1-D consolidation model with stress dependent recompression index and coefficient of consolidation

Modèle de consolidation unidimensionnelle incluant des coefficients de re-compression et de consolidation dépendant des contraintes.

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ABSTRACT: According to Terzaghi's consolidation theory, the coefficient of consolidation, c_v is a composite parameter that depends on both the coefficient of permeability, k and coefficient of volume compressibility of the soil, m_v. Conventionally, k is related to void ratio, e by a $e-\log k$ relationship and m_v is calculated based on a bi-linear e–log σ'_v compressibility model with a recompression index, c_r and compression index, c_c at the over-consolidated and normally consolidated stress ranges, respectively. Based on the observations of an instrumented embankment site, this paper has shown that because of the inherent simplification of the soil compressibility using a constant c_r, the conventional model overestimates c_v and underestimates settlement at the lightly over-consolidated stress range near the pre-consolidation pressure. This paper further proposes a 1-D consolidation model with a stress dependent c_r in order to model appropriately the soil behavior at this stress range.

RÉSUMÉ : Suivant la théorie de la consolidation selon Terzaghi, le coefficient de consolidation c_v dépends de la perméabilité k et du coefficient de compressibilité volumique m_v. Usuellement, k est relié à l'indice des vides e par une relation $e-\log k$ et mv est calculé en utilisant un modèle bilinéaire e–log σ'_v incluant un coefficient de re-compression c_r dans le domaine sur-consolidé, puis un coefficient de consolidation c_c dans le domaine normalement consolidé. S'appuyant sur les observations d'un chantier instrumenté, cet article démontre que la simplification de la courbe de consolidation par le modèle bilinéaire tend à surestimer la valeur de c_r et sous-estimer les tassements dans le domaine légèrement sur-consolidé, proche de la contrainte de pré-consolidation. Cet article propose un modèle de consolidation unidimensionnelle incluant un c_r dépendant des contraintes qui représente plus correctement le comportement du sol dans le domaine proche de la contrainte de pré-consolidation.

KEYWORDS: compressibility; consolidation; permeability.

1 INTRODUCTION

According to Terzaghi's theory of 1-D consoldiation of soils, the coefficient of consolidation, c_v is thoertically related to the coefficient of volume compressibility, m_v and the coefficient of permeability, k by

$$c_v = k/(m_v\gamma_w) \qquad (1)$$

where γ_w is the unit weight of the water and m_v can be defined in tems of the compressibility index c_c, or the recompression index c_r, depending on the vertical effective stress σ'_v relative to the preconsoldiaiton pressure, σ'_p as follows:

$$m_v = \frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'_v} \quad \begin{cases} m_v = \dfrac{0.434C_r}{\sigma'_v(1+e_0)} & (\sigma'_v \leq \sigma'_p) \quad (2a) \\ m_v = \dfrac{0.434C_c}{\sigma'_v(1+e_0)} & (\sigma'_v > \sigma'_p) \quad (2b) \end{cases}$$

where e_0 is the initial void ratio. For the derivation of k in equation (1), Tavenas et al (1983) indicated that it can be related to the void ratio, e by:

$$e = e_0 + c_k \log (k / k_0) \qquad (3)$$

where c_k is the permeability index and k_0 is the initial permeability value. The data presented in Tavenas et al (1983), indicate that the above linear $e-\log k$ relationship holds irrespective of stress history. Substituting Equations (2) and (3) into (1), and following the derivation similar to that given in Walker et al (2012), a theoretical c_v versus σ'_v relationship can be obtained as shown in Figure 1. For the normally consolidated stress range, both k and m_v decrease rapidly with decreasing void ratio, hence c_v is fairly constant and this is consistent with the general understanding about the variation of c_v with pressure. For the over-consolidated stress range, the soil has a lower compressibility with less reduction of m_v with decreasing e whilst the rate of change of k with e remains the same as for the normally consolidated stress range. This results in an increasing c_v with σ'_v, followed by a sudden drop of c_v as the soil state changes from over-consoldiation (OC) to normal consoldiation (NC). This calculated c_v appears to be contradictory with the expected trend outlined in Figure 1 (Ladd and DeGroot, 2003), which show that $c_{v\,(OC)}$ decreases to a fairly constant $c_{v\,(NC)}$ value as the stress level increases up to σ'_p. Therefore, the soil model may overestimate the consolidation rate when the soil is in the slightly over-consolidated stress range. The authors considered that this inconsistency is due to the use of the simplified bi-linear recompression and compression e–log σ'_v curves in the model.

