Quasi-static RTM method and its application to the asymmetric consolidation of a layered transversely isotropic saturated soil

La méthode quasi-statique de matrix de réflexion et de transmission ainsi que son application au sol saturé d'isotropie transversale (TISS) soumis à une consolidation asymétrique

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ABSTRACT: The reflection-transmission matrix (RTM) method was originally developed for addressing the wave propagation in the layered elastic medium. In this study, the quasi-static RTM method for the layered transversely isotropic saturated soil (TISS) undergoing asymmetric consolidation is developed. To this aim, a partial differential equation system is established using the governing equations for the TISS first. Applying the Hankel-Laplace transform to the partial differential equation system furnishes the corresponding ordinary differential equation system. By using the general solution of the ordinary differential equation system, the RTM method for the layered TISS is established. To show the capacity of the proposed model, one numerical example for the consolidation of the layered TISS subjected to a horizontal force is presented.

RÉSUMÉ : La méthodeintitulé la Matrice de réflexion et de transmission(RTM)était initialement développéepour étudier la propagation des ondes dansdes milieux élastiques et superposés. Dans cette étude, on élabore uneméthode quasi-statiquebaséeà la RTMpour étudier le sol saturé d'isotropie transversale (TISS) soumis à une consolidation asymétrique. Premièrement, Un système d'équationsaux dérivées partielles est déduitaux équations fondamentales pour le TISS. Avec l'aide de la transformation de Hankel et de Laplace, les équations ci-dessus sont réduites dans un système d'équations différentielles ordinaires. La solution générale obtenue après avoir résolu ce system d'équations différentielles ordinaires nous aide à développe le RTMs pour le TISS. Enfin dedémontrer la capacité du modèle proposé, on présente un exemple numérique concernant la consolidation dusol saturé d'isotropie transversale (TISS) soumis à une force horizontale.

1 Introduction

Since the construction is built on soils, soil consolidation is thus an important problem to be dealt with when designing the construction in civil engineering. The consolidation theory for the saturated soil was developed originally by Terzaghi for the one-dimensional case, and then extended to the 3-D case by Biot (1941). Since then, Biot's consolidaiton theory has been used extensively to study the consolidation of the saturated soil. It is noted that natural soils tend to be stratified and the parameters of different layers are thus different. Hence, many methods have been developed to investigate the consolidation of the layered soil (Ai, 2008; Chiou & Chi, 1994; Christian & Boehmer, 1970; Senjuntichai & Rajapakse, 1995; Wang & Fang, 2002). Because of the deposition process, although most soils are isotropic in the horizontal plane, parameters in the horizontal direction are unusually different from those in the vertical direction. Hence, the transversely isotropic medium is a realistic model for the soil. Due to its semi-analytical feature, the transfer matrix (TM) method is an important approach for dealing with the consolidation of the layered transversely isotropic saturated soil (TISS) (Chen et al., 2005; Ai & Wang, 2009). However, it is well known that the presence of both positive and negative exponential terms in the transfer matrix may entail numerical instability when the thickness of the soil layers is large, which restrains the application of the method considerably. Hence, it is still necessary to develop efficient and numerically stable method to address the consolidation of the TISS.

To circumvent the difficulty associated with the TM method, the reflection-transmission matrix (RTM) method (Kennett, 1983), which was originally developed to deal with the wave propagation in the layered elastic medium, is proposed for the consolidation of the layered TISS in this study. To this end, a system of partial differential equations is established for the TISS first. Applying the Hankel-Laplace transform to the system of the partial differential equations furnishes a system of ordinary differential equations. Based on the general solution for the system of the ordinary differential equations, the RTM method of the layered TISS is established. One numerical example is used to show the capacity of the proposed RTM method.

2. Governing equations and the correspond-ing general solutions

In this section, the governing equations for the TISS are outlined first and then, by using the Hankel-Laplace transform method, the general solutions for the governing equations of the TISS are obtained.

