Probabilistic model for settlement prediction of cohesive soils in the Niger Delta region of Nigeria

Modèle probabiliste de prévision d'établissement de sols cohésifs dans la région du delta du Niger au Nigeria

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ABSTRACT: Settlement of building foundations on cohesive soils has been a major problem hindering infrastructure development in the Niger Delta region of Nigeria. An attempt is made in this research to develop probabilistic models for settlement prediction in order to aide preliminary analyses and design of foundations on the cohesive soils. Compressibility tests were done to determine settlement parameters and the Pearson's method was used to determine the appropriate distribution models. The beta distribution was found to be an appropriate distribution model for the behavior of Niger Delta clays with thickness greater than 3.5m, while the normal distribution proved to be the model for clays of thickness less than 3.5m. The beta distribution was also found to be an appropriate model for the unit settlement (cm/m) of Niger Delta clays. The mean unit settlement which can be used as a recommended design value for geotechnical works on Niger Delta clays was found to be 22.64 cm/m with a range of 5.11 to 51.31 cm/m.

RÉSUMÉ: Le règlement des fondations de bâtiments sur des sols cohésifs a été un problème majeur entravant le développement de l'infrastructure dans la région du delta du Niger au Nigeria. Dans cette recherche, on tente de développer des modèles probabilistes de prévision de l'établissement afin d'aider à l'analyse préliminaire et à la conception de fondations sur les sols cohésifs. Des tests de compressibilité ont été effectués pour déterminer les paramètres de colonisation et la méthode de Pearson a été utilisée pour déterminer les modèles de distribution appropriés. La distribution bêta s'est révélée être un modèle de distribution approprié pour le comportement des argiles Delta du Niger avec une épaisseur supérieure à 3.5 m, tandis que la distribution normale s'est révélée être le modèle pour les argiles d'épaisseur inférieure à 3.5m. On a également trouvé que la distribution bêta était un modèle approprié pour la colonisation unitaire (cm / m) des argiles du Delta du Niger. La valeur unitaire moyenne qui peut être utilisée comme valeur de conception recommandée pour les travaux géotechniques sur les argiles du Delta du Niger a été estimée à 22.64 cm/m avec une plage de 5.11 à 51.31 cm/m.

KEYWORDS: probabilistic model, settlement prediction, Niger delta, Nigeria, cohesive soil, clay, distribution model

1 INTRODUCTION.

It is common knowledge that the use of probabilistic modeling in geotechnical engineering has seen enormous development in recent times. For example, the Brazilian foundation code that was reviewed in 2010 defined the criteria for the use of probabilistic analysis in the design of foundation, and there is a perspective that such analyses are increasingly being used in practice in different parts of the world. In the same vein, probabilistic modeling was utilized in the design of the new lifting bridge being constructed across the river Oude Maas in the Rotterdam harbor area in the Netherlands (Indraratna and Correia, 2013).

Fenton and others (1996) also used probabilistic analysis to estimate probabilistic measures of total settlement under a single spread footing and of differential settlement under a pair of spread footings using a two-dimensional model combined with a Monte Carlo simulation. Earlier, Ejezie and Harrop-Williams (1984) used the Pearson's system in the probabilistic characterization of Southern Nigerian Soils.

Notable pioneer researchers namely, Lumb (1966), Hooper and Butler (1966) and Matsuo and Asaoka (1976); postulated that a number of material parameters encountered in geotechnical engineering follow the normal probability distribution. The major drawback to this assertion is that the soils used for their work were not from Africa.

In this work, probability modeling was carried out on settlements of clay soils which has been a major problem to infrastructures and building construction in the Niger Delta region of Nigeria.

Based on the difficulty in computing foundation settlement, an attempt is made in this research to develop probabilistic models for settlement prediction in the Niger Delta region of Nigeria to aide preliminary analysis and design of foundations placed on clayey soils.