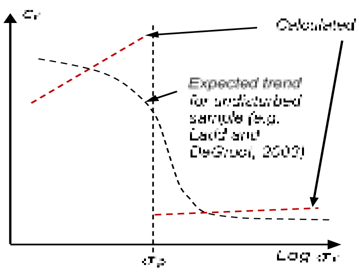

Figure 1. c_v calculated based on bi-linear e –log σ'$_v$ compressibility curves and void ratio dependent permeability function

This paper proposes a new soil model that consists of a curvilinear reompression curve in the lead up to a linear virginal compression line in the e–log σ'_v space. It is demonstrated that the proposed model is able to capture appropriately the c_v response at the lightly consolidated stress range.

2 CONSTITUTIVE RELATIONSHIP

A variable recompression index c_r with σ'_v is proposed as:

$$c_r(\sigma'_v) = c_c\left[\frac{(1-m)}{e^{(OCR-1)n}} + m\right] \tag{4}$$

where c_c is compression index and is considered as constant with stress; OCR is the over-consoldiation ratio, i.e. OCR = σ'_v / σ'_p; m is the ratio of initial c_r value to c_c, i.e. $c_r(\sigma'_{vo})$ / c_c; and n controls the rate of change of c_r with σ'_v. As to be discussed in Section 3, the parameter n can be obtained from oedometer test results. A higher n value (typically n = 4 or above) will give a more accute change in the compressibility curve from OC to NC states than a lower n value. The m_v as defined by Eq 2 can be rewritten as

$$m_v = \frac{1}{1+e_0}\frac{\partial e}{\partial log\sigma'_v}\frac{\partial log\sigma'_v}{\partial\sigma'_v} = \frac{1}{1+e_0}\frac{0.434}{\sigma'_v}\frac{\partial e}{\partial log\sigma'_v} \tag{5}$$

But $$c_r(\sigma'_v) = \frac{\partial e}{\partial log\sigma'_v} \quad (\text{for } \sigma'_v \leq \sigma'_p) \tag{6}$$

Substituting equations (4) and (6) into (5) gives

$$m_v = \frac{0.434c_c}{\sigma'_v(1+e_0)}\left[\frac{(1-m)}{e^{(OCR-1)n}} + m\right] \tag{7}$$

It can be seen from Eq. 7 that for OCR = 1, m_v reduces to volume compressibility for virgin consolidation defined by Eq. 2b.

In the proposed model, c_v is computed from the m_v defined in Eq. 7 and k defined in Eq. 3 using the relationship defined by Eq. 1. This soil model hs been incorporated in an in-house program LMCON, which solves the partial differential equation for 1-D consolidation via finite diference method. The resultant function for c_v can be appraised as follows.

Let $c_{v(OC)}$ and $c_{v(NC)}$ be the c_v values at the over-consolidated state and normally consolidated state, respectively. Also, as $c_{v(NC)}$ is fairly constant with $\sigma'_v \geq \sigma'_p$, the stress dependence of c_v in the recompression and compression zones can be expressed as

$$\begin{aligned}\frac{c_{v(OC)}}{c_{v(NC)}} &= \frac{k_{OC}}{m_{v(OC)}\gamma_w} \times \frac{m_{v(NC\ at\ \sigma'_p)}\gamma_w}{k_p} \\ &= \frac{k_{OC}}{k_p} \times \frac{\frac{0.434C_c}{\sigma'_p(1+e_0)}}{\frac{0.434c_c}{\sigma'_p(1+e_0)}\left[\frac{(1-m)}{e^{(OCR-1)n}}+m\right]} \\ &= \frac{k_{OC}}{k_p} \times \frac{1}{\left[\frac{(1-m)}{e^{(OCR-1)n}}+m\right]} \times \frac{1}{OCR}\end{aligned} \tag{8}$$