2.1 THE GOVERNING EQUATIONS FOR THE TISS

The equilibrium equations of the TISS in the absence of the body force in the cylindrical coordinate system are as follows:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0,$$
$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{z\theta}}{\partial z}+\frac{2\sigma_{r\theta}}{r}=0,$$
$$\frac{\partial\sigma_{zr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{z\theta}}{\partial\theta}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{zr}}{r}=0, \tag{1}$$

in which σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} are the total normal stress components in the r , θ and z directions, respectively; σ_{zr} , $\sigma_{z\theta}$ and $\sigma_{r\theta}$ are the shear stress components. The principle of effective stress can be expressed as follows:

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}'+\mathbf{p},\ \boldsymbol{\sigma}=\{\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{zr},\sigma_{z\theta},\sigma_{r\theta}\}^T,$$
$$\boldsymbol{\sigma}'=\{\sigma'_{rr},\sigma'_{\theta\theta},\sigma'_{zz},\sigma'_{zr},\sigma'_{z\theta},\sigma'_{r\theta}\}^T,$$
$$\mathbf{p}=\{p,p,p,0,0,0\}^T, \tag{2}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}'$ and $\mathbf{p}$ are the vectors of the total stresses, effective stresses and pore pressure, respectively. It is noted that the compressive normal stresses and pore pressure are considered to be positive in this study. The constitutive relation of the effective stresses of the TISS are as follows (Lu et al., 2016):

$$\sigma'_{rr}=c_{11}\varepsilon_{rr}+c_{12}\varepsilon_{\theta\theta}+c_{13}\varepsilon_{zz},$$
$$\sigma'_{\theta\theta}=c_{12}\varepsilon_{rr}+c_{11}\varepsilon_{\theta\theta}+c_{13}\varepsilon_{zz},$$
$$\sigma'_{zz}=c_{13}\varepsilon_{rr}+c_{13}\varepsilon_{\theta\theta}+c_{33}\varepsilon_{zz},$$
$$\sigma'_{zr}=2c_{44}\varepsilon_{zr},\ \sigma'_{z\theta}=2c_{44}\varepsilon_{z\theta},\ \sigma'_{r\theta}=2c_{66}\varepsilon_{r\theta}, \tag{3}$$

in which ε_{rr}, $\varepsilon_{\theta\theta}$, ε_{zz}, ε_{zr} , $\varepsilon_{z\theta}$ and $\varepsilon_{r\theta}$ are the strain components and are determined by the following expressions:

$$\varepsilon_{rr}=-\frac{\partial u_r}{\partial r},\ \varepsilon_{\theta\theta}=-\frac{u_r}{r}-\frac{\partial u_\theta}{r\partial\theta},\ \varepsilon_{zz}=-\frac{\partial u_z}{\partial z},$$
$$\varepsilon_{zr}=-\frac{1}{2}\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right),\ \varepsilon_{z\theta}=-\frac{1}{2}\left(\frac{\partial u_z}{r\partial\theta}+\frac{\partial u_\theta}{\partial z}\right),$$
$$\varepsilon_{\theta r}=-\frac{1}{2}\left(\frac{\partial u_r}{r\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\right), \tag{4}$$

where u_r , u_θ and u_z are the displacement components in the r, θ and z directions, respectively. The elasticity constants c_{11}, c_{12} , c_{13} , c_{33} and c_{44} , c_{66} in equation (3) have the following representations:

$$c_{11}=\frac{E_h\left(1-\eta\nu_{hv}^2\right)}{\left(1+\nu_h\right)\left(1-\nu_h-2\eta\nu_{hv}^2\right)},$$
$$c_{12}=\frac{E_h\left(\nu_h+\eta\nu_{hv}^2\right)}{\left(1+\nu_h\right)\left(1-\nu_h-2\eta\nu_{hv}^2\right)},$$
$$c_{13}=\frac{E_h\nu_{hv}}{1-\nu_h-2\eta\nu_{hv}^2},\ c_{33}=\frac{E_v\left(1-\nu_h\right)}{1-\nu_h-2\eta\nu_{hv}^2},$$
$$c_{44}=\mu_v,\ c_{66}=\frac{c_{11}-c_{12}}{2}=\mu_h,\ \eta=\frac{E_h}{E_v}=\frac{\nu_{vh}}{\nu_{hv}}, \tag{5}$$

in which E_h , E_v , μ_h and μ_v are the horizontal and vertical Young's moduli as well as the shear moduli, respectively; ν_h and ν_{hv} are the Poisson's ratios characterizing the horizontal strain in the plane of transverse isotropy due to the perpendicular horizontal strain and the vertical strain; ν_{vh} is the Poisson's ratio describing the vertical strain due to the horizontal strain.

