Time-dependent settlements for 3-D scenarios with partially penetrating and regularly distributed vertical strip drains

Tassements dépendantes du temps pour les cadres 3-D avec pénétrant partiellement drains verticaux et distribué régulièrement

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ABSTRACT: Recently, the installation of vertical strip drains regularly distributed along saturated clayey soils has demonstrated to be an efficient and economic technique to reach high grade of consolidation in acceptable times. For these scenarios, it is always possible to obtain analytical solutions for the average degree of consolidation (or settlements) expressed in terms of mathematical series of complicated management and more or less slow convergence. Using the network method as numerical technique, the simulation of a large number of 3-D practical scenarios in which the main parameters change has allowed the construction of universal curves for the average degree of consolidation as function of the three discriminated dimensionless groups that characterize these scenarios. These are the dimensionless time (or time factor), the ratio 'length of the drain/thickness of the soil' and the ratio of consolidation coefficients corrected by the form factor 'distance between vertical drains/thickness of the soil'.

RÉSUMÉ : Récemment, l'installation de drains verticaux régulièrement répartis tout au long de sols argileux a démontré être une technique efficace et économique pour atteindre de hautes degrés de consolidation en temps acceptables. Pour ces cadres, il est toujours possible d'obtenir des solutions analytiques pour le degré moyen de consolidation (ou tassements) en termes de séries mathématiques d'utilisation compliquée et convergence lente. En utilisant la méthode de réseaux en tant que technique numérique, la simulation d'un grand nombre de cas pratiques 3-D a permis de représenter les courbes universelles du degré moyen de consolidation en fonction des trois paramètres adimensionnels qui caractérisent ces cadres. Ce sont le facteur du temps, le rapport 'longueur du drain/épaisseur du sol' et le rapport des coefficients de consolidation corrigé par le facteur de forme 'distance entre les drains verticaux/épaisseur du sol'.

KEYWORDS: prefabricated vertical drains, settlement, average degree of consolidation, dimensionless groups

1 INTRODUCTION.

The soil consolidation is an essential issue for civil soil engineers whose fundamentals are well stablished in all soil mechanics texts, particularly for 1-D linear scenarios (Scott 1963, Taylor 1984, Berry and Reid 1987). But, on the one hand, the analytical solutions for 2-D and 3-D more complex scenarios in homogeneous soils, even may be derived analytically– not without difficulty–, are not easy to manage by practical engineers who need an approximate but reliable solution with low cost. These solutions are generally expressed in terms of mathematical series of functions whose slow convergence may need a large number of terms to ensure a precise solution. On the other hand, although the problem can be solved numerically by using standard commercial codes, this option requires a specialized knowledge beyond the scope of applied soil engineers.

Assuming homogeneous and anisotropic soil properties as well as the known Terzhagi's oedometer hypothesis (Terzaghi 1923), the design of a simple network model has been carried out in order to provide a precise and computationally fast solution by running it in a free code of circuit simulation such as Ngspice (Ngspice 2016).

The design of the model, both for a volume element and boundary conditions, is based on the formal equivalence between the equation of the circuit and the finite-difference differential equation that comes from the special discretization of the partial differential governing equation of the mathematical model. To do this, a suitable equivalence between the electrical and physical variables must be stablished. Since the equations are linear and contain very few terms, the rules needed for implementing the components of the model are very few and intuitive, only requiring basic acknowledge of the theory of the electric circuits.

In the last years, the experience in the use of the network model is extensive and covers applications to many engineering fields in which linear, non-linear and coupled problems are solved, always providing successfully results [Bég et al. 2008, Morales-Guerrero et al. 2012, Sánchez et al. 2012].

The application presented in this communication assumes vertical strip drains of fix transversal section and variable penetration depth with a rectangular distribution along the soil surface. The dimensionless groups that rule the problem are easily derived from the list of relevant variables formed by the lengths of the domain and the consolidation coefficients; horizontal and vertical consolidation coefficient are assumed to have a different value to approach real scenarios.

Applying discriminated dimensional analysis (Madrid and Alhama 2008, Alhama and Madrid 2012), which assumes the quantities of the same nature related to different spatial directions to be different, the three independent groups are: the ratio 'length of the drain/thickness of the soil', the ratio of consolidation coefficients corrected by the ratio 'distance between vertical drains/thickness of the soil' and the dimensionless time. The influence of the width of the drain has been deleted while the ratios of horizontal lengths have been fixed to constant standard values. As a function of these groups, universal curves of the average degree of consolidation are represented, point to point, by successive simuations. This representation provides the engineer with a valuable tool for the design of scenarios with partially penetrating drains.