where k_{OC} is the permeability at over-consolidated state and k_p is the permeability at pre-consolidation pressure. k_{OC}/k_p can be calculated based on a relationship similar to Eq. 3 as

$$\frac{e_{OC}-e_p}{c_k} = \log\left(\frac{k_{OC}}{k_p}\right)$$

Hence, $$\frac{k_{OC}}{k_p} = 10^{\frac{e_{OC}-e_p}{c_k}} \tag{9}$$

where e_{OC} is the void ratio within the recompression range and e_p is the void ratio at preconsolidation pressure that can be obtained from oedometer test. Combining Eq. 9 and Eq. 8 gives

$$c_{v(OC)} = c_{v(NC)}10^{\frac{e_{OC}-e_p}{c_k}} \times \frac{1}{\left[\frac{(1-m)}{e^{(OCR-1)n}}+m\right]} \times \frac{1}{OCR} \tag{10}$$

Eq. 10 can be used to obtain, as a preliminary, the varying $c_{v(OC)}$ value with OCR if $c_{v(NC)}$ is known and assumed constant, which is not necessarily the case. Note that LMCON calculates c_v from m_v (Eq. 7) and k (Eq. 3) via Eq. 1, instead of using Eq. 10.

3 CRS TESTS ON BALLINA CLAY

Constant rate of strain (CRS) oedometer tests on high quality piston-sampled Ballina clay (Pineda *et al.*, 2016) were simulated using the proposed model. Figure 2 shows the plots of index properties, compression ratio (CR= c_c/(1+e_0)) and recompression ratio (RR= c_r/(1+e_0)) with reduced level (RL) obtained from CRS tested samples. Figure 3 shows the normalized compression curves from representative CRS tests within the lower soft clay (undrained shear strength S_u of about 15kPa) between RL-4m and RL-10 m. The curves exhibit non-linear response in the normally consolidated range with largest deformation just passing σ'_p and then reduces with increasing σ'_v, which is the consequence of progressive soil restructuration. Using the conventional Casagrande's graphical approach, the assessed RR, CR and p_c' are 0.05, 0.65 and 80kPa, respectively. Note that for the purpose of trial embankment prediction over the Ballina clay as outlined in Section 4, the stress range is less than 200 kPa after the application of 3 m high embankment load, hence justifying the use of a constant CR for this stress range just passing σ'_p.

Using the proposed model, a varying c_r with σ'_v (Eq. 4) is employed while maintaining a constant c_c for the virgin consolidation curve. The fitted parameters of Eq. 4 for the expermental data are n = 0.65 (similar to RR/CR ratio in the bi-linear model) and m = 5 (by fitting the curvature approaching σ'_p). The proposed model is considered to give a better representation of the soil compressibility at the lightly over-consolidated stress range. The conventional bi-linear e–log σ'_v model is only an approximation at the OC – NC transition, and may potentially underestimate settlement as depicted in Figure 3.

As discussed in Pineda *et al.*, 2016, The CRS tests were conducted at a displacement rate of 0.004 mm/min. The σ'_p indicated from these tests are higher than when tested at a slower displacement rate, but no important changes in the shape of the compressibility curves. Pineda suggested to apply a reduction factor of 0.84 on σ'_p to correct for the strain rate effect.