According to the Darcy's law, the rate of the fluid flow in the r, θ and z directions have the following expressions:

$$q_r=-k'_h\frac{\partial p}{\partial r},\ q_\theta=-k'_h\frac{\partial p}{r\partial\theta},\ q_z=-k'_v\frac{\partial p}{\partial z} \tag{6}$$

in which, $k'_h=k_h/\gamma_w$ and $k'_v=k_v/\gamma_w$; k_h and k_v denotes the permeabilities of the soil in the horizontal and verti cal directions, respectively; γ_w is the specific weight of the pore water. Using equation (6), th e seepage continuity condition for the TISS can be written as:

$$\frac{\partial e}{\partial t}+k'_h\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta}\right)p+k'_v\frac{\partial^2 p}{\partial z^2}=0,$$
$$e=-\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}\right), \tag{7}$$

where e denotes the dilatation of the soil skeleton and t is the time variable.

2 .2 The general solutions for the governing equations of the TISS

Using equations $(3)_4$ and $(4)_4$, the following equation is obtained:

$$\frac{\partial u_r}{\partial z}=-\frac{\partial u_z}{\partial r}-\frac{\sigma_{zr}}{c_{44}}. \tag{8}$$

Similarly, using equations $(3)_5$ and $(4)_5$, one has the following equation:

$$\frac{\partial u_\theta}{\partial z}=-\frac{\partial u_z}{r\partial\theta}-\frac{\sigma_{z\theta}}{c_{44}}. \tag{9}$$

By using equations $(3)_3$, $(4)_1$, $(4)_2$ and $(4)_3$, the following equation is derived:

$$\frac{\partial u_z}{\partial z}=-\frac{c_{13}}{c_{33}}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_\theta}{r\partial\theta}\right)+\frac{\left(p-\sigma_{zz}\right)}{c_{33}}. \tag{10}$$

Substituting equation $(6)_3$ into equation $(7)_1$ and using equation (10) in the resulting equation, one has

$$\frac{\partial q_z}{\partial z}=\left(\frac{c_{13}}{c_{33}}-1\right)\frac{\partial}{\partial t}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_\theta}{r\partial\theta}\right)-$$
$$\frac{1}{c_{33}}\frac{\partial}{\partial t}\left(p-\sigma_{zz}\right)+k'_h\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)p. \tag{11}$$

Substituting equations (4) into equation (3), the stresses components σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ can be represented by the displacement components. Inserting the resulting expressions for σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ into equation $(1)_1$, the following equation is obtained:

$$\frac{\partial\sigma_{zr}}{\partial z}=\left(c_{11}-\frac{c_{13}^2}{c_{33}}\right)\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right)+\frac{c_{66}}{r^2}\frac{\partial^2 u_r}{\partial\theta^2}$$
$$-\frac{c_{13}}{c_{33}}\frac{\partial\sigma_{zz}}{\partial r}+\left(\frac{c_{11}+c_{12}}{2}-\frac{c_{13}^2}{c_{33}}\right)\frac{1}{r}\frac{\partial^2 u_\theta}{\partial r\partial\theta}$$
$$+\left(\frac{c_{12}-3c_{11}}{2}+\frac{c_{13}^2}{c_{33}}\right)\frac{1}{r^2}\frac{\partial u_\theta}{\partial\theta}+\left(\frac{c_{13}}{c_{33}}-1\right)\frac{\partial p}{\partial r}. \tag{12}$$

