON-VERTICAL EQUIPOTENTIAL LINE ON THE SEEPAGE QUANTITY AND EXIT GRADIENT

For this assessment, seepage quantity and exit gradient estimated by two method of fragments methods, [method 1- Griffiths (1984) and method 2 - Madanayaka and Sivakugan (2016)] , are compared with solutions given by the full numerical modeling of cofferdams.

5.1 *Effect of non-vertical equipotential line due to cofferdam width and depth of the cut-off wall*

Extensive number of cofferdam geometries were studied for the isotropic (R=1) and no excavation (α=0) conditions. Here the range of s/T studied was from 0.1 to 0.9 while LR/T changed from 0.1 to 1. Figures 3 and 4 show the comparisons of seepage quantity and exit gradient respectively, for three selected cofferdam widths (LR/T=0.1, 0.4 & 0.8). In both comparisons, seepage quantity and exit gradient values predicted by the method of fragments are in good agreement with the actual values given by full numerical modeling.

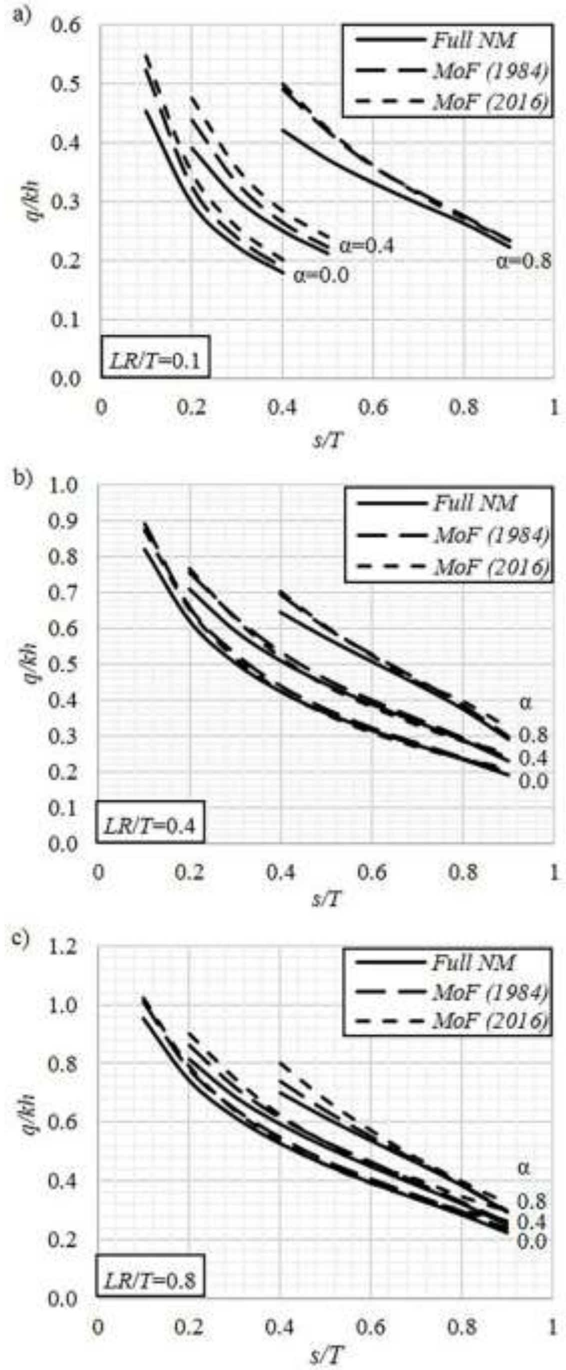

Figure 3. Comparison of the seepage quantity.

Predictions according to the method 1 are slightly higher than the actual values (i.e. on the conservative side). However their deviations from the actual values become lesser with increasing cut-off wall embedded depth and cofferdam width; hence the accuracy of the method of fragments improves, in line with the equipotential line behavior discussed in Section 4.1. The worst situations were encountered when cut-off walls were relatively short ($s/T \leq 0.2$). 22 cofferdam geometries were studied for this condition ($s/T \leq 0.2$ and LR/T = 0.1 to 1), and the average deviations of the seepage quantity and exit gradient according to the method 1 are about 8% and 12%, respectively. In other situations ($s/T > 0.2$), 68 cofferdam geometries were studied, and observed deviations are only about 4% and 3%, for seepage quantity and exit gradient, respectively.