2 THE MATHEMATICAL AND NETWORK MODELS.

Figures 1 and 2 show the physical scheme of the problem: the soil drains by both the top surface and the vertical drain (rest of the boundaries are assumed impermeable) and consolidates due to the effect of a surface load (q_o) in a 3-D process with water flow in rectangular coordinates.

The mathematical model, once assumed homogeneous, anisotropic soil and the usual hypotheses of Terzaghi's consolidation theory, is formed by the following equations:

Governing equation:

$$\frac{\partial u}{\partial t} = c_{v,z}\frac{\partial^2 u}{\partial z^2} + c_{v,x}\frac{\partial^2 u}{\partial x^2} + c_{v,y}\frac{\partial^2 u}{\partial y^2} \quad (1)$$

Initial condition:

$$u_{(x,y,z,t=0)} = q_o \quad (2)$$

Draining boundaries:

$$u_{(x,y,z=H,t)} = 0 \quad (3)$$

$$u_{(x=[0,c],y=0,z=[H-d,H],t)} = 0 \quad (4)$$

$$\frac{\partial u}{\partial n}_{(\text{rest of the boundaries})} = 0 \quad (5)$$

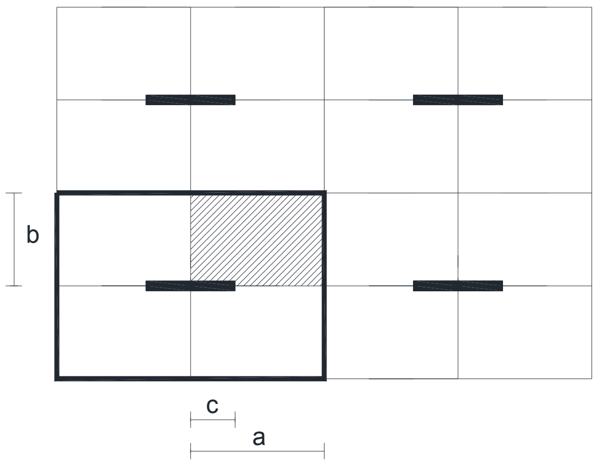

Figure 1. Physical scheme of the 3-D linear rectangular consolidation problem with vertical drains (squared distribution).

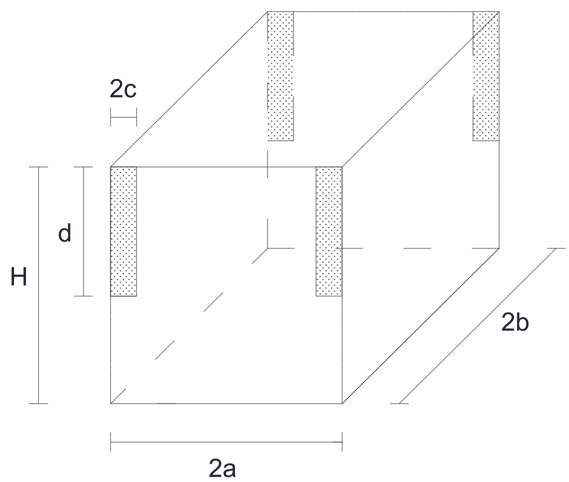

Figure 2. Vertical section of the domain.

In these equations, u is the excess pore pressure whereas $c_{v,x}$, $c_{v,y}$ and $c_{v,z}$ are the corresponding consolidation coefficients.

The implementation of the network model follows the steps of the network method (González-Fernández 2002). Figure 3 shows the model of the volume element whose 3-D coupling with adjacent cells is made by means of ideal electrical contacts. Boundary conditions are added by simple devices: resistors of very high value and constant voltage generators of zero value for the impermeable and draining boundaries, respectively.

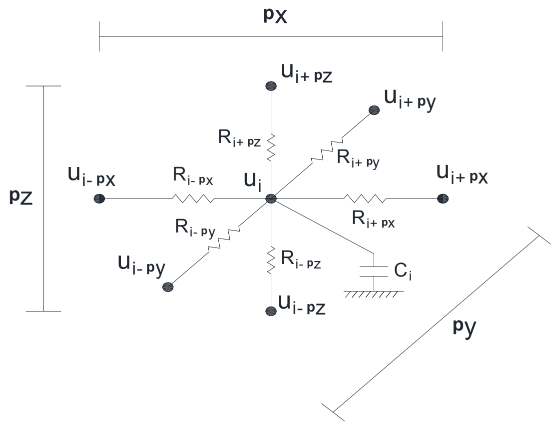

Figure 3. Network model of the elemental cell (3-D rectangular consolidation).