Likewise, using equations (3), (4) and $(1)_2$, one has the following equation for $\sigma_{z\theta}$:

$$\frac{\partial\sigma_{z\theta}}{\partial z}=-\left(\frac{c_{13}^2}{c_{33}}-c_{66}-c_{12}\right)\frac{\partial^2 u_r}{r\partial r\partial\theta}-\left(\frac{c_{13}^2}{c_{33}}-c_{11}\right)$$

$$\times\frac{1}{r^2}\frac{\partial u_\theta{}^2}{\partial^2\theta}-\frac{c_{13}}{c_{33}}\frac{\partial\sigma_{zz}}{r\partial\theta}+c_{66}\left(\frac{\partial^2 u_\theta}{\partial r^2}+\frac{\partial u_\theta}{r\partial r}-\frac{u_\theta}{r^2}\right)$$

$$-\left(1-\frac{c_{13}}{c_{33}}\right)\frac{1}{r}\frac{\partial p}{\partial\theta}-\left(\frac{c_{13}{}^2}{c_{33}}-c_{11}-c_{66}\right)\frac{\partial u_r}{r^2\partial\theta} \tag{13}$$

With equation $(1)_3$, one has the following equation for σ_{zz}:

$$\frac{\partial\sigma_{zz}}{\partial z}=-\frac{\partial\sigma_{zr}}{\partial r}-\frac{\sigma_{zr}}{r}-\frac{\partial\sigma_{z\theta}}{r\partial\theta}. \tag{14}$$

Finally, by equation $(6)_3$, the following equation for p is obtained:

$$\frac{\partial p}{\partial z}=-\frac{1}{k_v'}q_z. \tag{15}$$

For the layered TISSS, if the external forces are symmetrical with respect to the x-axis, then, the above variables have the following expansions with respect to the coordinate θ:

$$\{u_r, u_z, \sigma_{zz}, q_z, p\}=\sum_{m=0}^{\infty}\{u_{rm}, u_{zm}, \sigma_{zzm}, q_{zm}, p_m\}(r,z,t)\cos m\theta,$$

$$\{u_\theta, \sigma_{z\theta}\}=\sum_{m=0}^{\infty}\{u_{\theta m}, \sigma_{z\theta m}\}(r,z,t)\sin m\theta. \tag{16}$$

For convenience of subsequent derivations, we define the following m-th order variables:

$$u_{pm}=u_{rm}+u_{\theta m},\quad u_{dm}=u_{rm}-u_{\theta m},$$

$$\sigma_{pm}=\sigma_{zrm}+\sigma_{z\theta m},\quad \sigma_{dm}=\sigma_{zrm}-\sigma_{z\theta m}. \tag{17}$$

For the TISS undergoing consolidation, the following m-th order eight-dimension state vector is defined:

$$\boldsymbol{\psi}_m(r,z,t)=\left\{\mathbf{u}_m^T(r,z,t), \mathbf{f}_m^T(r,z,t)\right\}^T,$$

$$\mathbf{u}_m(r,z,t)=\left\{u_{pm}, u_{dm}, u_{zm}, q_{zm}\right\}^T,$$

$$\mathbf{f}_m(r,z,t)=\left\{\sigma_{pm}, \sigma_{dm}, \sigma_{zm}, p_m\right\}^T. \tag{18}$$

If the layered TISS is subjected to a point horizontal force along the x-axis, only the terms corresponding to $m=1$ in equation (16) are present. Since all the variables contain only the terms for $m=1$, the subscript 1 of the following variables is suppressed.

To derive the general solutions for the TISS, the following n-th order Laplace-Hankel transform are introduced (Sneddon, 1972)

$$\tilde{\phi}^{(n)}(\xi,z,s)=\int_0^\infty\int_0^\infty e^{-st}rJ_n(r\xi)\phi(r,z,t)\mathrm{d}r\mathrm{d}t,$$

$$\phi(r,z,t)=\frac{1}{2\pi\mathrm{i}}\int_0^\infty\int_{\alpha-i\infty}^{\alpha+i\infty}\xi e^{st}J_n(r\xi)\tilde{\phi}^{(n)}(\xi,z,s)\mathrm{d}\xi\mathrm{d}s, \tag{19}$$

where the ~ denotes the Hankel-Laplace transform and the superscript n denotes the order of the transform; $J_n(*)$ is the n-th order Bessel function; ξ and s are the transform parameters corresponding to the Hankel and Laplace transforms, respectively.