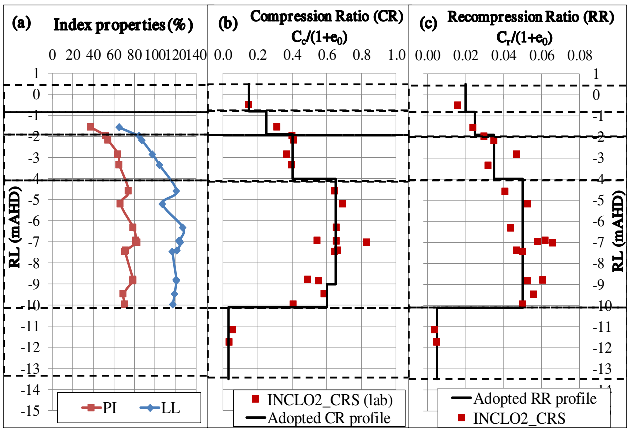

Figure 2. Plots of (a) index properties; (b) CR vs. RL; (c) RR vs. RL

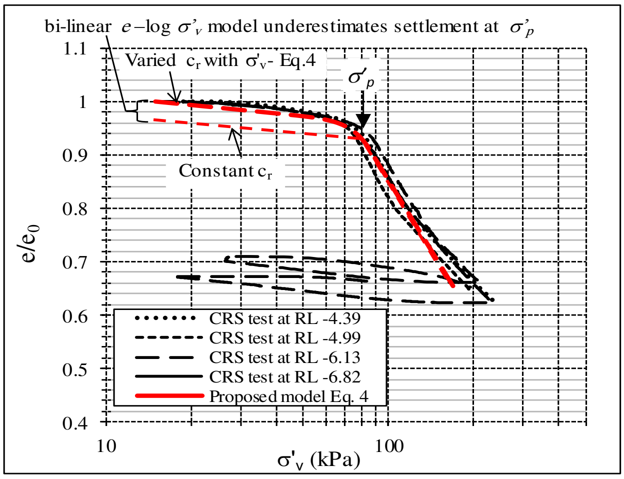

Figure 3. Normalised compressibility curves from CRS compared with proposed model and convention bi-linear e–log σ'_v model

To define the permeability function (Eq. 3) as part of the inputs to LMCON, the e_0 and k_0 values are typically 3.0 and 1E-9 m/s, respectively, for the soft soil between RL-4 m and RL-10 m, as shown in Figure 4. The c_k is about 1 according to CRS data presented in Pineda *et al.*, 2016.

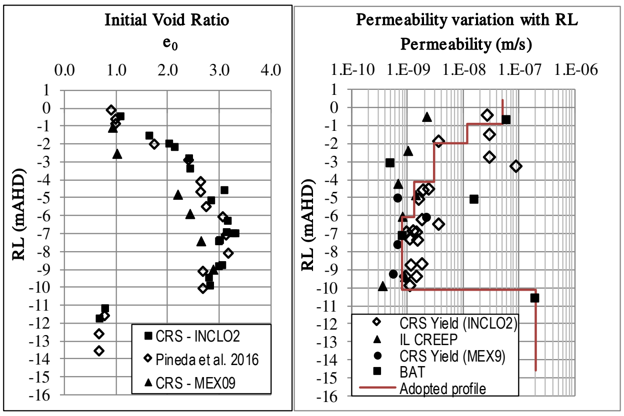

Figure 4. Initial void ratio and initial permeability from CRS tests

Figure 5 shows the plot of c_v versus σ'_v normalized with in-situ σ'_p, for the CRS tests within soft soil between RL-4 m and RL-10 m. It can be seen that c_v reduces as σ'_v increases up to σ'_p and beyond which c_v is fairly constant. The $c_{v(NC)}$ at the normally consolidated stress range is about 0.4 – 0.7 m²/year, which is consistent with the empirical correlation proposed by NAVFAC (1971) as shown in Figure 6 for a liquid limit (LL) of up to about 120% (see Figure 2a for index properties). Also shown in Figure 5 are the simulation of c_v with increasing σ'_v. LMCON with varying c_r gives agreeable c_v responses at the OC stress range as well as post-yielding. Conversely, the conventional bi-linear e–log σ'_v compressiblity model gives an inconsistent and opposite trend to the laborary results, in which c_v increases with σ'_v until σ'_p is reached, followed by an abrupt drop in value at the NC stress range.