2 DATA ACQUISITION AND ANALYSIS

Undisturbed clay soil samples were collected from fifty-three local government areas located in nine states in the Niger Delta region of Nigeria. The states include Abia, Akwa Ibom, Bayelsa, Cross River, Delta, Edo, Imo, Ondo and Rivers state. The soil samples were greyish brown in colour with bluish grey mottling. They were predominantly soft to stiff clays inter-bedded with seams of fine sand. The samples of the soil obtained during the field survey were subjected to oedometer test in the laboratory. Tests on the soil samples were carried out in accordance with BS 1377 standard code of practice (Methods of Testing Soil for Civil Engineering purposes and Earth Works). The Consolidation settlement was evaluated using the following equations (Smith, 1984; Raj, 2008).

After computing the consolidation settlement for the various soil sample, descriptive statistical analysis was then carried out on the data and the Pearson's (1894) Method is used to determine the appropriate distribution model. The Pearson's method involves the determination of the coefficients of skewness (β_1) and Kurtosis (β_2). A parameter K is next introduced as a function of these coefficients as follows:

$$K=\frac{\beta_1\left(\beta_2+3\right)^2}{4\left(2\beta_2-3\beta_1-6\right)\left(4\beta_2-3\beta_1\right)} \tag{1}$$

The K value is then used to select the specific distribution type that best represents the sample data based on the scale in Figure 1. The Pearson's system predicts the Beta distribution if K < 0 (Type 1). Gamma if K = ±∞ (Type III), Lognormal if K = 1 (Type V), and Normal if K = 0 and β_2 = 3 (transition between Types II and VII).

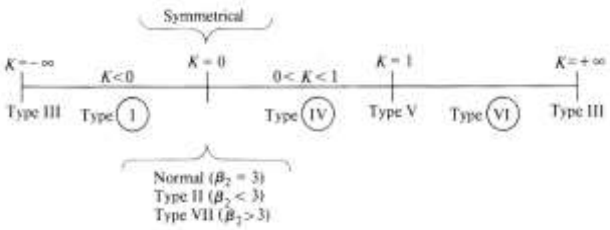

Fig. 1: The Criterion K (After Elderton and Johnson, 1969)

3 RESULTS AND DISCUSSION

Table 1 summarizes the probability distributions of foundation settlement of Niger Delta clays as determined by the Pearson system. After carrying out the descriptive statistics and the probabilistic modeling of settlement of each states and the Niger Delta as a whole the results are tabulated in Table 1 below.

Table 1: Probability Distributions of foundation settlement of Niger Delta clays as determined by the Pearson system

	Shallow Depth (<3.5m)	**Deep Depth (> 3.5m)**
ABIA	-	TYPE IV -NORMAL
AKWA IBOM	-	TYPE II - NORMAL
BAYELSA	TYPE II -NORMAL	-
CROSS RIVER	-	TYPE II - NORMAL
DELTA	TYPE II - NORMAL	-
EDO	TYPE IV NORMAL	-
IMO	-	TYPE I - BETA
ONDO	TYPE I - BETA	-
RIVERS	TYPE II - NORMAL	-
NIGER DELTA	TYPE II - NORMAL	TYPE I - BETA

3.1 *Probability Density Function Of Settlement Of Deep Depth Of Clay In The Niger Delta*

The beta distribution approximation of deep settlement in the Niger Delta is obtained as follows:

The data ranges from a = 37.21 to b = 627.5, and Mean, $\overline{X}$ = 214.6 (2)

Standard derivation S_x = 136.92 (3)

Sample Variance = S_x^2 = 136.92² = 18746.77 (4)

For beta distribution

$$\bar{x}=\frac{\bar{x}-a}{b-a}=\frac{214.6-37.21}{627.5-37.21}=0.3 \tag{5}$$

$$\tilde{V}=\left(\frac{S_x}{b-a}\right)^2=\left(\frac{136.92}{627.5-37.21}\right)^2=0.054 \tag{6}$$

This gives:

$$\alpha=\frac{\tilde{x}^2}{\tilde{V}}\left(1-\tilde{x}\right)\left(1+\tilde{x}\right)=\frac{0.3^2}{0.054}\left(1-0.3\right)\left(1+0.3\right)=1.52 \tag{7}$$

$$\beta=\frac{\alpha+1}{\tilde{x}}-\left(\alpha+2\right)=\frac{1.52+1}{0.3}-\left(1.52+2\right)=4.88 \tag{8}$$