Using equations (8)-(15), and the expressions for the variables obtained by retaining only the $m=1$ terms in equation (16), and applying the Laplace-Hankel transform method, the following ordinary differential equation system of the state vector is obtained as follows:

$$\frac{\mathrm{d}\tilde{\boldsymbol{\psi}}(\xi,z,s)}{\mathrm{d}z}=\tilde{\mathbf{A}}(\xi,s)\tilde{\boldsymbol{\psi}}(\xi,z,s),$$

$$\tilde{\boldsymbol{\psi}}(\xi,z,s)=\left\{\tilde{u}_p^{(2)}\ \tilde{u}_d^{(0)}\ \tilde{u}_z^{(1)}\ \tilde{q}_z^{(1)}\ \tilde{\sigma}_p^{(2)}\ \tilde{\sigma}_d^{(0)}\ \tilde{\sigma}_{zz}^{(1)}\ \tilde{p}^{(1)}\right\}^T, \tag{20}$$

where $\tilde{\mathbf{A}}(\xi,s)$ is the coefficient matrix in the transformed domain. To obtain the general solution to equation (20), the eigen values and eigen vectors of the matrix $\tilde{\mathbf{A}}(\xi,s)$ should be determined in advance. The eigen equation for the transformed coefficient matrix $\tilde{\mathbf{A}}(\xi,s)$ is as follows:

$$\left[\tilde{\mathbf{A}}(\xi,s)-\lambda\mathbf{I}_{8\times8}\right]\mathbf{v}=\mathbf{0}, \tag{21}$$

where λ and $\mathbf{v}$ are the eigen value and eigen vector of the matrix $\tilde{\mathbf{A}}(\xi,s)$; $\mathbf{I}_{8\times8}$ is the eight by eight identity matrix. Calculating the determinant of the coefficient matrix of equation (21) yields the characteristic equation for the eigen value of the matrix $\tilde{\mathbf{A}}(\xi,s)$:

$$\left(\lambda^6+b_2\lambda^4+b_1\lambda^2+b_0\right)\left(\lambda^2-\lambda_h{}^2\right)=0, \tag{22}$$

in which the coefficients b_0, b_1, b_2 and λ_h can be obtained using the expression for the matrix $\tilde{\mathbf{A}}(\xi,s)$.

For the solution to equation (22), three cases may occur (Lu et al., 2016). In summary, for all the cases, the general solution to equation (20) can be written uniformly in the following form:

$$\tilde{\boldsymbol{\psi}}(\xi,z,s)=\sum_{i=1}^{4}E_i^*\mathbf{V}_ie^{\gamma_i z}+\sum_{i=5}^{8}E_i^*\mathbf{V}_ie^{-\gamma_{i-4}z}, \tag{23}$$

in which γ_i ($i=1\sim4$) is the real part of the i-th eigen value; E_i^* ($i=1\sim8$) is the arbitrary constants; $\mathbf{V}_i$ ($i=1\sim8$) is the eight-dimensional vector which can be represented by the eigen-vectors and eigen-values of the matrix $\tilde{\mathbf{A}}(\xi,s)$. Note that unlike the case for the wave propagation problem, $\mathbf{V}_i$ generally depends on the vertical coordinate, and its concrete representation depends on the roots of equation (22) (Lu et al., 2016). It is noted that to obtain physically reasonable RTMs, the coordinate z in this study should be the vertical coordinate in the global coordinate system. Using the general solution given by equation (23), and following the procedure to establish the RTM model for the layered saturated soil undergoing consolidation, the RTM method for the layered TISS subjected to a horizontal point force can be established.