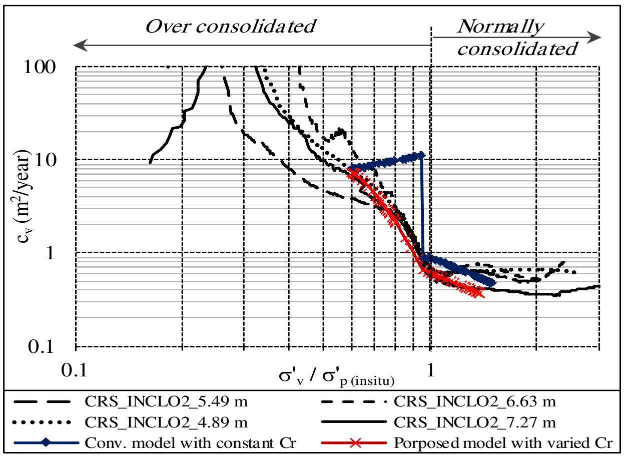

Figure 5. Variation of c_v with σ'_v during CRS loading (Pineda *et al 2016*)

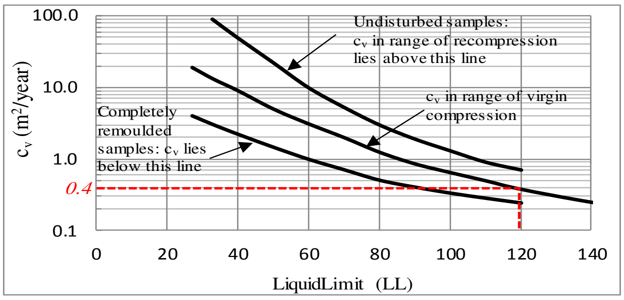

Figure 6. NAVFAC (1971) correlation of c_v vs. LL

4 TRIAL EMBANKMENT ON BALLINA CLAY.

A 3 m high embankment was constructed over Ballina clay treated with prefabricated vertical drains (PVD). The embankment was thoroughly instrumented with vibrating wire piezometers (VWP), settlement plates, extensometers, horizontal profile gauge and inclinometers to measure the behavior of the underlying 8.5 - 10m thick very soft estuarine silt/ clay deposits during staged loading and three years of subsequent consolidation. Figure 7 shows the geotechnical section of the instrumented embankment. This paper focuses on the extensometer and VWP installed in the middle of the embankment.

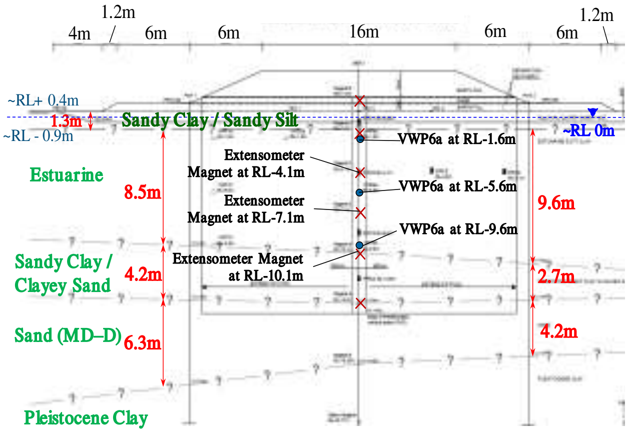

Figure 7. Geotechnical section of the instrumented trial embankment

The soil stratigraphy and much of the soil deformation and drainage properties have been outlined in Section 2. In addition, Figure 8a shows the σ'_p profile assessed from CRS and oedometer tests. While σ'_p of the CRS are higher than that of the oedometer tests, good agreement is indicated after σ'_p of the CRS results are corrected for strain rate effect by applying a reduction factor of 0.84. Also shown in Figure 8a is the in-situ σ'_v with RL. The corresponding OCR with RL is shown in Figure 8b.