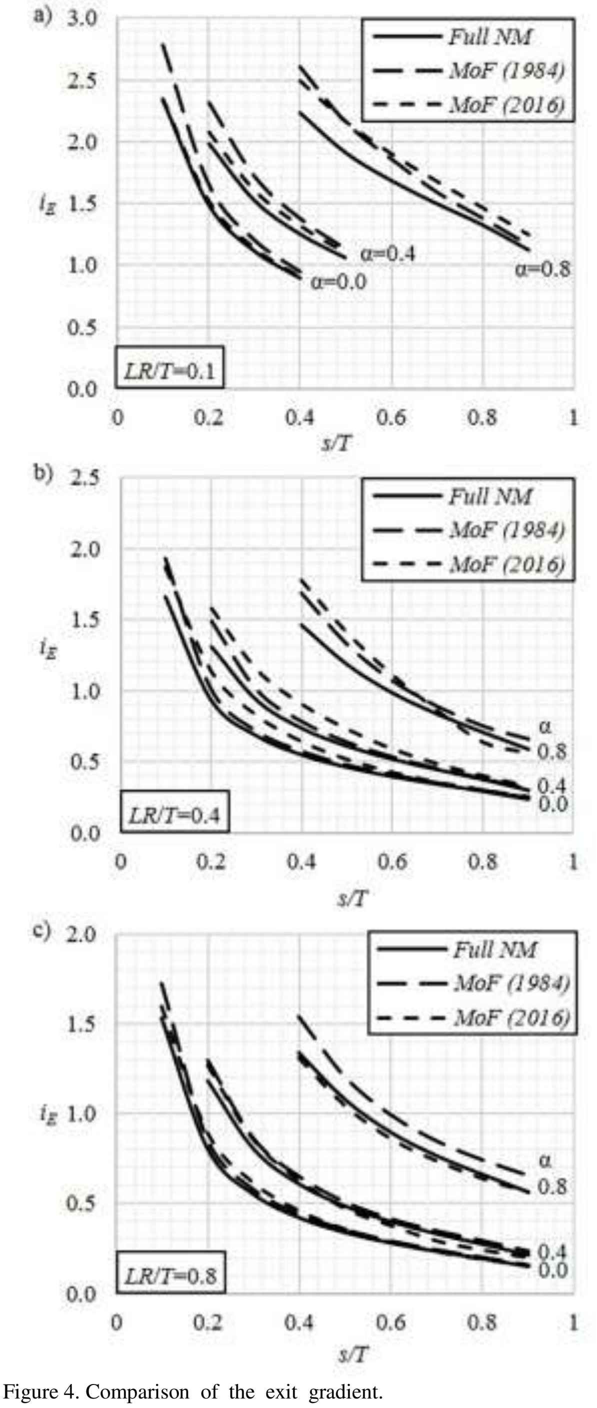

Figure 4. Comparison of the exit gradient.

Seepage quantity and exit gradient predicted by the method 2 show a similar trend to predictions by method 1. When the cut-off walls were short ($s/T \leq 0.2$), predicted seepage quantity by the proposed expressions in method 2 (Eqs. 5 and 6) were higher with the average deviation of 9%. In the case of exit gradient, Eq.7 provides higher values for most of the geometries (20 out of 22) with an average deviation of 13%. Predicted values for other two geometries are lower, with average deviation less than 3%. When the s/T ratio is greater than 0.2, form factor expressions predict higher seepage quantity for most of the cases (52 out of 68) with an average deviation of about 5% while the predictions for rest of the geometries are lower with the average deviation of about 1%. In exit gradient comparison, the expression proposed provide higher values than actual for 56 geometries out of 68 with an average deviation of 8%. For other geometries, predicted exit gradients are lower, with the average deviation less than 4%.

5.2 *Effect of non-vertical equipotential line due to excavation depth*

Comparison of the seepage quantity and exit gradient for two excavation depths ($\alpha = 0.4$ and 0.8) are also shown in Figures 3 and 4 respectively. When $\alpha = 0.4$, the deviation of the method of fragments based seepage quantity and exit gradient from the actual values follow a trend similar to that observed with no excavation condition ($\alpha = 0$) in general. Hence it verified the conclusion made at Section 4.1 (i.e., effect of the excavation depth is not significant on the validity of the assumption). For $\alpha = 0.8$, the deviations of seepage quantity and exit gradient are slightly higher than for no excavation condition; however the error decreases with increasing s/T ratio for all LR/T values. This is stated in the conclusion made in Section 4.2 that the effect of the excavation depth on the validity of the assumption decreases with increasing depth of cut-off wall.

6. SUMMARY AND CONCLUSION

The method of fragments relies on the assumption that the equipotential line at the tip of the sheet pile is vertical, which divides the flow domain into two fragments. An assessment is made in this paper on the validity of this assumption and the effects of any violation on the computed values of flow rate and the exit gradient.

It is shown that this equipotential line can be far from vertical, especially for low values of L/T and s/T. However, it is shown that the method of fragments [Griffiths (1984) and Madanayaka and Sivakugan (2016)] charts and expressions are adequate for reasonable estimates of the flow rate and exit gradient provided $s/T > 0.2$. When $s/T \leq 0.2$, method of fragments still provides conservative solutions, but their level of accuracy is low when compared to the geometries with $s/T >$ 0.2. However, the situations where $s/T \leq 0.2$ are of no practical significance and hence this should not be a concern to practicing engineers.

7. REFERENCES

Banerjee S. 1993. Design Charts for double walled cofferdams. *Journal of Geotechnical Engineering ASCE* 119(2), 214-222.

Bouchelghoum F. and Benmebarek N. 2011. Critical hydraulic head loss assessment for a circular sheet pile wall under axisymmetric seepage conditions. *Studia Geotechnica et Mechanica* 33(4), 3-23.

Griffiths D. V. 1984. Rationalized charts for the method of fragments applied to confined seepage. *Geotechnique* 34(2), 229-238.

Harr M. E. 1962. *Groundwater and seepage*. New York: McGraw-Hill.

Harza L. 1935. Uplift and seepage under dams on sand. *Transactions of the American Society of Civil Engineers* 100(1), 1352-1385.

King G. and Cockroft J. 1972. The geometric design of long cofferdams. *Geotechnique* 22(4), 619-633.

Madanayaka T. A. and Sivakugan N. 2016. Approximate equations for the method of fragment. *International Journal of Geotechnical Engineering* 10(3), 297-303.

Pavlovsky N. N. 1956. *Collected Works*. Leningrad, Russia: Doklady Akademii Nauk USSR.

Sivakugan N. and Alaghbari M. Y. S. (1993). *Method of fragments - quick solutions to seepage problems.* Environmental Management : Geo-Water and Engineering Aspects, Wollongong.