The values of the capacitors and resistors between nodes (García, 2016) are:

$$R_{i+\Delta z} = R_{i-\Delta z} = \frac{(\Delta z)^2}{2c_{v,z}} \quad (6)$$

$$R_{i+\Delta x} = R_{i-\Delta x} = \frac{(\Delta x)^2}{2c_{v,x}} \quad (7)$$

$$R_{i+\Delta y} = R_{i-\Delta y} = \frac{(\Delta y)^2}{2c_{v,y}} \quad (8)$$

$$C_i = 1 \quad (9)$$

The specification of each one of the components of the model follows intuitive and basic rules. Note that the volume element has only two kinds of electrical devices (resistor and capacitor) whose information reduces to the nodes in which they are located and the value of the component. The same occurs for the two components of the boundary conditions. The numeration of the nodes can be choose by the user as convenience. Once the whole network model is implemented, its simulation in Ngspice does not require mathematical manipulations.

After introducing the estimate time window the code adjusts continuously the time step in order to optimize the total computing time; time step enlarges with a smooth answer and shortened with sharp changes.

The simuation solutions are given to the user in tabulated form (excess pore pressure at any point of the network and local flow at the boundaries of each volumen element) so that subroutines of data treatment are required for a graphical representation. For this work we have used Matlab (Matlab 2015) subroutines.

3 NUMERICAL SIMULATIONS. UNIVERSAL CURVES

Following the discriminate nondimensionalization process (Madrid and Alhama 2008) and assuimg that, in general, the coefficients of consolidation in the horizontal plane are equal ($c_{v,x}$=$c_{v,y}$), it can be found that the proposed problem depends on five dimensionless groups (García, 2016). These, derived from the relevant list of variables {t(s), a(m), b(m), c(m), d(m), H(m), $c_{v,h}$(m^2/s) and $c_{v,z}$(m^2/s)}

are the following:

$$\pi_1 = \left(\frac{t\ c_{v,z}}{H^2}\right),\ \pi_2 = \left(\frac{c_{v,z}a^2}{c_{v,h}H^2}\right),$$
$$\pi_3 = \left(\frac{a}{b}\right),\ \pi_4 = \left(\frac{c}{a}\right),\ \pi_5 = \left(\frac{d}{H}\right) \qquad (10)$$

where π_1 is the time factor, π_2 is the corrected ratio between vertical and horizontal consolidation coefficients to give it a discriminate character, π_3 is the ratio 'distance between vertical drains/separation between rows of drains', π_4 is the ratio 'drain width/distance between vertical drains' and π_5 is the depth of penetration of the drain in relation to the total thickness of the soil. The drainage boundaries, equations (3) and (4), u=0 (a first kind condition), as well as the impermeable boundaries, equation (5) (second class homogeneous condition), do not provide any other group to the solution.

In this work, and in order to obtain universal solutions which depend on the minimum number of parameters, it has been assumed that the separation between drains is the same that the distance between rows of drains, so that π_3 takes a value of 1. Moreover, being the usual width of a prefabricated vertical drain 0.1 m and taking a 2 m separation between drains, the values of π_4 turns out to be 1/20. In this way, the problem only depends on the groups π_2 and π_5, as well as the time factor.

Figures 4, 5 and 6 show the temporal evolution of the average degree of consolidation (the ratio between the averaged surface settlement for a certain time and the final settlement at the end of the consolidation process) in function on the parameters π_2 and π_5. With the aim of covering most of the real scenarios, a wide range of values has been chosen for π_2, going from 0.1 to 10, while for π_5 three forms of penetration of the drain have been considered: 25% and 50% penetration in the soil layer (partially penetrating drain) and 100% penetration (fully penetrating), Figures 4, 5 and 6, respectively.

In view of the obtained results, as expected, it is verified that increasing the monomial π_5 (d/H) diminishes the consolidation time, π_1. Also, as π_2 decreases consolidation time is reduced, while for high values of this monomial all the curves converge to the same consolidation time. Physically, this is fairly simple to understand. On the one hand, as π_5 rises the depth of penetration of the drain is greater, which implies an increase of the water flow in horizontal direction, accelerating the consolidation process. On the other hand, as π_2 rises the flow becomes predominantly 1-D vertical and, therefore, the process will take longer.