3 Numerical results and corresponding analyses

In the preceding sections, only the solution in the transformed domain is obtained by the RTM method for the layered TISS. To retrieve the solution in the physical domain, the inverse Hankel and Laplace transform has to be performed. The inversion of the Hankel transform is accomplished by numerical integration, while the inversion of the Laplace transform is conducted by the method proposed by Schapery (1962). For convenience of presenting numerical results, a reference length L_a, a reference shear modulus μ_a and a reference coefficient of permeability k_a are introduced. The non-dimensional geometrical and physical quantities can thus be defined as follows:

$$z^*=\frac{z}{L_a},\quad r^*=\frac{r}{L_a},\quad t^*=\frac{k_at}{L_a},\quad u_i^*=\frac{u_i}{L_a},$$

$$\sigma_{ij}^{*}=\frac{\sigma_{ij}}{\mu_a}\ ,\quad p^{*}=\frac{p}{\mu_a}\ ,\quad q^{*}=\frac{q}{k_a}\ ,\quad F_h^{*}=\frac{F_h}{\mu_a L_a^2} \tag{24}$$

where the quantity with a asterisk is the dimensionless quantity.

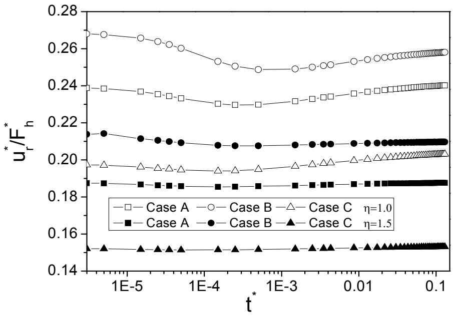

Figure 1 Variation of the non-dimensional displacement u_r^* of the layered saturated soil at the receiver for different values of η .

In this section, we use one example to show the capacity of the proposed RTM method. In the example, the layered TISS is composed of three layers: two overlying layers and the underlying half space. The thickness of the two overlaying layers are $h_1 = 6.0$ m and $h_2 = 6.0$ m. The horizontal point force F_h is located at $z_s = 9$ m, and the receiver point is located at $r_R = 6$ m, $\theta_R = 0$, and $z_R = 7$ m. Other soil parameters are given in Table1 1. In calculation, the thickness, the shear modulus and the coefficient of the vertical permeability of the first layer are chosen as L_a , μ_a and k_a . The influence of the ratio $\eta = E_h / E_v$ on the radial displacement u_r^* is shown in Figure 1. Figure 1 shows that with the increasing time, the horizontal displacement of the soil becomes stable. For the same value of η , the horizontal displacement of the soil for the softer middle layer case is the largest among the three, while that of the harder middle layer case is the smallest. Also, the horizontal displacement of the soil decreases with increase in η . Further, the range of the variation of the horizontal displacement with time becomes narrow with increasing η .

Table 1 Parameters of the three-layered TISS half-space soil

layer	h / m	E_v / Pa			k_v, k_h / m·s^{-1}	γ_w / N·m^{-3}	μ_v / Pa	ν_h	ν_{hv}
		Case A	Case B	Case C					
1	6.0	2.0×10^7	2.0×10^7	2.0×10^7	3.0×10^{-7}	9.8×10^3	3.0×10^7	0.4	0.4
2	6.0	2.0×10^7	1.0×10^7	4.0×10^7	3.0×10^{-7}	9.8×10^3	3.0×10^7	0.4	0.4
3	-	2.0×10^7	2.0×10^7	2.0×10^7	3.0×10^{-7}	9.8×10^3	3.0×10^7	0.4	0.4

4 Conclusions

By the researches conducted in this study, the RTM method for the layered half-space TISS has been developed. By the researches conducted in this study, the following conclusions can be drawn.

- The RTM method can be extended to deal with the consolidation of the layered TISS subjected to a horizontal point force.
- With the proposed RTM method, the problem of the mismatched positive and negative exponential terms is solved and the consolidation of the layered saturated soil with thick layers can be dealt with effectively.
- The proposed method can be used to investigate the consolidation of the layered TISS directly. Besides, it can also be used to address the structure-soil interaction problem associated with the layered TISS.

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