Consequently,

$$\mathrm{B}\left(\alpha+1,\beta+1\right)=\frac{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}{\Gamma\left(\alpha+\beta+2\right)}=\frac{\Gamma\left(2.52\right)\Gamma\left(5.88\right)}{\Gamma\left(8.4\right)} \tag{9}$$

Using the Gamma function table,

$$\Gamma\left(2.52\right)=1.52\Gamma\left(1.52\right)=1.52\times0.88704=1.348 \tag{10}$$

$$\Gamma\left(5.88\right)=1.88\left(2.88\right)\left(3.88\right)\left(4.88\right)\Gamma\left(1.88\right) \tag{11}$$

$$=102.52\times0.95507=97.91$$

$$\Gamma\left(8.4\right)=1.4\left(2.4\right)\left(3.4\right)\left(4.4\right)\left(5.4\right)\left(6.4\right)\left(7.4\right)\Gamma\left(1.4\right)$$

$$=12855.13\times0.88726=11405.84 \tag{12}$$

$$\therefore\ \mathrm{B}\left(\alpha+1,\beta+1\right)=\frac{1.348\times97.91}{11405.84}=0.0116 \tag{13}$$

The beta distribution function is given by

$$f\left(x\right)=\frac{1}{\left(b-a\right)\mathrm{B}\left(\alpha+1,\beta+1\right)}\left(\frac{x-a}{b-a}\right)^{\alpha}\left(\frac{b-x}{b-a}\right)^{\beta} \tag{14}$$

$\therefore$ the required probability density function for the beta distribution approximation of settlement of deep depth in the Niger Delta is given by

$$f\left(x\right)=\frac{1}{7\left(0.0116\right)}\left(\frac{x-37.21}{590.29}\right)^{1.52}\left(\frac{627.5-x}{590.29}\right)^{4.88} \tag{15}$$

Table 2 below presents the summary information relative to the grouped experimental data and beta approximation.

Table 2: Summary information relative to the grouped experimental data and beta approximation

DEEP	**Frequency**	**Mid-point**	***f*(*x*)**	**Relative Frequency**
0 - 100	5	50	0.032335362	0.2272727
100 - 200	6	150	0.353161237	0.2727273
200 - 300	4	250	0.295740029	0.1818182
300 - 400	6	350	0.119138872	0.2727273
400 - 500	0	450	0.020823871	0
500 - 600	0	550	0.000534800	0
600 - 700	1	650	0.000000001	0.0454545

A plot of the frequency against settlement for deep depth of clay showing both the original data and beta approximation f(x) is shown in Figure 2.

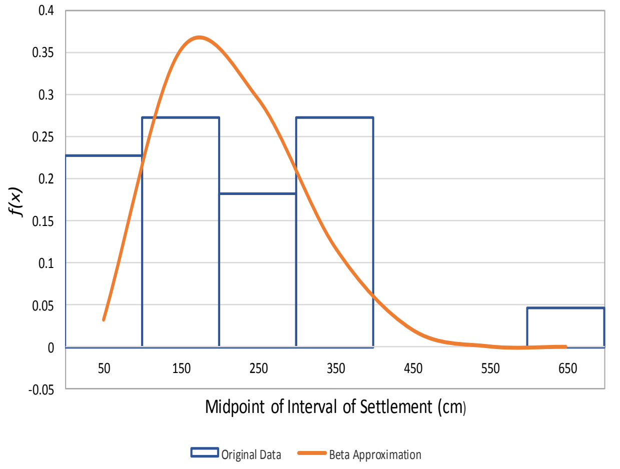

Fig. 2. Plot of frequency against settlement (cm) for deep depth of clay showing both the original data and beta approximation

From Figure 2 it can be seen that the beta approximation is a good fit for the probabilistic modeling.

3.2 *Probability Density Function Of Settlement Of Shallow Depth Of Clay In The Niger Delta*

A plot of the frequency against settlement for shallow depth of clay showing both the original data and normal approximation $f_R(x)/\Delta z$ is shown in Figure 3.