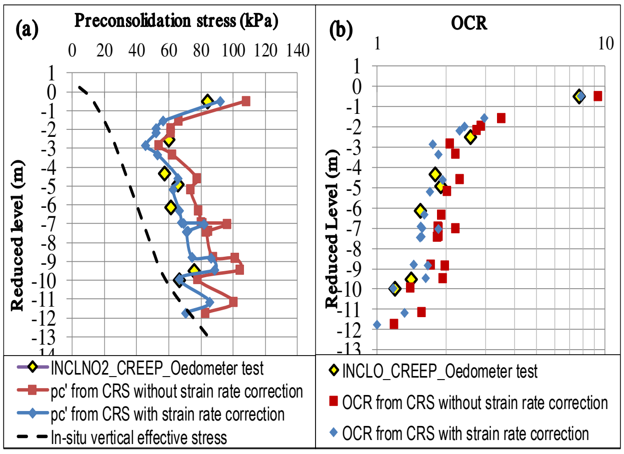

Figure 8. Pre-consolidation pressure and over consolidation ratio profile

Prefabricated vertical drains were installed at the trial embankment in a square pattern 1.2 m apart, to a depth of 15 m. The PVD installation has caused remoulding of the soft clay surrounding the drains. Indraratna *et al.* (2015) have conducted research of smear zone characteristics of actual drain installation at the trial embankment site by studying the extracted soil samples collected around the drains. The study has indicated that the smear zone was about 5-7 times greater than the equivalent dimensions of the mandrel, or about 11 times greater than the drain well radius. In addition, the ratio of in-situ permeability to disturbed permeability in the smear zone is about 1.7 -2.

The authors have conducted a Class A prediction and subsequently a back-analysis for the trial embankment at Ballina. Details of the works have been provided in Chan *et al.* (2017). This paper provides highlights of the back-analysis results, and focuses only on the behavior of estuarine clay between RL-4 m and RL-10 m beneath the centerline of the embankment.

Figure 9 shows the measurement of the excess pore pressure at RL-5.6m (see Figure 7 for piezometer location), which has been corrected for the increase in total head due to the settlement of the piezometer under load. Also shown in the figure are the predictions of excess pore pressure using the conventional bi-linear e–log σ'_v compressiblity model and the proposed model with varied c_r with σ'_v ($<\sigma'_p$) that has been incorporated in LMCON. Note that whilst both models are for 1D consolidation, the vertical stress that is applied on each discretized soil layer has been factored by the Boussinseq stress influence factor to account for the effect of 2D loading.

The conventional model gives a much faster pore pressure dissipation at the onset of consolidation when $\sigma'_v < \sigma'_p$, leading to an overall under-prediction of excess pore pressure compared to the measurements. This under-prediction is due to the over-estimation of c_v at the OC stress range outline in Figure 5. Conversely, the proposed model employed in LMCON gives a more agreeable pore pressure response with the measurements.

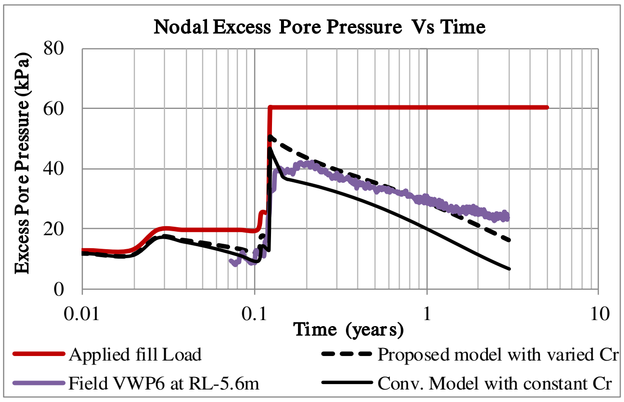

Figure 9. Comparison of predictions and measured excess pore pressure

Figure 10 shows the measured soil compression between RL-7 m and RL-10 m, which has been obtained from the measured settlements of the extensometer magnets installed at the corresponding reduced levels. LMCON gives very good prediction of the soil compression under load. In contrast, the conventional model underestimates the compression by about 10%. This is due to the inherent simplification of the conventional model for the transition from OC to NC at σ'_p as delineated in Figure 3.