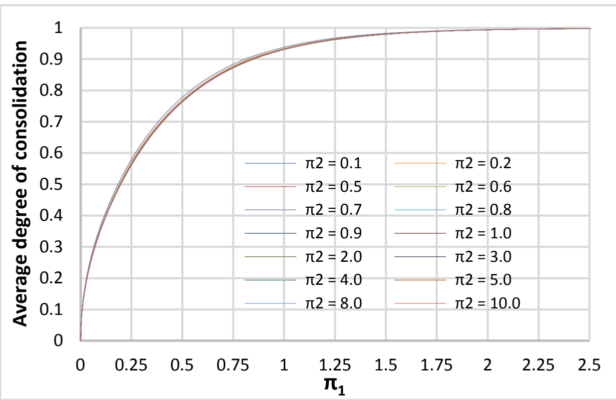

Figure 4. Average degree of consolidation as a function of π_1 and π_2. 3-D rectangular domains with vertical drains. a/b = 1, c/a = 0.05 y d/H = 0.25.

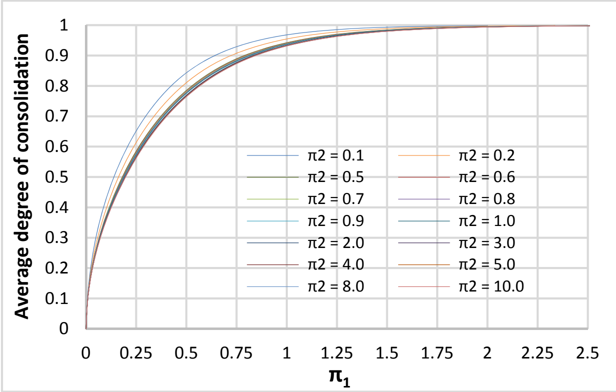

Figure 5. Average degree of consolidation as a function of π_1 and π_2. 3-D rectangular domains with vertical drains. a/b = 1, c/a = 0.05 y d/H = 0.5.

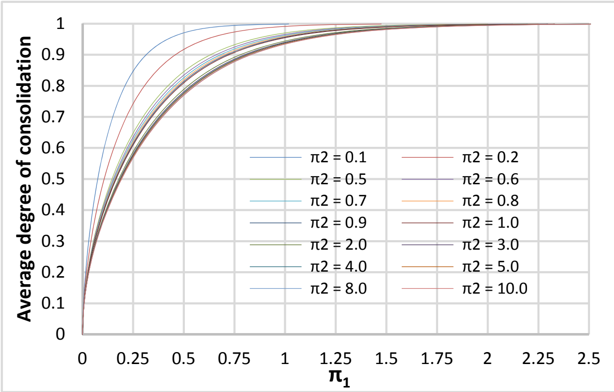

Figure 6. Average degree of consolidation as a function of π_1 and π_2. 3-D rectangular domains with vertical drains. a/b = 1, c/a = 0.05 y d/H = 1.

Numerical simulations have been done taking values of grid size of $N_x = N_y = N_z = 20$. Computing times for this reticulation is of the order of 10 minutes. Due to the powerful mathematical algorithms and interpolation routines integrated in the software Ngspice it can be ensure that the former grid size provides very reliable results. In fact, for a grid of 40×40×40 maximum deviation of the average degree of consolidation is below 1% with respect to the grid 20×20×20, while computing times increases considerably. In order to design the circuit it has been used the software Matlab through which have been created programming routines that allow to obtain the circuit file for Ngspice automatically.

Note, finally, that the group $\pi_1 = (t\ c_{v,z}/H^2)$ would have been chosen in other forms, for example $\pi_1 = (t\ c_{v,h}/a^2)$ or $\pi_1 = (t\ c_{v,h}/b^2)$; however, the different universal representation of the results would have produced the same values of the average degree of consolidation for the same value of real time.

4 COMMENTS AND CONCLUSIONS

Reliable numerical solutions for 3-D soil consolidation scenarios with partially or totally penetrating vertical drains have been obtained by using the network method, providing universal curves for the average degree of consolidation. This unknown results to be influenced by the depth of penetration of the drains, as well as the ratio between the vertical and horizontal consolidation coefficients (duly corrected by the form factor 'distance between vertical drains/thickness of the soil').

For the derivation of the independent groups that rule the solution (from the list of relevant variables and using the known rules of dimensional analysis) it has been introduced the concept of discrimination, which allows to consider any of the form factors formed by ratios between horizontal and vertical lengths (such as a/d, b/d, a/H, b/H…), as well as the ratio between horizontal and vertical consolidation coefficients, as non-dimensional groups. In fact, any of these form factors join to the ratio of consolidation coefficients to form a real dimensionless 'discriminated' group. As a result, the independent groups are reduced from three to two (since a/b and c/a have constant values for the studied scenario), a number of groups for which universal representation is relatively easy to carry out with charts obtained, point to point, by numerical simulations.

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