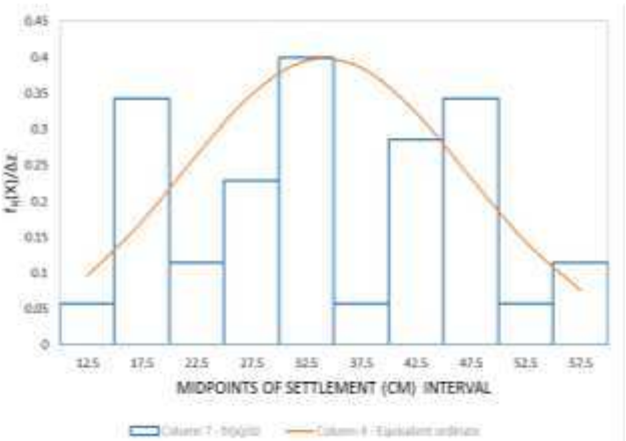

Fig. 3: Plot of frequency against settlement (cm) for shallow depth of clay showing both the original data and normal approximation

From Figure 3 we can conclude that the fit of the normal distribution to the data is very good. Therefore the standard normal distribution with the following probability density function and cumulative density function can be used. The normal density function to be used is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \qquad -\infty < x < \infty \tag{16}$$

where μ and σ are the mean and standard deviation, respectively. The corresponding distribution function is given by:

$$F(x) = P(X \le x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(\upsilon-\mu)^2/2\sigma^2} dv \tag{17}$$

3.3 *Probabilistic Modeling of Unit Settlement (cm/m) of Clay in the Niger Delta*

Table 3 below shows the descriptive statistical result of the analysis of the unit settlement (cm/m) of clay in the Niger Delta. The coefficients of kurtosis and skewness presented in this table were used with the Pearson System to conclude that the Beta Distribution approximation was a better model in carrying out probabilistic modeling of unit settlement (cm/m) of clay layers in the Niger Delta.

Table 3: Descriptive Statistics of Unit Settlement of Clay in the Niger Delta

DESCRIPTIVE STATISTICS OF UNIT SETTLEMENT OF CLAY IN THE NIGER DELTA	
Mean	22.64477273
Mode	31.52
Standard Deviation	8.908018856
Sample Variance	79.35279995
Kurtosis	-0.218402933
Skewness	0.095685793
Range	46.2
Minimum	5.11
Maximum	51.31
Sum	1992.74
Count	88
Pearson's K Value	0.126244848

The beta distribution approximation of unit settlement of clay in the Niger Delta is obtained as follows:
The data ranges from a = 5.11 to b = 51.31, and Mean

$$\bar{X} = 22.64 \tag{18}$$

Standard derivation $S_x = 8.91$ (19)

Sample Variance = $S_x^2 = 8.91^2 = 79.35$ (20)

For beta distribution:

$$\bar{x} = \frac{\bar{x}-a}{b-a} = \frac{22.64-5.11}{51.31-5.11} = 0.38 \tag{21}$$

$$\tilde{V} = \left(\frac{S_x}{b-a}\right)^2 = \left(\frac{8.91}{51.31-5.11}\right)^2 = 0.037 \tag{22}$$

which gives:

$$\alpha = \frac{\tilde{x}^2}{\tilde{V}}(1-\tilde{x})(1+\tilde{x}) = \frac{0.38^2}{0.037}(1-0.38)(1+0.38) = 3.34 \tag{23}$$

$$\beta = \frac{\alpha+1}{\tilde{x}} - (\alpha+2) = \frac{3.34+1}{0.38} - (3.34+2) = 6.08 \tag{24}$$

Consequently,

$$\mathrm{B}(\alpha+1,\beta+1) = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} = \frac{\Gamma(4.34)\Gamma(7.08)}{\Gamma(11.42)} \tag{25}$$