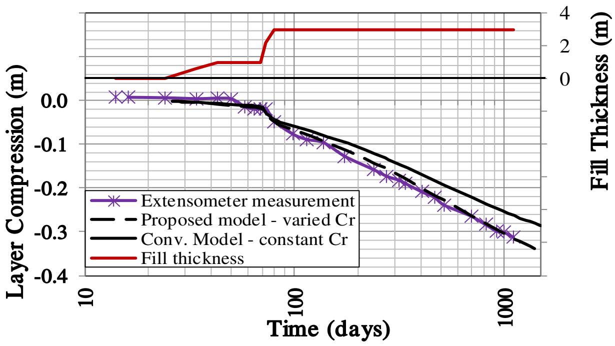

Figure 10. Comparison of predictions and extensometer results between RL-7m and RL-10m

Figure 11 shows the measured soil compression between RL-4 m and RL-7 m obtained from extensometer results. Both the conventional and proposed models underestimate the soil compression. This is attributed to plastic deformation associate with soil yielding at shallow depth during fill placement (Chan *et al.* 2017), which is unable to be captured by the present models for 1D consolidation. Nevertheless, the proposed model gives good prediction in terms of the slope of the compression-time curve post-plastic yielding. Moreover, the prediction given by the proposed model is closer to the measurements than that of the conventional model

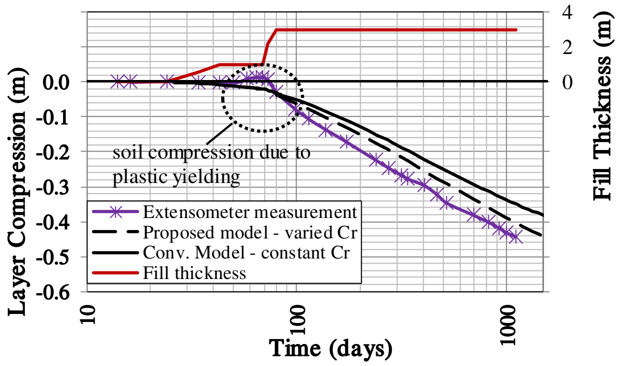

Figure 11. Comparison of predictions and extensometer results between RL-4m and RL-7m

5 CONCLUSION

A new 1-D soil compressibility curve consisting of a stress dependent recompression index c_r at the over-consolidated stress range has been developed. This compressibility model, when used in conunction with a log-linear void-ratio – permeability function, is able to capture implicitly the trend of reduction of c_v with σ'_v at the over-consolidated stress range, which has been verified with high quality CRS oedometer test results. Conversely, the conventional compressibility model with constant c_r will give an opposite trend of increasing c_v with σ'_v at the over-consolidated stress range. Furthermore, the newly proposed soil model will capture slightly more settlement than that of the conventional model at the transition from OC to NC soil states. The predictive capability of the proposed soil model has been demonstrated by numerical simulation of settlement and excess pore water pressure monitored during the staged loading and 3 years of subsequent consolidation for a trial embankment constructed over Ballina Clay treated with PVD.

6 REFERENCES

Chan, K., Poon, B. & Perera. D. 2017. Numerical prediction and back analysis of embankment using simple calculations – practitioner's approach. *Computer and Geotechnics.* In print.

Indraratna, B., Perera, D., Rujikiatkamjorn, C. & Kelly, R. 2015. Soil disturbance analysis due to vertical drain installation. *Proc. Institution of Civil Eng. Geotech. Engineering*, 168 (3), 236-246.

Ladd, C.C. and DeGroot, D.J. 2003. Recommended Practice for Soft Ground Site Characterization. The Arthur Casagrande Lecture, *Proceedings of the 12th Panamerican Conference on Soil Mechanics and Geotechnical Engineering*, Boston, MA, Vol. 1, pp. 3-57.

NAVFAC 1971. *Soil mechanics, foundations and earth structures. Design Manual DM-7, Naval Facilities Engineering Command.*

Pineda, J.A, Suwal, L. P., Kelly R. B., Bates L., and Sloan S. W, 2016. Characterisation of Ballina clay, *Géotechnique*, 66: 556-577

Tavenas, R., Jean, P., Leblond, P., and Leroueil, S. 1983. The permeability of natural soft clays, Part II: Permeability characteristics. *Can. Geot. J.*, 20(4), 645-660.

Walker R., Indraratna B. and Rujikiatkamjorn C. (2012). Vertical drain consolidation with non-Darcian flow and void-ratio-dependent compressibility and permeability. *Géotechnique* 50 (1), 89-97.