Using the Gamma function table,

$$\Gamma(4.34) = 1.34(2.34)(3.34)\Gamma(1.34) \tag{26}$$

$$= 10.4729 \times 0.8922 = 9.344$$

$$\Gamma(7.08) = 1.08(2.08)(3.08)(4.08)(5.08)(6.08)\Gamma(1.08)$$

$$= 871.897 \times 0.9597 = 836.76 \tag{27}$$

$$\Gamma(11.42)=1.42(2.42)(3.42)(4.4)2(5.42)(6.42)(7.42)\times(8.42)(9.42)(10.42)\Gamma(1.42)=11084625.4\times0.8864=9825411.9 \quad (28)$$

$$\therefore\ \mathrm{B}(\alpha+1,\beta+1)=\frac{9.344\times836.76}{9825411.9}=0.0008 \quad (29)$$

The beta distribution function is given by:

$$f(x)=\frac{1}{(b-a)\mathrm{B}(\alpha+1,\beta+1)}\left(\frac{x-a}{b-a}\right)^{\alpha}\left(\frac{b-x}{b-a}\right)^{\beta} \quad (30)$$

$\therefore$ the required probability density function for the beta distribution approximation of unit settlement of clay layers in the Niger Delta is given by:

$$f(x)=\frac{1}{10(0.0008)}\left(\frac{x-5.11}{46.2}\right)^{3.34}\left(\frac{51.31-x}{46.2}\right)^{6.08} \quad (31)$$

Table 4 presents the summary information relative to the grouped experimental data and beta approximation.

Table 4: Summary information relative to the grouped experimental data and beta approximation

Unit Settlement cm/m	Frequency	Mid-points x	$f(x)$	Relative Frequency
0 - 5	0			
5 - 10	8	7.5	0.00448287	0.090909
10 - 15	11	12.5	0.09463759	0.125
15 - 20	22	17.5	0.23061192	0.25
20 - 25	8	22.5	0.27084004	0.090909
25 - 30	14	27.5	0.19788254	0.159091
30 - 35	23	32.5	0.09263197	0.261364
35 - 40	1	37.5	0.02480470	0.011364
40 - 45	0	42.5	0.00260998	0
45 - 50	0	47.5	2.4426E-05	0
50 - 55	1	52.5	0	0.011364

A plot of the frequency against unit settlement (cm/m) of clay in the Niger Delta showing both the original data and beta approximation f(x) is shown in Figure 4.

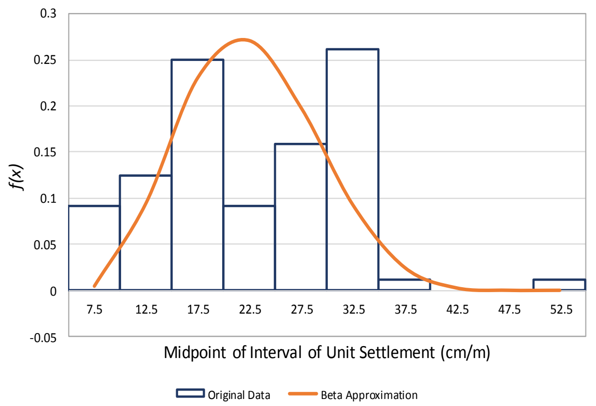

Fig. 4. Plot of frequency against unit settlement (cm/m) of Niger Delta clay showing both the original data and beta approximation

From Figure 4 it can be seen that the beta approximation is a good fit for the probabilistic modeling.

4 CONCLUSION

Based on the results of this study, the following conclusions can be drawn:

1. The beta distribution is an appropriate distribution model to use in studying settlement variability for Niger Delta clay layers greater than 3.5m.
2. The normal distribution is an appropriate distribution model to use in studying settlement variability for Niger Delta clay layers less than 3.5m.
3. The beta distribution is an appropriate distribution model to use in studying unit settlement (cm/m) variability for Niger Delta clays.
4. The mean unit settlement of Niger Delta clay is 22.64 cm/m with a minimum value of 5.11 cm/m and a maximum value of 51.31 cm/m. The mean unit settlement of 22.64 cm/m can be used as a recommended design value for geotechnical design works on Niger Delta clays.

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